# Modal Parameter Identification Method for Fuel Tube with Spacer Grids

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### 1. Introduction

Nuclear fuel rods are exposed to axial flow in a reactor, and flow-induced-vibration due to the flow usually causes fretting wear in the fuel rods. Thus a prior knowledge about dynamic properties of a fuel rod in air condition or flow condition is very valuable to estimate or understand fuel rod dynamic behavior in the reactor. Modal testing is a powerful method to measure structure dynamic properties, but the dynamic properties are acquired after some processing of the measured signal. Generally, supposing a structure as a linear second order dynamic system, the measured signals such as frequency responses can be represented as a ratio of two polynomials. A modal parameter identification using Legendre polynomial will be introduced in the paper. Since the Legendre polynomials satisfy the orthogonality condition, the method with the polynomials results to more reliable curve-fitting than ordinary polynomial method. A fuel tube with several grid assemblies is used as an objective structure and the modal parameters are identified with the suggested method.

# 2. Identification Methods and Results

#### 2.1 Previous Methods

When each mode is separated well, circle-fitting method in frequency domain is very useful to identify modal parameters [1]. If there exist closely spaced neighboring modes, assuming the measured frequency responses as summation of rational polynomials, usually least-square error minimization is used. In this case, identified results are very sensitive to the noise in the measured signals. Thus, to minimize noise effects, Forsythe polynomial is used [2,]. Also combining Prony method [3] and circle-fitting method, Jung[4] tried to improve the curve-fitting results.

# 2.2 Legendre Polynomial Curve-fitting Method

A measured frequency response function can be expressed in Laplace domain as,

$$H_{i,e}(\omega) = \sum_{k=1}^{n} \left[ \frac{r_k}{s - p_k} + \frac{r_k^*}{s - p_k^*} \right]_{s = j\omega}$$
(1)

where  $H_{i,e}$  is a measured frequency response at 'i' location when we excite at 'e' location.  $p_k$  means system pole and consists of damping and natural frequency.  $r_k$  is residue of the system and '\*' means

complex conjugate. The system with n degree of freedom, usually, can be written as a ratio of two polynomials,

$$H_{i,e} = \frac{\sum_{k=0}^{m} a_k s^k}{\sum_{k=0}^{2n} b_k s^k}$$
(2)

where  $a_k$  and  $b_k$  are real polynomial coefficients. Thus, an error function  $\varepsilon$  between the analytic frequency response and the measured one  $\widetilde{H}$  at a frequency is defined as

$$\varepsilon = \sum_{k=0}^{m} a_k s^k - \widetilde{H}_{i,e} \sum_{k=0}^{2n-1} b_k s^k - \widetilde{H}_{i,e} s^{2n}$$
(3)

Through minimization procedure of the error function, one can uniquely determine the polynomial coefficients  $a_k$  and  $b_k$ .

The identified results, as mentioned before, are wholly dependent on the used polynomials. Since Legendre polynomials are satisfying the orthogonality condition, recursive relations, and symmetry condition, least-squares fitting with the polynomials could provide reliable results. Legendre polynomials can be expressed in several ways, but this study used the following formula.

$$P_n(s) = \sum_{m=0}^{M} (-1)^m \frac{(2n-2m)!}{2^n m! (n-m)! (n-2m)!} s^{n-2m}$$
(4)

Where *M* is n/2 or (n-1)/2 when n is even number or odd number, respectively.

After determining all coefficients, based on the relation between Eq.(1) and Eq.(2), damped natural frequency, damping ratio, and modes can be found [1]. Consequently, we can estimate modal parameters with the above procedure.

### 3. Fuel Tube Modal Analysis Results

#### 2.1 Fuel Tube Structure and Test Configuration

Test assembly, fuel tube structure, is composed of 3 m fuel tube with 6 5x5 spacer grids. All the grids are clamped to the rigid stand [5]. The exciter in the figure excites the tube at the 4-th span from the bottom. Impedance head which is attached in the tip of the steel stinger measures exciting force and acceleration of the point. An accelerometer measures acceleration of the mid point in each span.

### 2.2 Test Results

Since the fuel tube motion cannot be rigidly restricted by dimples and spring in each grid, we can only obtain noise contaminated signal when random excitation, which is very normal exciting method, is applied. Figure 1 is the measured frequency response function at the exciting point, and is resulted from random excitation. Figure 2 is the coherence function of the measured frequency response function, but the coherence level is roughly low especially for lower frequency band. Low coherence means that the system is not linear. It seemed that the non-linearity comes from fuel tube support condition at spacer grid. Since the grid spring and dimple just contacts the tube, friction and impact can be happened when a gap develops.

Avoiding random excitation, expecting high signal to noise, sinusoidal excitation (sine sweep test) was preferred. Figure 3 shows the measured frequency response function when sinusoidal excitation is applied. Clearer signal can be seen in Figure 3 rather than in Figure 1.

Finally, with the above curve-fitting method, natural frequencies and their dampings can be estimated as Table 1 for the first three modes.



Figure 1. Point frequency response function from random excitation



Figure 2. Coherence of the point frequency response function



Figure 3. Point frequency response function from sinusoidal excitation

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Mode number	1	2	3
Natural freq.(Hz)	136.1	163.8	174.2
Damping(%)	0.28	0.11	0.49

# 4. Conclusion

parameter identification Modal method with Legendre polynomial is suggested. Since the polynomials are satisfying orthogonality, symmetricity, and recursive relations, identification method with the polynomials provides reliable results rather than a method with ordinary polynomials. We measured vibration signal of fuel tube with spacer grids, and some modal parameters can be identified. It is notified that the proposed method cannot cover wide frequency band, but examples show that the suggested method is applicable in a limited frequency band.

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