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Development of a Perturbation Theory Module for Triangular-z Geometry

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1. Introduction

Perturbation theory is used to calculate efficiently the reactivity changes caused by the perturbation in composition or neutron cross sections and by which reactivity coefficients such as fuel Doppler coefficients and sodium void coefficients are produced in fast reactor analysis [1, 2].

The perturbation theory code PERT-K [3] for hexagonal geometry based on nodal expansion method had been developed. But only the first order perturbation of the PERT-K code was validated and the accuracy in the outer core region was not sufficient.

We have developed a perturbation theory module for Triangular-Z (TRI-Z) geometry to enhance the accuracy of reactivity calculation. To validate the TRI-Z geometry perturbation theory module we compared the results of TRI-Z model with those of direct calculation by the DIF3D code [4], and compared the accuracy of TRI-Z model with that of Hexagonal-Z (HEX-Z) model.

2. Methods and Results

2.1 Methods of Perturbation

The methods of perturbation theory can be started from the neutron diffusion equations [5]. The adjoint diffusion equation of the unperturbed state can be used in the form as:

$$-\nabla D_g \nabla \psi_g + (\Sigma_{a,g} + \sum_{g' \neq g} \Sigma_{s,g \to g'}) \psi_g = \sum_{g' \neq g} \Sigma_{s,g \to g'} \psi_{g'} + \frac{\nu \Sigma_{f,g}}{k_{eff}} \sum_{g'} \chi_{g'} \psi_{g'} \quad (1)$$

where ψ_g is the adjoint flux. The diffusion equation of the perturbed state is written in the following form:

$$-\nabla D_g^* \nabla \phi_g^* + (\Sigma_{a,g}^* + \sum_{g' \neq g} \Sigma_{s,g \to g'}^*) \phi_g^* = \sum_{g' \neq g} \Sigma_{s,g' \to g}^* \phi_{g'}^* + \frac{\chi_g}{k_{eff}^*} \sum_{g'} \nu \Sigma_{f,g'}^* \phi_{g'}^* \quad (2)$$

where ϕ_g^* is the perturbed flux and perturbed cross sections are defined by:

$$D_g^* = D_g + \delta D_g$$

$$\Sigma_{a,g}^* = \Sigma_{a,g} + \delta \Sigma_{a,g}$$

$$\Sigma_{s,g \to g'}^* = \Sigma_{s,g \to g'} + \delta \Sigma_{s,g \to g'}$$

$$\nu \Sigma_{f,g}^* = \nu \Sigma_{f,g} + \delta \nu \Sigma_{f,g}$$

Using equations (1) and (2) the reactivity change can be written as:

$$\frac{1}{k_{eff}} - \frac{1}{k_{eff}^*} = \frac{1}{\sum_g \nu \Sigma_{f,g} \phi_g^* \cdot \sum_{g'} \chi_{g'} \psi_{g'}} \times \left[-\sum_g \delta D_g \nabla \psi_g \bullet \nabla \phi_g^* - \sum_g \delta \Sigma_{a,g} \phi_g^* \psi_g - \sum_g \psi_g \sum_{g' \neq g} \delta \Sigma_{s,g \to g'} \phi_g^* + \sum_g \psi_g \sum_{g' \neq g} \delta \Sigma_{s,g' \to g} \phi_{g'}^* + \frac{1}{k_{eff}^*} \sum_g \chi_g \psi_g \sum_{g'} \delta \nu \Sigma_{f,g'} \phi_{g'}^* \right] \quad (3)$$

The expression of equation (3) is referred to as exact perturbation theory and the reactivity change can be calculated by integrating right hand side of equation (3).

The first order perturbation can be expressed by replacing ϕ_g^* by ϕ_g of equation (3) and replacing k_{eff}^* by k_{eff} in the bracket of equation (3).

2.2 Validation of TRI-Z perturbation theory module

To validate the perturbation theory module for TRI-Z geometry we calculated the reactivity changes due to fuel cross sections and sodium density changes of KALIMER-600 core and compared the results with those of DIF3D direct calculation. We also compared the accuracy of TRI-Z model with HEX-Z model. Each calculation was performed with 9 neutron energy groups and 54 triangles per hexagon are used in TRI-Z model. Four cases of comparison were performed:

1) Fuel Doppler reactivity
2) Whole core sodium void reactivity
3) Local 100% sodium void reactivity
4) Inner core local partial void reactivity

Fuel Doppler reactivity in table 1 shows the reactivity changes due to the microscopic cross section changes of fuel by raising the fuel temperature 1350K and 1800K from 900K respectively. We can discern the accuracy of each model and method by comparing the reactivity difference between each perturbation calculation and DIF3D direct calculation. The accuracy of first order perturbation is decreasing as the amount of perturbation goes large, whereas exact perturbation keeps the accuracy. The accuracy of TRI-Z exact perturbation is

less than 1pcm compared to the DIF3D direct calculation.

Whole core sodium void reactivity in table 2 shows the reactivity changes in cases of 20, 50, 80 and 100% sodium coolant void respectively. The accuracy of TRI-Z exact perturbation is less than 33pcm and the relative error is less than 1%.

Local sodium void reactivity in table 3 shows the reactivity changes in cases of 100% sodium coolant voiding of inner, middle and outer core region respectively. The accuracies of TRI-Z exact perturbation in the inner and middle core region are 1pcm and 4pcm, and accuracy in the outer core is 12pcm. But the accuracy of HEX-Z exact perturbation in the outer core region is -121pcm. This shows that TRI-Z model is superior to HEX-Z model.

Table 4 shows that the accuracies of HEX-Z and TRI-Z exact perturbation in the inner core local partial void are 5pcm and 1pcm respectively.

Table 1. Fuel Doppler reactivity

Fuel temp. (K)	HEX-Z Ref. $\Delta\rho^{\dagger}$ (pcm)	HEX-Z $\Delta\rho$ diff., pcm (Relative error, %)		TRI-Z Ref. $\Delta\rho^{\ddagger}$ (pcm)	TRI-Z $\Delta\rho$ diff., pcm (Relative error, %)	
		1st order	exact		1st order	exact
1350	-244	-4 (2)	4 (-2)	-245	-5 (2)	-0.3 (0.1)
1800	-416	-13 (3)	2 (-0.4)	-417	-15 (4)	-0.4 (0.1)

† DIF3D direct calculation HEX-Z 900K, k=1.00704
‡ DIF3D direct calculation TRI-Z 900K, k=1.00632

Table 2. Whole core sodium void reactivity

Void (%)	HEX-Z Ref. $\Delta\rho^{\dagger}$ (pcm)	HEX-Z $\Delta\rho$ diff., pcm (Relative error, %)		TRI-Z Ref. $\Delta\rho^{\ddagger}$ (pcm)	TRI-Z $\Delta\rho$ diff., pcm (Relative error, %)	
		1st order	exact		1st order	exact
100	2138	-371 (-17)	-159 (-7)	2130	-221 (-10)	33 (2)
80	1722	-270 (-16)	-132 (-8)	1716	-151 (-9)	14 (0.8)
50	1083	-131 (-12)	-78 (-7)	1079	-57 (-5)	8 (0.78)
20	434	-38 (-9)	-30 (-7)	433	-9 (-2)	3 (0.75)

† DIF3D direct calculation HEX-Z 0% void, k=1.00207
‡ DIF3D direct calculation TRI-Z 0% void, k=1.00137

Table 3. Local 100% sodium void reactivity

Fuel region	HEX-Z Ref. $\Delta\rho^{\dagger}$ (pcm)	HEX-Z $\Delta\rho$ diff., pcm (Relative error, %)		TRI-Z Ref. $\Delta\rho^{\ddagger}$ (pcm)	TRI-Z $\Delta\rho$ diff., pcm (Relative error, %)	
		1st order	exact		1st order	exact
Inner	1200	-139 (-12)	-7 (-0.6)	1214	-134 (-11)	0.8 (0.1)
Middle	889	-105 (-12)	-32 (-4)	882	-77 (-9)	4 (0.1)
Outer	63	-159 (-253)	-121 (-192)	47	-42 (-88)	12 (26)

† DIF3D direct calculation HEX-Z 0% void, k=1.00207
‡ DIF3D direct calculation TRI-Z 0% void, k=1.00137

Table 4. Inner core local partial void reactivity

Void (%)	HEX-Z Ref. $\Delta\rho^{\dagger}$ (pcm)	HEX-Z $\Delta\rho$ diff., pcm (Relative error, %)		TRI-Z Ref. $\Delta\rho^{\ddagger}$ (pcm)	TRI-Z $\Delta\rho$ diff., pcm (Relative error, %)	
		1st order	exact		1st order	exact
80	959	-91 (-9)	-5 (-0.6)	970	-86 (-11)	0.6 (0.1)
50	595	-37 (-6)	-3 (-0.5)	602	-34 (9)	0.4 (0.1)
20	235	-7 (-3)	-1 (-0.5)	238	-5 (-88)	0.1 (0.1)

† DIF3D direct calculation HEX-Z 0% void, k=1.00207
‡ DIF3D direct calculation TRI-Z 0% void, k=1.00137

3. Conclusion

We developed a perturbation theory module for TRI-Z geometry to enhance the accuracy of reactivity calculation. We validated the TRI-Z geometry perturbation theory module by comparing the results of TRI-Z model with those of DIF3D direct calculation, and compared the accuracy of TRI-Z model with that of HEX-Z model.

The results show that TRI-Z exact perturbation keeps the accuracy when the amount of perturbation is large, whereas first order perturbation can not. TRI-Z model is more accurate than HEX-Z model especially in the outer fuel region. This implies that the developed TRI-Z module is more versatile than the HEX-Z module in PERT-K code to apply various perturbation problems.

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