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Dynamic Reliability Graph with General Gates

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1. Introduction

There are several methods to analyze system reliability, such as fault tree analysis, reliability graph, Markov chain, Bayesian network. Although fault tree analysis is the most widely used method, it is not an intuitive method. So, as a system becomes complex, a corresponding fault tree becomes much more complex.

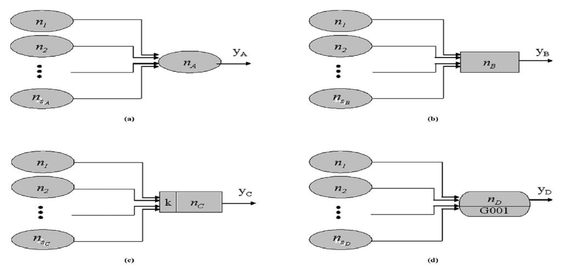
To overcome this shortcoming, reliability graph with general gates (RGGG) method was proposed [1]. By adding general gates to conventional reliability graph, it can possess expression power same as fault tree and be an intuitive method.

However, conventional fault tree and RGGG cannot capture the dynamic behavior of the system associated with time dependent events. Dynamic fault tree method was proposed [2], but it also has a shortcoming related to intuitiveness.

This paper describes how we can add dynamic properties to RGGG method.

2. Reliability graph with general gates

Reliability graph is the intuitive method, so it can model a system by one-to-one match graph. But the reason why it's not used widely is low expression power. It can express property of only OR gate. To overcome this limited expression power, RGGG which utilizes general gates was proposed. And by determining the probability table for each node, RGGG can be transformed to an equivalent Bayesian network and calculate the system reliability.



	$y_1 = 1$ (success)		$y_1 = 0$ (failure)	
	$y_2 = 1$ (success)	$y_2 = 0$ (failure)	$y_2 = 1$ (success)	$y_2 = 0$ (failure)
$y_A = 1$ (success)	$r_{1A} + r_{2A} - r_{1A}r_{2A}$	r_{1A}	r_{2A}	0
$y_A = 0$ (failure)	$1 - (r_{1A} + r_{2A} - r_{1A}r_{2A})$	$1 - r_{1A}$	$1 - r_{2A}$	1

Figure 1. Definition of gates for reliability graph with general gates and probability table for a node with OR gate when n =2. (a) OR gate (b) AND gate (c) k-out-of-n gate (d) general purpose gate. [1]

3. Dynamic reliability graph with general gates

Dynamic fault tree was proposed by adding dynamic gates to the conventional fault tree. Dynamic RGGG also needs additional nodes which have dynamic properties. In this section the additional dynamic nodes and the probability tables for each node are described.

3.1 Dynamic gates

Additional dynamic nodes for dynamic RGGG are proposed according to dynamic gates of dynamic fault tree. Those are priority-AND (PAND) gate, spare (WSP, CSP, HSP) gate, functional-dependency (FDEP) gate, sequence-enforcing (SEQ) gate. Each dynamic node is shown in figure 2. Alphabet *W* in figure 3(b) means the warm spare gate.

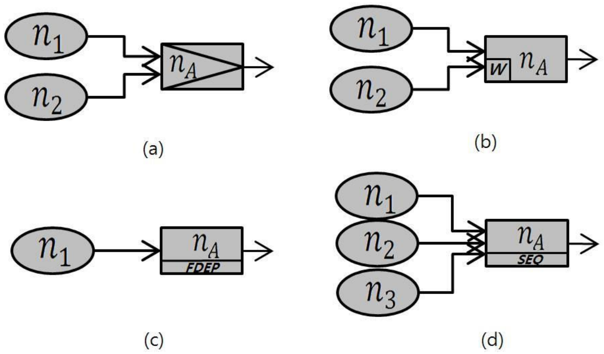


Figure 2. Dynamic nodes. (a) PAND gate (b) WSP gate (c) FDEP gate (d) SEQ gate.

3.2 Probability tables for dynamic nodes

In order to transform a dynamic reliability graph to an equivalent Bayesian network, the probability table corresponding to each dynamic gate should be derived. When determining the probability table, the discrete-time method [3, 4] is used. We divide the process time line into n same intervals.

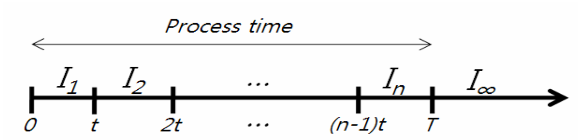


Figure 3. Discretization of process time.

Then output of each node is one of $\{I_1, I_2, \ldots, I_n, I_\infty\}$. I_k means that the node is failed in *k*th time interval and I_∞ means that the node is never failed. The probability that an arc a_{ij} from node n_i to n_j is failed in *k*th time interval is denoted as P_{ij}^k. When the cumulative failure distribution function of a_{ij} is $F_{ij}(t)$,

$$P_{ij}^{k} = \int_{(k-1)t}^{kt} \frac{dF(t)}{dt} dt \tag{1}$$

Using equation (1), the probability tables for dynamic nodes can be derived. Table 1 shows the probability tables when $n = 2$ for PAND node and WSP node which are shown in figure 2(a), 2(b). In table 1(b), $P_{ij}^{k\alpha}$ means the probability that an arc a_{ij} is failed in *k*th time interval and at that time the arc a_{ij} is in spare state. α is dormancy factor which is the ratio of spare state failure rate to working state failure rate (i.e. the dormancy factor of CSP is 0 and dormancy factor of HSP is 1).

The probability tables for the other nodes can be determined similarly and are not presented in this paper. If we derive the probability tables for all the static and dynamic nodes, it is possible to calculate the dynamic system reliability using software tools for Bayesian networks such as Hugin™ and MSBNx™.

(a)

	$n_1=I_1$			$n_1=I_2$			$n_1=I_\infty$		
	$n_2=I_1$	$n_2=I_2$	$n_2=I_\infty$	$n_2=I_1$	$n_2=I_2$	$n_2=I_\infty$	$n_2=I_1$	$n_2=I_2$	$n_2=I_\infty$
$n_A=I_1$	0	0	0	0	0	0	0	0	0
$n_A=I_2$	0	1	P_{2A}^2	0	P_{1A}^1	$P_{1A}^1P_{2A}^2$	0	P_{1A}^1	$P_{1A}^1P_{2A}^2$
$n_A=I_\infty$	1	0	$1-P_{2A}^2$	1	$1-P_{1A}^1$	$1-P_{1A}^1P_{2A}^2$	1	$1-P_{1A}^1$	$1-P_{1A}^1P_{2A}^2$

(b)

	$n_1=I_1$			$n_1=I_2$			$n_1=I_\infty$		
	$n_2=I_1$	$n_2=I_2$	$n_2=I_\infty$	$n_2=I_1$	$n_2=I_2$	$n_2=I_\infty$	$n_2=I_1$	$n_2=I_2$	$n_2=I_\infty$
$n_A=I_1$	1	$P_{2A}^{1\alpha}$	$P_{2A}^{1\alpha}$	P_{1A}^1	$P_{1A}^1P_{2A}^{1\alpha}$	$P_{1A}^1P_{2A}^{1\alpha}$	P_{1A}^1	$P_{1A}^1P_{2A}^{1\alpha}$	$P_{1A}^1P_{2A}^{1\alpha}$
$n_A=I_2$	0	$1-P_{2A}^{1\alpha}$	P_{2A}^2	$1-P_{1A}^1$	$1-P_{1A}^1P_{2A}^{1\alpha}$	$P_{1A}^1P_{2A}^2+(1-P_{1A}^1)P_{2A}^{2\alpha}$	P_{1A}^2	$P_{1A}^1(1-P_{2A}^{1\alpha})+P_{1A}^2$	$P_{1A}^1P_{2A}^2+P_{1A}^2P_{2A}^{2\alpha}$
$n_A=I_\infty$	0	0	$1-P_{2A}^{1\alpha}-P_{2A}^2$	0	0	$1-P_{1A}^1P_{2A}^{1\alpha}-P_{1A}^1P_{2A}^2-(1-P_{1A}^1)P_{2A}^{2\alpha}$	P_{1A}^3	P_{1A}^3	$1-P_{1A}^1P_{2A}^{1\alpha}-P_{1A}^1P_{2A}^2-P_{1A}^2P_{2A}^{2\alpha}$

Table 1. Probability table for a node with (a) PAND gate and (b) WSP gate when $n = 2$.

4. Conclusion

If we use this method, the shortcoming of original RGGG that it can model only static system can be overcome. As n increases, the obtained reliability becomes more exact. If we set n as infinity, we can derive the ideally accurate reliability. But as n increases, the execution time for calculating system reliability increases much more. However the system failure rate is usually very small, therefore n doesn't have to be large in order to derive nearly exact reliability. According to another paper about discrete-time method, it was shown that small values of n (from 1 to 5) are sufficient to get the order of magnitude of the system's reliability [4].

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