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A Rapidly Convergent Generalized Rebalance Method (GRM) for Discrete Ordinates Transport Equations

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1. Introduction

Recently, there have been some efforts to devise the effective acceleration methods[1,2,3,4] by using the diffusion equation derived without consistency with the transport equation and study theoretically the relation[5] between these methods and the conventional coarse mesh rebalance (CMR) equation. However, to our knowledge, there have been no effective methods giving a satisfied fast convergence for full range of mesh size. In this paper, a generalized rebalance method (GRM) having two parameters (at present, only fine-mesh case is considered) for solving the discrete ordinates transport problems is given. The parameters are related with the coupling coefficient and an interpolation. It is shown that for the step characteristic (SC) and the diamond difference (DD) schemes, this method with a choice of the optimal parameters gives a very fast convergence for full range of mesh size.

2. Theory and Methodology

In this method, to derive the acceleration equations, the following relationship between the interface angular flux and the cell average scalar fluxes are assumed:

$$\Psi^{l+1/2}_{i+1/2,m} = (\nu + \alpha^{+}_{i+1/2,m})\Psi^{l+1/2}_{i,m} + \alpha^{+}_{i+1/2,m}\Psi^{l+1/2}_{i+1,m}, \mu_m > 0$$
$$\Psi^{l+1/2}_{i+1/2,m} = \alpha^{-}_{i+1/2,m}\Psi^{l+1/2}_{i,m} + (\nu + \alpha^{-}_{i+1/2,m})\Psi^{l+1/2}_{i+1,m}, \mu_m < 0 \quad (1)$$

where ν is a parameter related with the diffusion coefficient. The coefficient α is updated by

$$\alpha^{+}_{i+1/2,m} = \frac{\Psi^{l+1/2}_{i+1/2,m} - \nu\Psi^{l+1/2}_{i,m}}{\Psi^{l+1/2}_{i,m} + \Psi^{l+1/2}_{i+1,m}}, \mu_m > 0 \quad (2)$$

Integrating of the first equation of Eq.(1) over μ after multiplying μ_m for $\mu_m > 0$, changing the index *l+1/2* in the partial current and in the scalar flux of the numerators by *l+1*, and using the interpolation of the scalar fluxes in the denominators give

$$J^{l+1,+}_{i+1/2} = \left[\frac{\sum_{\mu_m>0} w_m\mu_m(\nu+\alpha^{+}_{i+1/2,m})\Psi^{l+1/2}_{i,m}}{\tau\phi^{l}_{i} + (1-\tau)\phi^{l+1/2}_{i}}\right]\phi^{l+1}_{i} + \left[\frac{\sum_{\mu_m>0} w_m\mu_m\alpha^{+}_{i+1/2,m}\Psi^{l+1/2}_{i+1,m}}{\tau\phi^{l}_{i+1} + (1-\tau)\phi^{l+1/2}_{i+1}}\right]\phi^{l+1}_{i+1}, \quad (3)$$

where τ is an interpolation parameter. This partial current and its counterpart for opposite direction can be used to obtain the neutron balance equation and the resulting neutron balance equation has just a tri-diagonal form. These acceleration equations can be linearized to be analyzed by using the Fourier analysis method. The linearized form of the partial current given in Eq.(3) is given by

$$\frac{\delta J^{-,l+1}_{i+1/2}}{\delta\varepsilon}|_{\varepsilon=0} = \frac{1+\nu}{2}\frac{qt}{\sigma_a}[\varsigma^{J+1}_{i+1} - (\tau\varsigma^{J}_{i+1} + (1-\tau)\varsigma^{J+1/2}_{i+1})] + \frac{1-\nu}{2}\frac{qt}{\sigma_a}[\varsigma^{J+1}_{i} - (\tau\varsigma^{J}_{i} + (1-\tau)\varsigma^{J+1/2}_{i})] \quad (4)$$
$$+ \frac{q}{\sigma_a}\sum_{\mu_m<0} w_m|\mu_m|\varsigma^{J+1/2}_{i+1/2,m},$$

where $t = \sum_{\mu_m>0} w_m\mu_m$.

Some simple algebra shows that the linearized partial currents of FMR, CMFD, and p-CMFD can be obtained by choosing the parameter ν as follows;

$$\nu_{FMR} = 1.0, \quad (5a)$$

$$\nu_{f-CMFD} = \frac{1}{3\sigma ht} = \frac{D}{ht}, \quad (5b)$$

$$\nu_{p-CMFD} = 1 + \frac{D}{th} = 1 + \frac{1}{3\sigma ht}, \quad (5c)$$

where *D* is the diffusion coefficient (=1/(3σ)) and *h* is the mesh size. The relation of Eq.(5b) was derived in Ref. 5 independently. The quantity of this parameter (i.e., ν) times *t* has the same physical meaning as the *D/h*. The result of the spectral radius through the Fourier analysis is given by

$$\varpi = \frac{2\nu t[\tau + (1-\tau)g_0] - 2\nu t\tau\cos(\lambda h) - 2\nu t(1-\tau)g_0\cos(\lambda h) - 2g_1 + 2g_2}{2\nu t + \sigma_a h - 2\nu t\cos(\lambda h)},$$

$$x_{1m} = \frac{\sigma_{s0}[\sigma\cos^2(\lambda h/2) + (\frac{2\mu_m}{h} + \sigma\gamma_{i,m})\sin^2(\lambda h/2)]}{\sigma^2\cos^2(\lambda h/2) + (\frac{2\mu_m}{h} + \sigma\gamma_{i,m})^2\sin^2(\lambda h/2)}, \quad (6)$$

$$x_{2m} = \frac{\sigma_{s0}[\sigma\cos^2(\lambda h/2) - (\frac{2\mu_m}{h} + \sigma\gamma_{i,m})\sin^2(\lambda h/2)]}{\sigma^2\cos^2(\lambda h/2) + (\frac{2\mu_m}{h} + \sigma\gamma_{i,m})^2\sin^2(\lambda h/2)}$$

3. Numerical Analysis and Results

The results of the Fourier analysis through linearization are given in this section. For this analysis, a homogeneous infinite medium having the scattering ratio of 0.999 is considered. All analyzes are performed with S_{16} angular quadrature set. For the Diamond Difference (DD) scheme, the following five acceleration methods are inter-compared ; (1) FMR with τ=0.0, (2) FMR with τ=1.0, (3) CMFD with τ=0.0, (4) p-CMFD with τ=0.0, and (5) GRM with $\tau=\tau_{opt}$ and $\nu=\nu_{opt}$. The optimum values of the parameters τ and ν are obtained numerically with Eq.(6). Fig. 1 shows the spectral radii of these methods versus mesh size (mfp). As shown in this figure, all methods except for GRM with the optimal parameters have divergence for large mesh size region (σh>2.0) but GRM with the optimal parameters

has very fast convergence for all mesh sizes (the spectral radius is less than 0.2.). FMR has very narrow region of stable convergence near σh~1.0 and it with τ=1.0 has more wide region of stable convergence than FMR with τ=0.0. CMFD has good convergence for small mesh size region but its spectral radius increases rapidly as the mesh size approaches unity. Of all methods considered here except for GRM with the optimal parameters, p-CMFD has most wide region of stable convergence.

For the step characteristic (SC) scheme, the followings six methods are compared; (1) FMR with τ=0.0, (2) FMR with τ=0.0 for σh≤1.0 and τ=1.0 for σh>1.0 (FMR-1 in Fig. 2(a)), (3) CMFD with τ=0.0, (4) CMFD with τ=0.0 for σh≤1.0 and FMR with τ=1.0 for σh>1.0 (FMR/CMFD in Fig. 2(a)), (5) p-CMFD with τ=0.0, and (6) GRM with with $\tau=\tau_{opt}$ and $\nu=\nu_{opt}$. As shown in Fig. 2, FMR with τ=0.0 has very fast convergence for the optical thickness of a mesh larger than unity and its spectral radius decreases rapidly as the mesh size increases. However, for the mesh size (in mfp) less than unity, its spectral radius increases drastically and this method does not converge. The second method is a variation of FMR. In this method, the second parameter τ is set to unity for mesh size (in mfp) larger than unity. Fig. 2 shows that this effect of the second parameter is very significant in enhancing its convergence. The spectral radius drastically decreases as the mesh size increases. CMFD with τ=0.0 gives very fast convergence up to σh~1.5 but its spectral radius increases rapidly and becomes unstable for larger mesh size region. The fourth method that is a combination of the second and third methods has very fast convergence and its spectral radius is less than 0.225 for all mesh sizes. The p-CMFD method with τ=0.0 has very good convergence for all mesh sizes, its maximum value of spectral radius is 0.297. Of these all methods, GRM with the optimal parameters shows best convergence that is stable for all mesh sizes.

4. Conclusions

In this paper, a rapidly convergent generalized rebalance method for the discrete ordinates transport equations is presented. This method has two parameters that are related with the coupling coefficient and the interpolation. The convergence of this rebalance method with the Fourier analysis was analyzed for DD and SC. The results show that it is possible to obtain a very efficient acceleration method having very fast convergence for full range of mesh size by using the optimal parameters. However, the applicability of this method for practical problems has not been analyzed and it will be valuable if the convergence of this method for the problems divided into coarse meshes comprised of several fine meshes is analyzed in the future.

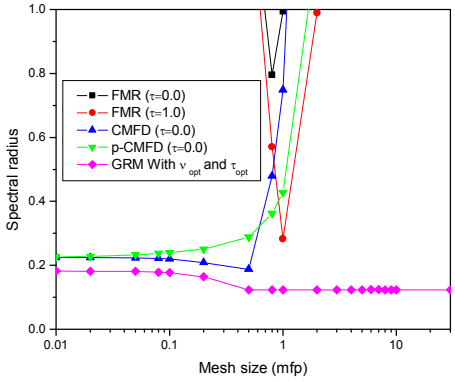

Fig. 1 Comparison of the spectral radii for DD

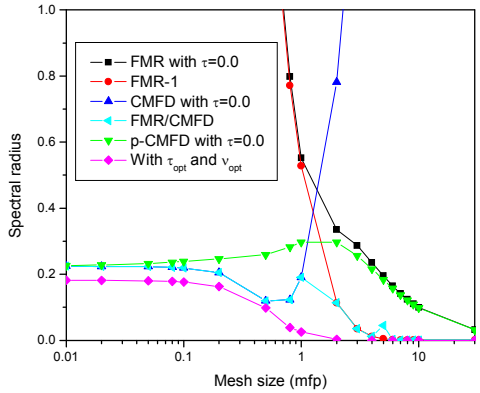

Fig. 2 Comparison of the spectral radii for SC

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REFERENCES

[1] N. Z. Cho, G. S. Lee, and C. J. Park, "On a New Acceleration Method for 3D Whole Core Transport Calculations," *Annual Meeting of AESJ*, March 2003, Japan.

[2] N. Z. Cho and G. S. Lee, "Comparison of CMFD and p-CMFD Acceleration Methods for Neutron Transport Calculations," *Proc. of Korean Nuclear Society*, Gyeongju, Korea, May 2003.

[3] N. Z. Cho and C. J. Park, "Comparison of Coarse Mesh Rebalance and Coarse Mesh Finite Difference Acceleration Methods for the Neutron Transport Calculations," *Proc. of Int. Conf. Nuclear Mathematical and Computational Sciences (M&C 2003)*.

[4] J. Y. Cho, et. al., "Cell-Based CMFD Formulation for Acceleration of Whole-Core Method of Characteristics Calculations," *Journal of Korean Nuclear Society*, **34**, 250 (2002).

[5] H. L. Hanshaw and E. W. Larsen, "Coarse-Mesh Rebalance and Diffusion Acceleration for S_N Problems," *Trans. Am. Nucl. Soc.*, **91**, 151, 2004.