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An Improved Monte Carlo Method for Solving Heat Conduction Problems with Complex Geometry

Jae Hoon Song and Nam Zin Cho
Korea Advanced Institute of Science and Technology
373-1 Kusong-dong, Yusong-gu
Daejeon, Korea, 305-701
nzcho@kaist.ac.kr

1. Introduction

There are many deterministic techniques to solve heat transfer problems. However, they are difficult to deal with problems having complex geometry. Because Monte Carlo method deals well with complicated geometries, it could be used to deal with such heat transfer problems.

Heat conduction is a diffusion process with the governing differential equation under no absorption, no fission and one speed condition [1, 2]. That is, the steady state differential equation of heat conduction for a stationary, isotropic solid is given by [1]

$$\nabla \cdot K(\vec{r})\nabla T(\vec{r}) + q'''(\vec{r}) = 0, \qquad (1)$$

where $K(\vec{r})$ =thermal conductivity, $q'''(\vec{r})$ =internal heat source. On the other hand, the steady state, one-speed neutron diffusion equation under isotropic scattering, no absorption, and no fission condition is given by [2]

$$\nabla \cdot \frac{1}{3\Sigma_s}\nabla \phi(\vec{r}) + S(\vec{r}) = 0, \qquad (2)$$

where ϕ =neutron flux, Σ_s =scattering cross section, S=internal neutron source.

While neutron diffusion is an approximation of neutron transport phenomena, inversely it is applicable to solve diffusion problems by a transport method, with $\Sigma_s = \frac{1}{3K(\vec{r})}$ and $S = q'''$.

Based on this idea, a Monte Carlo method of solving heat conduction problems was developed [3] which employs the MCNP code since MCNP is widely used as a Monte Carlo particle transport code [4]. However, it is not convenient to apply linear extrapolation near the boundary. To circumvent this inconvenience, an improved treatment using extended boundary for the boundary correction is introduced in this paper.

2. Limitation of Previous Method

The procedure of the method using linear extrapolation near the boundary follows [3]. First, find proper linear extrapolation point based on the smallest error. Second, translate MCNP result to the boundary condition value.

However, this method has a complicated procedure. The linear extrapolation point having the smallest error depends on the problem size so that it may be inconvenient to apply this method to real problems.

3. Improved Method for Heat Conduction Problems

The concept of the improved method using MCNP code is to extend boundary (of thickness τ) like the extrapolation distance. The reason why this extended boundary is used to solve heat conduction is because the transport solution is not met with diffusion solution near the boundary. Therefore, using the extended boundary, it gives effect that transport solution raised up near the boundary and this method is easy to handle. The detail procedure follows.

First, β, which is scaling factor [3], is chosen. This value determines how diffusivity of the problem that we solve. When β is increased, the problem becomes diffusive. Using larger β, differences between analytic solution and transport result obtained by using MCNP code become small, but the computing time is increased so that it is very important to find suitable β.

Because of this reason, α, problem size in terms of mean free path, is introduced as [3]:

$$\alpha = \frac{\beta}{3K} L \qquad (3)$$

This parameter is dimensionless. β is called scaling factor [3], K is thermal conductivity and L is the size of the problem. Based on this parameter, β is chosen and heat conduction problems can be solved by the MCNP code. The dependency of α is shown in Fig. 1.

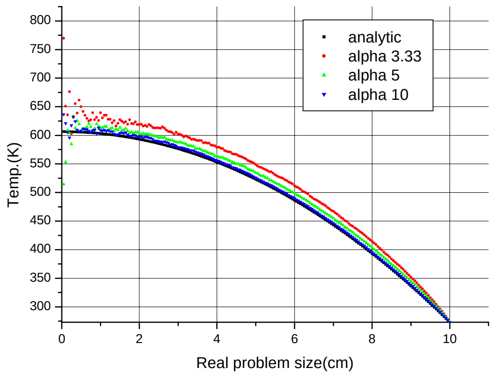

Fig. 1. MCNP results with q''' =10, K=0.5, depending on α, spherical one-dimensional problem, extended boundary distance used with 3, 2, and 1mfp respectively.

Second, it is required to determine the extended boundary. The relationship between the extended boundary and relative error between analytic solution and MCNP result are shown in Fig. 2.

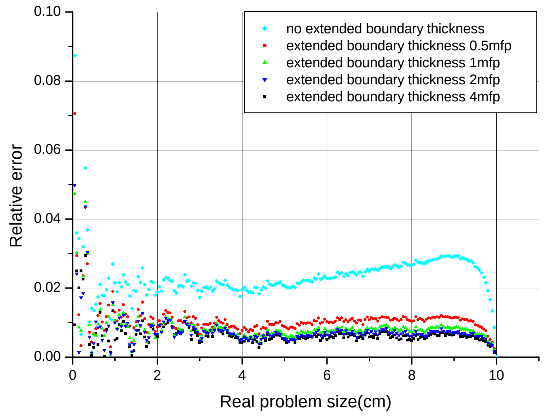

Fig. 2. The relationship between extended boundary distances and relative error in the problems with α =10, q''' =10, $K = 0.5$, spherical one-dimensional problem.

Table 1: Extended boundary distances and computing time for Fig. 2.

Extended boundary distance (mfp)	Computing time (sec)
0	61
0.5	69
1	78
2	94
4	137

The errors corresponding to each extended boundary were remarkably decreased up to 1mfp. However, the errors decrease little beyond 1mfp, but the computing time increases significantly beyond 1mfp. Therefore, in the view of effectiveness (relative error, time), τ = 1mfp extended boundary thickness was chosen.

Finally, it is necessary to translate MCNP result to the boundary condition value.

4. Comparison of Numerical Results in Problem Having Complex Geometry

The test problem is described in Table 1. The α values are 7 and 10. The heat source was distributed uniformly as q''' =10(w/cm^3). The boundary condition was used as $T(r=10) = 273K$. The geometry is spherical one-dimensional as in Fig. 3.

Table 2: Test Problem Description

Medium	R(cm)	K(thermal conductivity)
1	0<r<2	0.2
2	2<r<4	0.3
3	4<r<6	0.4
4	6<r<10	0.5

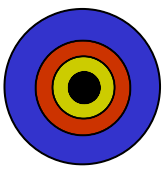

Fig. 3. Test Problem Description

The differences between analytic solution and MCNP result are shown in Fig. 4.

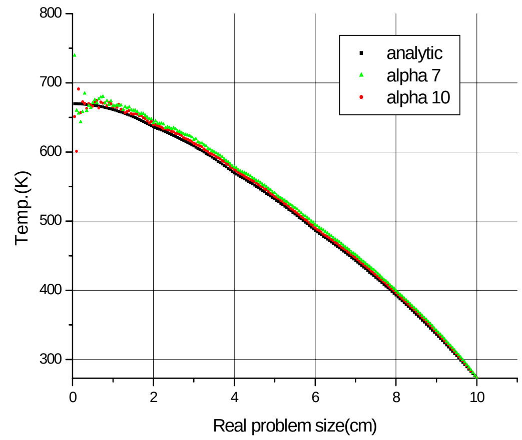

Fig. 4. Comparison of improved methods and analytic solution for spherical one-dimensional problem.

The black line is analytic solution. The green dots stand for α =7. In the α =7 case, the computing time is 303sec in the parallel computation with 4 CPUs (3.2GHz). Also the red dots stand for α =10. In this case, the computing time is 326sec under the same condition. In heterogeneous problems, K to be used for the purpose of assessing α in Eq. (3) is provided by the volume averaged heat conductivity, which is given by

$$\bar{K} = \frac{\sum V_i K_i}{\sum V_i}, \qquad (4)$$

due to media with different thermal conductivities. It is confirmed that when α is large, the accuracy was increased. On the other hand, it required longer computing time in order to obtain accurate result.

5. Conclusions

The previous method to solve heat conduction problem uses a complicated procedure of linear extrapolation near the boundary. The improved method, which uses extended boundary, was introduced to solve heat conduction problems in complicated geometry. Using the improved method, it is confirmed that the results of MCNP and analytic solutions are in close agreement.

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