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Non-Linear Equation of Motion of an LPM Type CRDM and Phase Plane Solutions

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1. Introduction

An LPM (Linear Pulse Motor) type drive mechanism is the one of the candidates for CRDM (Control Rod Drive Mechanism) of an integral reactor SMART (System-integrated Modular Advanced ReacTor)[1]. The LPM type CRDM has four sets of electric magnets in it, which are operated in sequences to move the reactivity control elements upward or downward. The absence of any mechanical parts reduces the mechanical losses and makes the responses faster. However, since the total weight of the moving part is held by electric magnet forces only, careful consideration regarding the stability of the driving system is required in design stages to prevent undesirable disturbances in driving procedures and by external dynamic loads. A non-linear equation of motion of the LPM type CRDM is derived and the solutions on phase plane are studied.

2. Non-Linear Equation of Motion

The LPM type CRDM consists of concentrically assembled parts and the gaps are filled with water as shown in Fig. 1. The holding and driving force from the electric magnets can be represented with a sinusoidal function. And the water filling the gaps provides viscous damping forces. The non-linear equation of motion of the system is as follows;

$$m\frac{d^2y}{dt^2} + c\left(\frac{dy}{dt} - \frac{dz}{dt}\right) + F_M \sin\frac{2\pi}{T}(y-z) = -mg \qquad (1)$$

where m, c, and F_M are the total mass of the moving parts, viscous damping coefficient, and maximum driving force of the motor respectively. The displacement of the mover is y and that of the basement is z. And T is a step period, and g is the gravitational acceleration. Introducing a coordinate transform and a sinusoidal base acceleration, results in Eq. (4).

$$u = \frac{2\pi}{T}(y-z), \qquad \ddot{z} = A\sin\Omega t \qquad (2, 3)$$

$$\frac{d^2u}{dt^2} + 2\zeta\omega_n\frac{du}{dt} + \omega_n^2 \sin u = -\omega_n^2\frac{mg}{F_M} - \omega_n^2\frac{mA}{F_M}\sin\Omega t \qquad (4)$$

where ω_n is the linear natural frequency, and ζ is the linear damping ratio;

$$\omega_n^2 = \frac{2\pi}{T}\frac{F_M}{m}, \qquad 2\zeta\omega_n = \frac{c}{m} \qquad (5\text{-a, b})$$

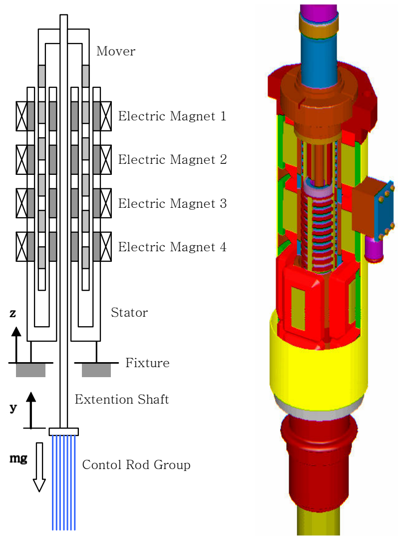

Fig. 1. Schematic diagram and 3D model view of the LPM type CRDM

Introducing a time scaling with Eq. (6) into Eq. (4), results in a non-dimensional non-linear equation of motion, Eq. (7).

$$\omega_n t = \tau \qquad (6)$$

$$\frac{d^2u}{d\tau^2} + 2\zeta\frac{du}{d\tau} + \sin u = -\gamma - \alpha\sin\beta\tau \qquad (7)$$

where γ is the ratio of the weight (gravitational force) to the maximum driving force of the linear motor, and α is the ratio of the maximum excitation force from the base to the maximum driving force of the linear motor, and β is the ratio of the excitation frequency to the linear natural frequency of the system;

$$\gamma = \frac{mg}{F_M}, \quad \alpha = \frac{mA}{F_M}, \quad \beta = \frac{\Omega}{\omega_n} \qquad (8\text{-a, b, c})$$

3. Solutions on Phase Plane

The solution curves of the equation of motion on the phase planes are calculated numerically with the Runge-Kutta method. To study the dynamic characteristics and stability of the system, the external excitation term is set to be zeros. Fig. 2 shows the solution curves on phase planes when $\gamma = 0.2$, $\alpha = 0$, and with some damping ratios. The horizontal and vertical axes are the scaled non-dimensional relative displacement and velocity of the mover of the CRDM;

$$u^* = u/\pi \quad \text{and} \quad v^* = \frac{1}{\pi}\frac{du}{d\tau} \qquad (9\text{-a, b})$$

The solution curves show that the phase portrait is a function of the damping ratio but there are invariant characteristic points on the horizontal axis such as center points, u_c, and saddle points, u_s, and they are periodically repeated as follows;

$$u_c = 2n\pi - u_0\,, \qquad u_s = (2n-1)\pi + u_0 \qquad \text{(10-a,b)}$$

where $u_0 = \min(\sin^{-1}\gamma)$ and $n = 0, \pm 1, \pm 2, \Lambda$.

The phase portrait of the undamped case, Fig. 3-(a), shows that there are small islands of stable region in which the solution curves create closed orbits surrounding a center point. Any unstable solution curve starting with a positive velocity flows down through between the stable regions to the lower part and changes its direction to a negative one. It means that if the mover and with the control rods of the LPM slips off from a stable region by an accident, it continuously slips down into the reactor core with an unbounded speed and the reactivity of the core will be unintentionally eliminated. The downward step movement is impossible when $\gamma > 0.2$ and without a damping. The unit step downward stroke of the control rods corresponds to a positive value of 0.5 in the horizontal axis of the phase plane, and it is out of bound for a stable region when $\gamma > 0.2$ without a damping.

Introducing a damping term, the stable region is enlarged. As one can see in Fig. 3-(b), the damping term breaks the closed orbits and creates spiral stable solution curves converging to center points. And the remaining unstable solutions are bounded by a certain slip velocity depending on the damping ratio. There is no unstable solution, when the damping ratio is greater than 0.0786, as shown in Fig. 3-(c), (d).

4. Conclusions

A non-linear dynamic equation of motion was derived, and solutions on the phase planes were studied. The results show that the ratio of the weight of the moving parts to the maximum driving force of the linear motor and the system damping ratio are important design variables for the stability of the system.

ACKNOWLEDGMENTS

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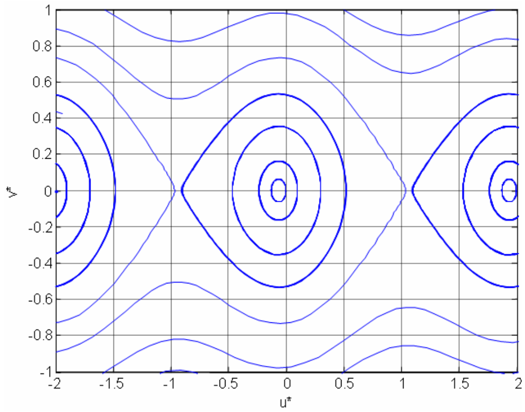
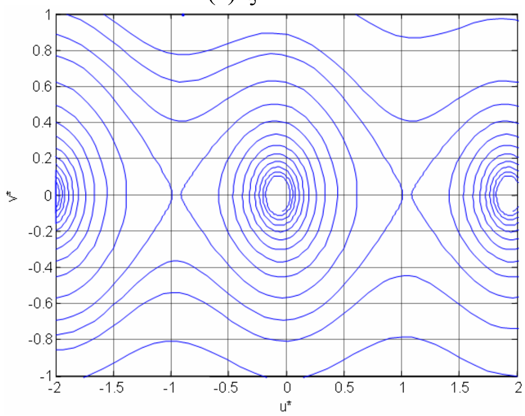
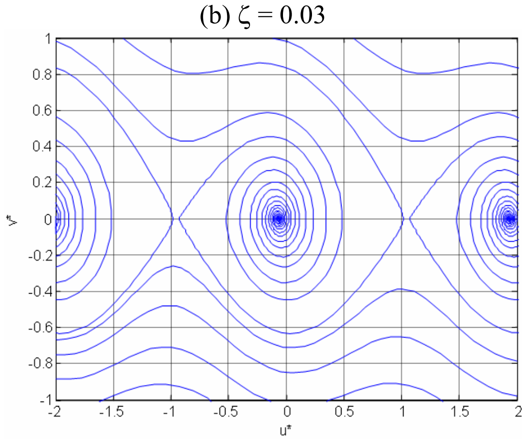


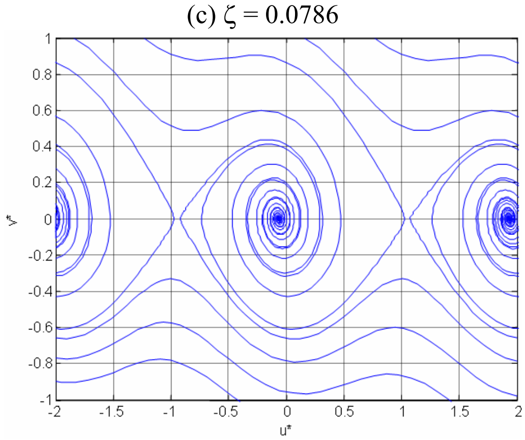
(a) $\zeta = 0.0$



(b) $\zeta = 0.03$



(c) $\zeta = 0.0786$



(b) $\zeta = 0.1$

Fig. 2. Solution curves in phase plane of the systems when $\gamma = 0.2$, $\alpha = 0$.