Transactions of the Korean Nuclear Society Spring Meeting
Chuncheon, Korea, May 25-26 2006

Simulation of Cavity Flow by the Lattice Boltzmann Method using Multiple-Relaxation-Time scheme

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1. Introduction

Recently, the lattice Boltzmann method(LBM) has gained much attention for its ability to simulate fluid flows, and for its potential advantages over conventional CFD method. The key advantages of LBM are, (1) suitability for parallel computations, (2) absence of the need to solve the time-consuming Poisson equation for pressure, and (3) ease with multiphase flows, complex geometries and interfacial dynamics may be treated. The LBM using relaxation technique was introduced by Higuerea and Jimenez[1] to overcome some drawbacks of lattice gas automata(LGA) such as large statistical noise, limited range of physical parameters, non-Galilean invariance, and implementation difficulty in three-dimensional problem. The simplest LBM is the lattice Bhatnager-Gross-Krook(LBGK)[2] equation, which based on a single-relaxation-time(SRT) approximation. Due to its extreme simplicity, the lattice BGK(LBGK) equation has become the most popular lattice Boltzmann model in spite of its well-known deficiencies, for example, in simulating high-Reynolds numbers flow. The Multiple-Relaxation-Time(MRT) LBM was originally developed by D'Humieres[3]. Lallemand and Luo[4] suggests that the use of a Multiple-Relaxation-Time(MRT) models are much more stable than LBGK, because the different relaxation times can be individually tuned to achieve 'optimal' stability.

A lid-driven cavity flow is selected as the test problem because it has geometrically singular points in the flow, but geometrically simple. Results are compared with those using SRT, MRT model in the LBGK method and previous simulation data using Navier-Stokes equations for the same flow conditions.

In summary, LBM using MRT model introduces much less spatial oscillations near geometrical singular points, which is important for the successful simulation of higher Reynolds number flows.

2. Methods and Results

2.1 LBM with SRT model

LBM method solves the microscopic kinetic equation for artificial particle distribution function *f(x,v,t)*, where *x* and v is the particle position and velocity vector, respectively, in phase space (x,v) and time t, where the macroscpic quantities (velocity and density) are obtained through moment integration of *f(x,v,t)*. The most popular LBM is the SRT LBGK model[2], and listed as follows:

$$f_i(\vec{x}+\vec{e}\Delta t, t+\Delta t) - f_i(\vec{x},t) = -\frac{1}{\tau}[f_i(\vec{x},t) - f_i^{eq}(\vec{x},t)]$$

where $f_i(\vec{x},t)$ and $f_i^{eq}(\vec{x},t)$ are the particle distribution function and equilibrium particle distribution function of the *i*th discrete particle velocity v_i, respectively, $\vec{e}$ is a discrete velocity vector. Note that Δt is the advancing time step and τ is the collision relaxation time. The D2Q9 model is used in the current study for simulating the cavity flow. Let $c = \Delta x / \Delta t = \Delta y / \Delta t$ be the lattice streaming speed for isothermal near-incompressible flows, the equilibrium distribution function can be derived as the following form[2]

$$f_i^{eq}(\vec{x},t) = \rho w_i [1 + \frac{3}{c^2}\vec{e_i}\cdot\vec{u} + \frac{9}{2c^4}(\vec{e_i}\cdot\vec{u})^2 - \frac{3}{2c^2}\vec{u}\cdot\vec{u}]$$

where w_i is a weighting factor and $\vec{u}$ is the fluid velocity. In addition, the valus of the weighting factors are w_i =4/9, i=0; w_i =1/9, i=1,2,3,4; w_i =1/36, i=5,6,7,8. The density and velocities can be computed simply by moment integration as

$$\rho = \sum_i f_i$$

$$\rho\vec{u} = \sum_i \vec{e_i} f_i$$

Application of the multi-scale technique(Chapma-Enskog expansion) yields the Navier-Stokes equation with the pressure $p = \rho c_s^2$, where $c_s = c/\sqrt{3}$, and an advection term with Galilean invariance. The viscosity of the simulated fluid is $\nu = (\tau - 1/2\Delta t)c_s^2$. In general, Eq.(1) is solved in two steps:

collion step:

$$f_i^*(\vec{x},t+\Delta t) = f_i(\vec{x},t) - \frac{1}{\tau}[f_i(\vec{x},t) - f_i^{eq}(\vec{x},t)]$$

streaming step:

$$f_i(\vec{x}+\vec{e}\Delta t, t+\Delta t) = f_i^*(\vec{x}, t+\Delta t)$$

which is known as the LBGK method. Note that, in the above, * denotes the post-collision values.

2.2 LBM with MRT model

Recently, Lallemand and Luo[4] have performed detailed theoretical analysis on the dispersion, dissipation and stability characteristics of a generalized lattice Boltzmann equation model proposed by d'Humieres[3]. They have found that the MRT model is equivalent to the SRT model in the long wave-length (low wave number) limit for macroscopic variables of interest in various simple flows through the linearized analyis. Difference between two relaxation models is identified as a high-order effect(short wavelength limit), which can hardly detected in simple flows. It is well known that geometrically and mathematically singular points can adversely affect the flow solution in short wavelength limit. For convection-dominated flows, the local difference near the singularities may also lead to large differences in flow regimes far away. Thus, it is important to understand how the solution using MRT model is different from that using SRT model. In addition, it is potentially useful to compute flows at high-Reynolds numbers using MRT model in LBM.

2.3 Boundary conditions

How to properly implement the wall boundary conditions within LBM framework is still an ongoing research topic[5]. The most popular scheme is the so-called 'bounce-back' scheme, which has been argued that it is only of first-order accuracy as compared with of second-order accuracy for LBM formulation. However, it was recently shown that the error is sufficiently small if the relaxation parameter is chosen to be close enough to 0.5[5].

In the current study, the upper moving plate velocity U=0.1, considering the validity of using LBM in simulating near-incompressible flows. We have assumed equilibrium distribution function at the upper moving plate, which is computed by substituting the uniform velocity into Eq.(2) and the initial density assignment. After streaming, the velocity at the top plate is reinforced to be the uniform plate velocity and then the equilibrium distribution function is reevaluated using the fixed plate velocity and the updated density at the plate.

2.4 Results

We compare various macroscopic variables(*u, p*) of interest. Results from the LBM using MRT and SRT models are compared with those of N-S solvers by Ghia et al[6](Figure 1). It is clearly shown that the difference of velocity distributions between the current study and Ghia et al.[6] is very small. Also the difference between MRT and SRT models is nearly undistinguished up to *Re*=1000. Generally, the overall flow structures (streamlines) predicted by the SRT and MRT models are very similar to those predicted by Ghia et al.[6], except some differences near the corners. As Reynolds number increases, there exists obvious pressure 'jiggles' around the upper two cavity corners using SRT model, especially due to the geometrical singularity at this corner. The situation even worse at Re=2000, but using the MRT model, there is no such pressure 'jiggles'. In addition, the SRT model, as compared with MRT model, is not converged well at high Reynolds Number(Re~5000). Finally, The MRT model is more suitable than SRT model for treating flow around geometrical singularity and high-Reynolds number flows.

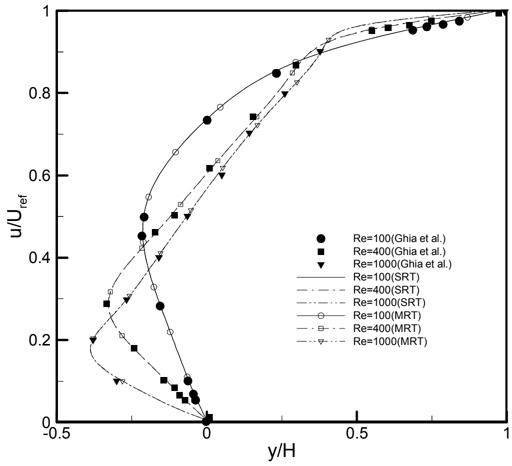

Figure 1. Velocity profile for u at x/L=0.5.

3. Conclusion

The lid-driven cavity flow is simulated by LBGK method using MRT scheme. Results are then compared with those by LBM using SRT, MRT scheme and previous published data using N-S solver[6]. In general, results using MRT and SRT techniques are both in good agreement with those using N-S solver for Re=100~1000. We can conclude that MRT scheme is superior to SRT scheme in simulating high-Reynolds number flows with geometrical singularity due to the different relaxation rates for different physical modes embedded in MRT scheme.

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