

## A Heat Transfer Correlation in a Vertical Upward Flow of CO<sub>2</sub> at Supercritical Pressures

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### 1. Introduction

Heat transfer data has been collected in the heat transfer test loop, named SPHINX (Supercritical Pressure Heat Transfer Investigation for NeXt generation), in KAERI [1]. The facility primarily aims at the generation of heat transfer data in the flow conditions and geometries relevant to SCWR (SuperCritical Water-cooled Reactor). The produced data will aid the thermo-hydraulic design of a reactor core. The loop uses carbon dioxide, and later the results will be scaled to the water flows. The heat transfer data has been collected for a vertical upward flow in a circular tube with varying mass fluxes, heat fluxes, and operating pressures. The results are compared with the existing correlations and a new correlation is proposed by fine-tuning the one of the existing correlations.

### 2. Experimental Setup

The detailed description of the test facility can be found in [1]. Fig. 1 is the dimensions of the test section and the locations of the measuring points. The test section is a circular tube with an inside diameter of 4.4 mm. The thickness of the tube wall is 0.89 mm. The tube is made of Inconel 625. The tube is attached to the loop in a vertical direction, and is uniformly heated by direct current.

CO<sub>2</sub> flows upward in the tube at supercritical pressures. Tests are conducted with varying mass fluxes, heat fluxes, and inlet temperatures at different pressures. The inlet temperature is changed to cover a sufficient range below the pseudo-critical temperature in terms of a reduced temperature. Table 1 shows the range of test conditions.

### 3. Results

Fig. 2 shows how closely the existing correlations predict the experiment result for a normal heat transfer case. The compared correlations are listed in Table 2. The correlations are used in a comparison of test data using R-22 [2]. The R-22 test is conducted in a test section of the same geometry with our experiment. The error between the correlation and the experiment is calculated as Eq. (1) and (2).

$$\overline{R.E.} = \left( \sum_{\text{data points}} R.E. \right) / N_{\text{total}} \quad (1)$$

$$\sigma_{R.E.}^2 = \left[ \sum_{\text{data points}} (R.E. - \overline{R.E.})^2 \right] / N_{\text{total}} \quad (2)$$

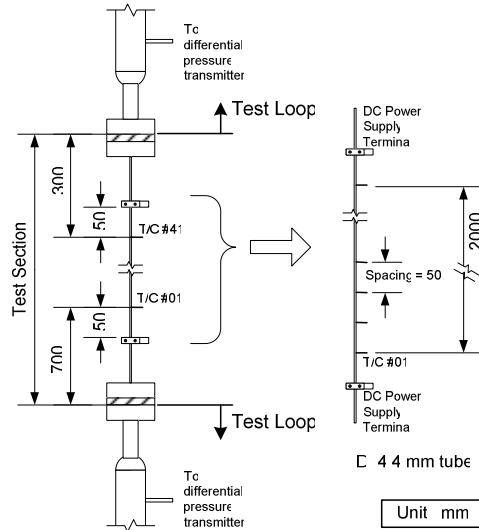


Figure 1. The test section and measuring locations

Table 1. Range of the test conditions in the tube

Condition	Unit	Value
Inlet pressure	MPa	7.75, 8.12, 8.85 (1.05, 1.1, 1.2 P <sub>crit</sub> respectively)
Inlet temperature	°C	5 ~ 30 (T/T <sub>crit</sub> = 0.91 ~ 0.99)
Fluid temperature	°C	5 ~ 80 (T/T <sub>crit</sub> = 0.91 ~ 1.15)
Mass flux	kg/m <sup>2</sup> s	400, 500, 750, 1000, 1200
Heat flux	kW/m <sup>2</sup>	Up to 150

Critical temperature and pressure of CO<sub>2</sub> = 30.98 °C, 7.38 MPa

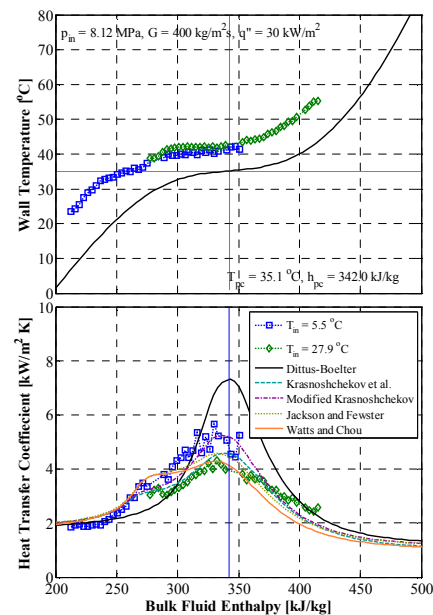


Fig. 2. Comparison of heat transfer coefficients from the experiment and the existing correlations for normal heat transfer cases.

Table 2. Correlations compared with the measured heat transfer coefficient

**Krasnoshchekov and Protopopov[2,5]**

$$Nu = \frac{hD}{k_b} = Nu_0 \left( \frac{\rho_w}{\rho_b} \right)^{0.3} \left( \frac{c_p}{c_{pb}} \right)^n$$

$$\text{where } n = \begin{cases} 0.4 & \text{for } T_b < T_w < T_{pc} \text{ and } 1.2T_{pc} < T_b < T_w \\ 0.4 + 0.2 \left[ \left( \frac{T_w}{T_{pc}} \right) - 1 \right] & \text{for } T_b < T_{pc} < T_w \\ 0.4 + 0.2 \left[ \left( \frac{T_w}{T_{pc}} \right) - 1 \right] \left\{ 1 - 5 \left[ \left( \frac{T_b}{T_{pc}} \right) - 1 \right] \right\} & \text{for } T_{pc} < T_b < 1.2T_{pc} \text{ and } T_b < T_w \end{cases}$$

$$Nu_0 = \frac{\xi/8 Re_b Pr_b}{12.7 \sqrt{\xi/8} (Pr_b^{2/3}) + 1.07}, \quad Re_b = \frac{GD}{\mu_b}, \quad Pr_b = \frac{c_{pb} \mu_b}{k_b},$$

$$\overline{c_{pb}} = \frac{i_w - i_b}{T_w - T_b}, \quad \xi = (1.82 \log Re_b - 1.64)^{-2}$$

**Jackson and Fewster[2,3,5]**

$$Nu = \frac{hD}{k_b} = 0.0183 Re_b^{0.82} \overline{Pr_b}^{-0.5} \left( \frac{\rho_w}{\rho_b} \right)^{0.3} \quad \text{where } \overline{Pr_b} = \frac{c_p \mu_b}{k_b}$$

where  $R.E. = (h_{exp} - h_{cor})/h_{cor} \times 100$  is a relative percent error.

The calculated errors are summarized in Table 3. The standard deviations are about 20 %. The modified Krasnoshchekov and Protopopov correlation shows the least mean error among the correlations in Table 2, while the Watts and Chou correlation was the best one in the test using R-22 [2]. The Jackson and Fewster correlation shows the second least error with its much simpler form. Thus, the Jackson and Fewster correlation is fine-tuned to minimize the mean error. The new correlation is:

$$Nu = 0.0186 Re_b^{0.83} \overline{Pr_b}^{-0.52} \left( \frac{\rho_w}{\rho_b} \right)^{0.29} \quad (3)$$

The proposed correlation shows a reduced mean error and standard deviation (the last row in Table 3). Fig. 3 illustrates the prediction performance of the existing correlations and the proposed correlation. The suggested correlation predicts about 95 % of data points within a  $\pm 30$  % error.

**4. Conclusion**

The heat transfer data were produced for the tube of 4.4 mm ID. The data were compared with the existing correlations on a normal heat transfer in supercritical pressure flows. A new correlation is proposed by fine-tuning the Jackson and Fewster correlation to the experiment results. The suggested correlation shows an improved prediction over the existing correlations and predicts 95 % of the data points within a  $\pm 30$  % error.

**REFERENCES**

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 [2] T. Yamashita, S. Yoshida, H. Mori, M. Ohno, H. Komita, and K. Nishida, "Heat Transfer to a Supercritical Pressure

**Modified Krasnoshchekov and Protopopov[2,5]**

$$Nu = \frac{hD}{k_b} = 0.0183 Re_b^{0.82} Pr_b^{0.5} \left( \frac{\rho_w}{\rho_b} \right)^{0.3} \left( \frac{c_p}{c_{pb}} \right)^n$$

where n is calculated as the Krasnoshchekov and Protopopov correlation

**Watts and Chou for vertical flow[2,4]**

$$Nu_{var p} = \begin{cases} \left[ 1 - 3000 \frac{\overline{Gr_b}}{Re_b^{2.7} Pr_b^{0.5}} \right]^{0.295} & \text{for } \frac{\overline{Gr_b}}{Re_b^{2.7} Pr_b^{0.5}} < 10^{-4} \\ \left[ 7000 \frac{\overline{Gr_b}}{Re_b^{2.7} Pr_b^{0.5}} \right]^{0.295} & \text{for } \frac{\overline{Gr_b}}{Re_b^{2.7} Pr_b^{0.5}} \geq 10^{-4} \end{cases}$$

$$\text{where } Nu_{var p} = 0.021 Re_b^{0.8} \overline{Pr_b}^{-0.55} \left( \frac{\rho_w}{\rho_b} \right)^{0.35}$$

$$\overline{Gr_b} = \frac{\rho_b (\rho_b - \rho_m) g D^3}{\mu_b^2}, \quad \rho_m = \frac{1}{T_w - T_b} \int_{T_b}^{T_w} \rho dT$$

Table 3. Deviations of the experimental data from the predicted ones by the correlations in Table 2.

Correlations	$\overline{R.E.}$	$\sigma_{R.E.}$
Krasnoshchekov & Protopopov	11	22
Modified Krasnoshchekov & Protopopov	6	20
Jackson and Fewster	9	19
Watts and Chou	18	21
The newly suggested correlation	-2	15

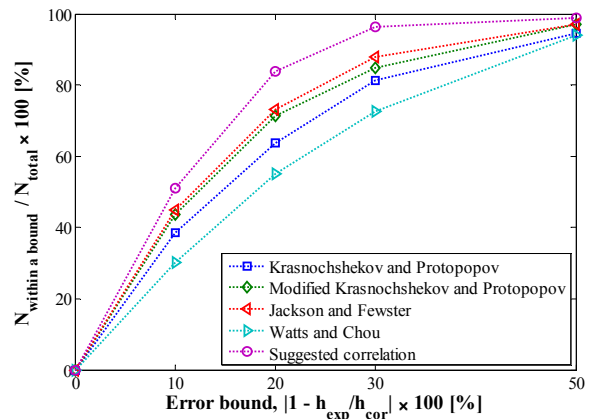


Fig. 3. Fractional numbers of the data points within specified error bounds.

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 [5] I. L. Pioro and R. B. Duffey, Literature Survey of Heat Transfer and Hydraulic Resistance of Water, Carbon Dioxide, Helium and Other Fluids at Supercritical and Near-Critical Pressures, AECL-12137, FFC-FCT-409, pp.103- 111.