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The AFEN Method in Cylindrical (r,θ,z) Geometry for Pebble Bed Reactors – Extension to Treatment of Void Regions –

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1. Introduction

Recently, we extended the analytic function expansion nodal (AFEN) method developed quite extensively in Cartesian (x,y,z) geometry and in hexagonal-z geometry to the treatment of the full three-dimensional cylindrical (r,θ,z) geometry for pebble bed reactors(PBRs).[1] The AFEN methodology in this geometry as in hexagonal geometry is "robust", due to the unique feature of the AFEN method that it does not use the transverse integration.

The transverse integration in the usual nodal methods, however, leads to an impasse[2], that is, failure of the azimuthal term to be transverse-integrated over r-z surface. The recent work reported in Ref. 3 is an attempt in this class of transverse integration nodal methods but it involves several unjustified assumptions and approximations in the formulation.

The typical pebble bed reactors have void regions in the top and side regions of the core. Ref. 4 provides finite diffusion coefficients for void regions (with zero other cross reactions) so that the void regions could be modeled by diffusion theory. This paper presents an optional treatment of the void regions in the core based on AFEN methodology.

2. Basic Theory and Method

The AFEN formulation in the (r,θ,z) coordinates system starts from the following multi-group diffusion equations in a homogenized node (see Fig. 1) :

$$-\nabla^2\vec{\phi}(r,\theta,z)+[\Lambda]\vec{\phi}(r,\theta,z)=0, \qquad (1)$$

where

$$[\Lambda]=[D]^{-1}\left([\Sigma]-\frac{1}{k_{eff}}[\chi][\nu\Sigma_f]\right),$$

for solid (nonvoid) regions.

All the notations are standard. The equations can be decoupled as follows:

$$\frac{\partial^2\vec{\phi}}{\partial r^2}+\frac{1}{r}\frac{\partial\vec{\phi}}{\partial r}+\frac{1}{r^2}\frac{\partial^2\vec{\phi}}{\partial\theta^2}+\frac{\partial^2\vec{\phi}}{\partial z^2}-[\Lambda]\vec{\phi}=0. \qquad (2)$$

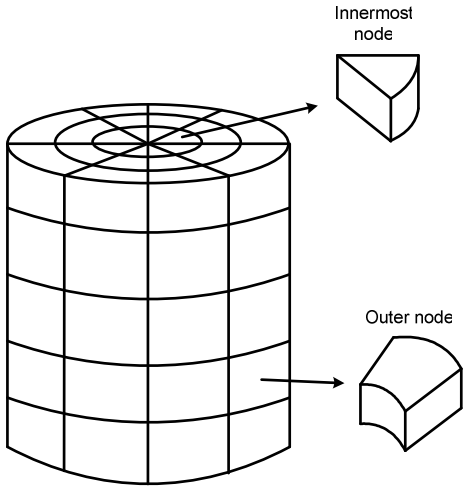

Fig. 1 Node shape in (r,θ,z) coordinates system

A general solution to Eq. (2) can be represented in terms of analytic basis functions that can be obtained using the method of separation of variables. For practical implementation, we choose the solution of a node expressed in a finite number of terms.

$$\begin{aligned}\vec{\phi}(r,\theta,z)=&\overrightarrow{A_0}+\sinh(\sqrt{[\Lambda]}z)\overrightarrow{A_1}+\cosh(\sqrt{[\Lambda]}z)\overrightarrow{A_2}\\&+I_0(\sqrt{[\Lambda]}r)\overrightarrow{A_3}+K_0(\sqrt{[\Lambda]}r)\overrightarrow{A_4}\\&+I_1(\sqrt{[\Lambda]}r)\sin(\theta)\overrightarrow{A_5}+I_1(\sqrt{[\Lambda]}r)\cos(\theta)\overrightarrow{A_6}\\&+K_1(\sqrt{[\Lambda]}r)\sin(\theta)\overrightarrow{A_7}+K_1(\sqrt{[\Lambda]}r)\cos(\theta)\overrightarrow{A_8}\\&+zI_0(\sqrt{[\Lambda]}r)\overrightarrow{A_9}+zK_0(\sqrt{[\Lambda]}r)\overrightarrow{A_{10}}\\&+\sinh(\sqrt{[\Lambda]}z)\ln(r)\overrightarrow{A_{11}}+\cosh(\sqrt{[\Lambda]}z)\ln(r)\overrightarrow{A_{12}},\end{aligned} \qquad (3)$$

where $\overrightarrow{A_i}$, $i=0,1,\cdots,12$, are expansion coefficient vectors.

Note that each term in Eq. (3) is an analytic solution of Eq. (2). The thirteen coefficients in Eq. (3) are made to correspond to the thirteen nodal unknowns for a node : i) one node average flux, and ii) twelve half-interface average fluxes (two half-interface average fluxes for each of the six surfaces).

The AFEN formulation for void nodes starts from the following multi-group "diffusion" equations :

$$-[D]\nabla^2\vec{\phi}(r,\theta,z)=0, \qquad (4)$$

without the second term in Eq. (1). This is the model implied in Ref. [4] for void regions with specified "equivalent" diffusion coefficients and zero cross sections.

The type of equations (4), i.e., Laplace equation, appears quite extensively in electromagnetic theory in physics [5].

A general solution to Eq.(4) can be represented in terms of analytic basis functions that can be obtained using the same method of separation of variables employed in Eq.(1):

$$\begin{aligned}\vec{\phi}(r,\theta,z) = {} & \vec{A_0} + rSin(\theta)\vec{A_1} + rCos(\theta)\vec{A_2} \\ & + \frac{1}{r}Sin(\theta)\vec{A_3} + \frac{1}{r}Cos(\theta)\vec{A_4} \\ & + Sin(\frac{z}{100})I_0(\frac{r}{100})\vec{A_5} + Cos(\frac{z}{100})I_0(\frac{r}{100})\vec{A_6} \\ & + Sin(\frac{z}{100})K_0(\frac{r}{100})\vec{A_7} + Cos(\frac{z}{100})K_0(\frac{r}{100})\vec{A_8} \\ & + z\vec{A_9} + Log(r)\vec{A_{10}} + zLog(r)\vec{A_{11}}. \end{aligned} \quad (5)$$

Note that each term in Eq. (5) is an analytic solution of Eq. (4). The twelve coefficients in Eq. (5) are made to correspond to the twelve nodal unknowns for a node : i) twelve half-interface average fluxes (two half-interface average fluxes for each of the six surfaces). Note that for a void node we use twelve nodal unknowns, since we exclude the node average flux from the nodal unknowns.

3. Numerical Results and Conclusions

This was implemented in the TOPS code. The TOPS code was used to solve the OECD/PBMR-400 benchmark problem, shown in Fig. 2. It is a two-group problem with void regions above the annular core and between outer reflector and core barrel and $j^- = 0$ boundary condition (specified as BLACK boundary condition). Detailed configurations and cross sections are shown in Ref. [4].

To describe void regions in the benchmark problem, OECD/NEA provided directional dependent diffusion coefficients. Table I shows the results on k_{eff} with r and z directional diffusion coefficients on side and top void regions of TOPS and VENTURE, respectively. In addition, we provided results using very small amount of absorption cross sections (1E-9 cm^{-1} for fast group and 1E-8 cm^{-1} for thermal group) on void regions by the TOPS code.

Table I. Results of the OECD/PBMR-400 benchmark problem

	k_{eff}	diff. (pcm)
VENTURE[(a)]	0.99941	reference
TOPS[(b)]	0.99945	4
TOPS (with small Σ_a)[(b)]	0.99945	4

[(a)] r-z (580x2900) calculation
[(b)] r-θ-z (20x4x29) calculation

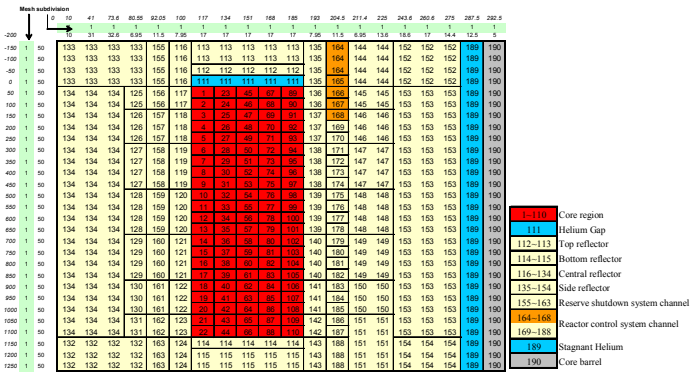

Fig. 2. OECD/PBMR-400 benchmark problem

The results show that the void calculation option based on AFEN methodology works correctly.

Acknowledgment

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