

Wavelet-Theoretic Method for Solution of Neutron Transport Equation

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1. Introduction

In recent decades, wavelet theory has been well developed and widely applied to various fields. Cho and Park^[1] firstly developed the wavelet Galerkin method for solving the neutron diffusion equation in one- and two-dimensional geometry. Then, based on this work, Nasif^[2] obtained the connection coefficients analytically, which lead to stability of the wavelet-based method. Both of them obtained high accuracy for the solution of strongly heterogeneous problems.

In this paper, we describe a wavelet theory method to solve the neutron transport equation. We utilize Daubechies' wavelet^[3] bases to expand the angular variables of second-order neutron transport equation, Self-Adjoint Angular Flux (SAAF) equation, and transform the SAAF equation into a set of partial differential equations in terms of wavelet expansion coefficients which contain spatial variables only. These spatially dependent partial differential equations are solved by use of Finite Element Method (FEM). Finally, numerical results of a test problem are given to demonstrate the validity of the method.

2. Wavelet Method for Neutron Transport Equation

The multi-group SAAF equation derived by Morel and McGhee^[4] can be written as:

$$-\Omega \cdot \nabla (\Sigma_{t,g} - S_d^g)^{-1} \Omega \cdot \nabla \phi_g + (\Sigma_{t,g} - S_d^g) \phi_g = \sum_{\substack{g'=1 \\ g' \neq g}}^G S_o^{g'-g} \phi_{g'} + q_g - \Omega \cdot \nabla (\Sigma_{t,g} - S_d^g)^{-1} \left(\sum_{\substack{g'=1 \\ g' \neq g}}^G S_o^{g'-g} \phi_{g'} + q_g \right) \quad (1)$$

where S_d denotes the within-group block of the multi-group scattering matrix and S_o denotes the between-group block of the multi-group scattering matrix.

We consider a two-dimensional neutron transport problem in a rectangular region and use the following notations for convenience,

$$\mu = \cos \theta \in [-1, 1] \quad \text{and} \quad \eta = \cos \varphi \in [-1, 1] \quad (2)$$

So the transport operator can be written as

$$\Omega \cdot \nabla = \mu \frac{\partial}{\partial x} + \eta \sqrt{1 - \mu^2} \frac{\partial}{\partial y} \quad (3)$$

In our wavelet method, the flux is expanded by use

of Daubechies' scaling functions as follows:

$$\phi_g(x, y, \mu, \eta) = \sum_{j=1}^p \sum_{k=1}^p \psi_{jk,g}(x, y) v_{n,j}(\mu) v_{n,k}(\eta) \quad (4)$$

where $v_{n,j}(\mu)$ is scaling function and $p = 2^{n+1} + 2N - 2$.

Substituting eq.(3) and eq.(4) into SAAF, applying the Galerkin method to the above equation, multiplying an individual wavelet on both sides and integrating in the angular domain, and using the following orthogonality properties of wavelets,

$$\int_{-1}^1 \int_{-1}^1 v_{n,m}(\mu) v_{n',m'}(\mu) v_{n,m}(\eta) v_{n',m'}(\eta) d\mu d\eta = \begin{cases} 1 & \text{if } m = m' \text{ and } n = n', \\ 0 & \text{otherwise,} \end{cases} \quad (5)$$

we obtain the following equations

$$\frac{1}{(\Sigma_{t,g} - S_d^g)} \sum_{j=1}^p \sum_{k=1}^p (A_{jj'kk'} \frac{\partial^2 \psi_{j'k',g}}{\partial x^2} + B_{jj'kk'} \frac{\partial^2 \psi_{j'k',g}}{\partial x \partial y} + C_{jj'kk'} \frac{\partial^2 \psi_{j'k',g}}{\partial y^2}) + (\Sigma_{t,g} - S_d^g) \psi_{jk,g} = Q_{jk,g}(x, y) + \frac{1}{(\Sigma_{t,g} - S_d^g)} \sum_{j=1}^p \sum_{k=1}^p (D_{jj'kk'} \frac{\partial Q_{j'k',g}}{\partial x} + E_{jj'kk'} \frac{\partial Q_{j'k',g}}{\partial y}) \quad (6)$$

where the coefficients $A \sim E$ can be readily calculated by use of Gaussian quadrature.

Two types of boundary condition are considered.

(1) Zero flux boundary condition

$$\phi_g(\vec{r}_b, \Omega) = 0 \quad \Rightarrow \quad \psi_{jk,g} = 0 \quad (7)$$

(2) Reflective boundary condition

$$\nabla \phi_g(\vec{r}_b, \Omega) = 0 \quad \Rightarrow \quad \frac{\partial \psi_{jk,g}}{\partial n} \Big|_{r=r_b} = 0 \quad (8)$$

There are many types of numerical methods to solve eq. (6). Here, we choose FEM with unstructured-meshes. Details of this procedure are not covered in this paper.

3. Numerical Results

A two-dimensional fixed source problem^[5] was solved by use of the method. The geometry and unstructured-meshes are shown in Fig.1. The reflective boundary conditions are used for all outer boundaries.

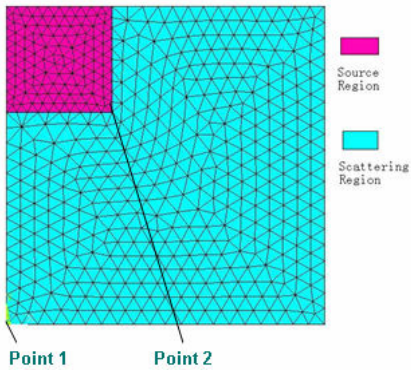


Fig. 1 Geometry and meshes

The results of spatial flux distribution along the bottom boundary are shown in Fig.2. The flux calculated by the S_N code TWOTRAN exhibits oscillations attributed to the ray effects. The wavelet method ($N=4, n=4$) results show better agreements with reference^[5] than P_N (P5) method^[6].

We compare the flux distributions at point 1 and 2 along azimuthal direction obtained by the P_N method and wavelet method as shown in Fig.3 and Fig.4. From the figures, we can see that the angular flux varies greatly along azimuthal direction. The results of the two methods show good agreement except for the obvious discrepancies at peak points.

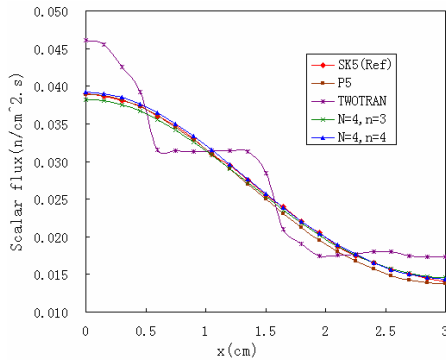


Fig.2 Flux distribution along bottom boundary

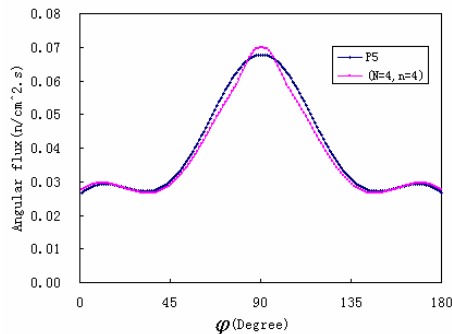


Fig.3 Flux distribution along φ at point 1

We also compare the computational efficiency of the two methods. P5 calculation takes 9.8 seconds, and

wavelet calculations with $N=4, n=3$ and $N=4, n=4$ take 14.5 seconds and 21.4 seconds, respectively. P7 calculation takes 22.3 seconds without obvious improvement of accuracy with respect to the reference. Thus, we may conclude that wavelet method can present high accuracy with competitive speed.

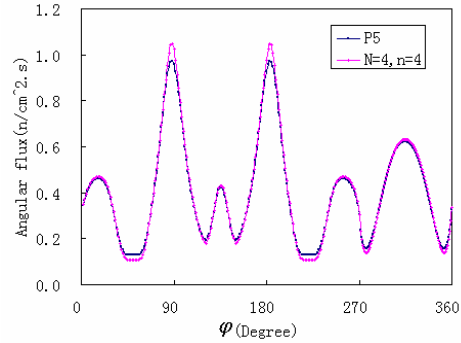


Fig.4 Flux distribution along φ at point 2

4. Conclusions and Suggestions

We developed a wavelet theory method to solve the neutron transport equation. In this method, Daubechies' wavelet bases are utilized to expand the angular variables of the neutron flux. A numerical test shows that the wavelet method may be used to solve the neutron transport equation with high precision and acceptable speed.

This work is just the beginning of research on application of wavelet theory to neutron transport equation. There are several problems which need to be improved, for example, vacuum boundary condition, acceleration method and stability of iteration. Further numerical tests are warranted.

Acknowledgment

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