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Estimation of Parameters of CCF with Staggered Testing

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1. Introduction

Common cause failures are extremely important in reliability analysis and would be dominant to risk contributor in a high reliable system such as a nuclear power plant. Of particular concern is common cause failure (CCF) that degrades redundancy or diversity implemented to improve a reliability of systems. Most of analyses of parameters of CCF models such as beta factor model, alpha factor model, and MGL(Multiple Greek Letters) model deal a system with a non-staggered testing strategy. Non-staggered testing is that all components are tested at the same time (or at least the same shift) and staggered testing is that if there is a failure in the first component, all the other components are tested immediately, and if it succeeds, no more is done until the next scheduled testing time. Both of them are applied in the nuclear power plants. The strategy, however, is not explicitly described in the technical specifications, but implicitly in the periodic test procedure. For example, some redundant components particularly important to safety are being tested with staggered testing strategy. Others are being performed with non-staggered testing strategy. This paper presents the parameter estimator of CCF model such as beta factor model, MGL model, and alpha factor model with staggered testing strategy. In addition, a new CCF model, rho factor model, is proposed and its parameter is presented with staggered testing strategy.

2. Estimation of Parameters of CCF Model

Consider a system of m identical components with staggered test. The total failure probability Q_t of a component in a common cause group of m components is

$$Q_t = \sum_{k=1}^{m} {}_{m-1}C_{k-1} Q_k$$

The likelihood estimator for Q_k which is the probability of a basic event involving k specific components, is given as

$$Q_k = \frac{n_k}{N_k}$$

n_k is the number of events involving k components in a failed state and N_k is the number of demands on any k components in the common cause group. In the testing strategy a number of testing works in a certain period is N_D. At each testing work one component is tested. If the test of the first component succeeds, no more test work is done until the next scheduled testing work. It means that there is no common cause failure for ${}_{m-1}C_{k-1}$ groups of k components. If, however, the first test fails, all other components are tested. Therefore, the total number of tests on any group of k components N_k is given as follows.

$$\begin{aligned} N_k &= \left(N_D - \sum_{j=1}^{m} n_j\right) {}_{m-1}C_{k-1} + \left(\sum_{j=1}^{m} n_j\right) {}_{m-1}C_{k-1} + \left(\sum_{j=1}^{m} n_j\right) {}_{m-1}C_k \\ &= N_D\, {}_{m-1}C_{k-1} + n_t\, {}_{m-1}C_k , \end{aligned}$$

$$where,\quad k = 1, ..., m-1,\ and\ N_m = N_D,\ n_t = \sum_{j=1}^{m} n_j$$

Through this equation, we can obtain Q_k with n_k and N_k, and Q_t with Q_k and N_D.

$$\begin{aligned} Q_t &= \sum_{k=1}^{m} {}_{m-1}C_{k-1} Q_k \\ &= \sum_{k=1}^{m} \left(\frac{n_k}{N_D + n_t s}\right),\ where\ s = \frac{m}{k} - 1 \end{aligned}$$

2.1 Estimation of the parameter of beta factor model

The beta factor model consists of the total component failure probability Q_t and β parameter. By definition of beta factor model, Q_k is described with Q_t and parameter β as follows.

$$\begin{aligned} Q_k &= (1-\beta) Q_t, & k &= 1 \\ &= 0, & m &> k > 1 \\ &= \beta Q_t, & k &= m \end{aligned}$$

Beta factor parameter with staggered testing strategy is obtained as follows.

$$\beta = \frac{\sum_{k=2}^{m} {}_{m-1}C_{k-1} Q_k}{\sum_{k=1}^{m} {}_{m-1}C_{k-1} Q_k} = \frac{\sum_{k=2}^{m} \left(\frac{n_k}{N_D + n_t s}\right)}{\sum_{k=1}^{m} \left(\frac{n_k}{N_D + n_t s}\right)}$$

2.2 Estimation of the parameters of MGL

The MGL model consists of the total component failure probability Q_t and β, γ, δ parameters. By definition of MGL model, Q_k is described with Q_t and parameters, β, γ, δ as follows:

$$Q_k = \frac{1}{{}_{m-1}C_{k-1}} \prod_{i=1}^{k} \rho_i (1-\rho_{i+1}) Q_t$$
$$where,\ \rho_1 = 1,\ \rho_2 = \beta,\ \rho_3 = \gamma,\ \rho_4 = \delta$$

MGL parameter with staggered testing strategy is obtained as follows.

$$\rho_i = \frac{\sum_{k=i}^{m} {}_{m-1}C_{k-1} Q_k}{\sum_{k=i-1}^{m} {}_{m-1}C_{k-1} Q_k} = \frac{\sum_{k=i}^{m} \left(\frac{n_k}{N_D + n_t s} \right)}{\sum_{k=i-1}^{m} \left(\frac{n_k}{N_D + n_t s} \right)}$$
$$where,\quad i = 2,3,4$$

2.3 Estimation of the parameters of alpha factor model

The alpha factor model consists of the total component failure probability Q_t and α_i parameter. By definition of alpha model, Q_k is described with Q_t and parameter α_i as follows.

$$Q_k = \frac{m}{{}_{m-1}C_{k-1}} \frac{\alpha_k}{\alpha_t} Q_t\ ,\quad where,\ \alpha_t = \sum_{k=1}^{m} k\alpha_k$$

Alpha factor model parameter with staggered testing strategy is obtained as follows:

$$\alpha_k = \frac{{}_m C_k Q_k}{\sum_{k=i}^{m} {}_m C_k Q_k} = \frac{(s+1)\left(\frac{n_k}{N_D + n_t s} \right)}{\sum_{k=1}^{m} (s+1)\left(\frac{n_k}{N_D + n_t s} \right)}$$
$$where,\quad k = 1,\ldots,m$$

2.4 Estimation of the parameters of rho factor model

In this paper, we propose a new CCF model, rho factor model. It is similar to MGL model. It consists of the total component failure probability Q_t and ρ_i parameter which is defined as follows.

$$\rho_i = \frac{\sum_{k=i}^{m} {}_{m-1}C_{k-1} Q_k}{\sum_{k=1}^{m} {}_{m-1}C_{k-1} Q_k}\ ,\ where,\ i = 1,\ldots,m$$

Q_k is described with Q_t and parameter ρ_i as follows.

$$Q_k = \frac{1}{{}_{m-1}C_{k-1}} (\rho_k - \rho_{k+1}) Q_t,$$
$$where,\ k = 1,\ldots,m,\ \rho_1 = 1,\ \rho_{m+1} = 0,$$

The rho parameter with staggered testing staggered is described as follows:

$$\rho_i = \frac{\sum_{k=i}^{m} \left(\frac{n_k}{N_D + n_t s} \right)}{\sum_{k=1}^{m} \left(\frac{n_k}{N_D + n_t s} \right)},\ where,\ i = 1,\ldots,m$$

3. Estimation of Parameter of beta Factor Model

Consider a system of 2 identical components with staggered testing. Beta factor is obtained as follows.

$$\beta = \frac{\sum_{k=2}^{2} \left(\frac{n_k}{N_D + n_t s} \right)}{\sum_{k=1}^{2} \left(\frac{n_k}{N_D + n_t s} \right)} = \frac{\frac{n_2}{N_D}}{\frac{n_1}{N_D + n_1 + n_2} + \frac{n_2}{N_D}}$$
$$\approx \frac{n_2}{n_1 + n_2},\quad where,\quad N_D >> n_1 + n_2$$

Consider a system of 2 identical components with non-staggered testing, in which all components are tested simultaneously. In this case beta factor is as follows.

$$\beta_{ns} = \frac{\sum_{k=2}^{2} kn_k}{\sum_{k=1}^{2} kn_k} = \frac{2n_2}{n_1 + 2n_2}$$

We can see that estimates of beta factor are different depending on what testing scheme is performed. The ratio of two beta factors is as follows.

$$\frac{\beta_{ns}}{\beta} = \frac{\frac{2n_2}{n_1 + 2n_2}}{\frac{n_2}{n_1 + n_2}} = 2\frac{n_1 + n_2}{n_1 + 2n_2} \approx 2,\quad where\ \ n_1 >> n_2$$

We can see that the staggered test estimator is approximately a factor of 2 smaller than the non-staggered test estimator.

3. Conclusion

This paper presents the parameter estimator for CCF model such as beta factor model, MGL model, alpha factor model, a new CCF model, rho factor model, which is similar to MGL, with staggered testing strategy. We can see that the staggered test estimator is approximately a factor of 2 smaller than the non-staggered test estimator.

REFERENCES

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