

# Dependent Failures: An Analysis Methodology

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## 1. Introduction

Dependent failures including common cause failures are extremely important in reliability analysis and would be dominant to risk contributor in a high reliable system such as a nuclear power plant. Of particular concern is common cause failure (CCF), which is a kind of dependent failure that degrades the redundant or diversity implemented to improve the reliability of systems. The previous CCF analyses such as beta factor method, alpha factor method, and MGL method can treat only the dependent failures between homogeneous (the same) components in a redundant system. However, following a general definition of CCF given by Mosles et al., it is "... a subset of dependent events in which two or more component fault states exist at the same time, or in a short time interval, and are direct results of a shared cause." Therefore, a new dependent failure analysis including existing CCF methods should be developed for dealing with the dependent failures among homogeneous and heterogeneous components in a redundant system. This paper presents a new method to deal with these dependent failures. The proposed analysis method is generalized common cause failure analysis for a multiple system.

## 2. Dependent Failure Models

Consider the redundant system with four diverse trains with one pump and one valve shown in Fig. 1. Where pumps or valves are identical and functionally diverse each other, the normal CCF method separately handles CCF for group 1 or group 2. However, in reality there is the possibility that eight components affect each other or share coupling factors, which cause simultaneous failures. Such factors include block maintenance, test, and external environmental conditions (e.g. temperature, vibration, and humidity). Therefore it is more general to analyze dependent failures among all components in one group.

In this paper two types of dependent failure models are proposed for dealing with a multiple system with homogeneous and heterogeneous components.

Firstly, the following equation, described with MGL notation, is the proposed dependent failure model

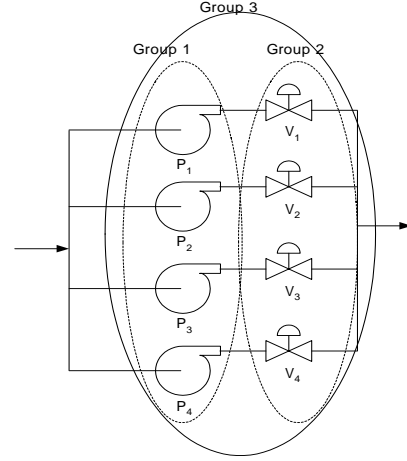


Fig. 1. A typical redundant system.

$$Q_t = \frac{1}{mN_D} \sum_{i=0}^m \sum_{j=0}^m (i+j) \cdot n_{i,j} = Q_{0t} + Q_{pt}$$

$$Q_{0t} = \frac{1}{mN_D} \sum_{j=0}^m j \cdot n_{0,j} = Q_{00} + Q_{01} + \dots + Q_{0m}, \quad n_{00} \equiv 0,$$

$$Q_{0v} = \frac{Q_{0t}}{C_{v-1}^{m-1}} (1 - \sigma_{v+1}) \left( \prod_{i=1}^v \sigma_i \right), \quad \text{where } v = 1, 2, \dots, m$$

$$\sigma_1 = 1, \quad \sigma_2 = \beta_v, \quad \sigma_3 = \gamma_v, \quad \dots, \quad \sigma_{m+1} = 0,$$

$$\beta_v = \frac{\sum_{j=2}^m j \cdot n_{0,j}}{\sum_{j=1}^m j \cdot n_{0,j}}, \quad \gamma_v = \frac{\sum_{j=3}^m j \cdot n_{0,j}}{\sum_{j=2}^m j \cdot n_{0,j}}, \quad \delta_v = \frac{\sum_{j=4}^m j \cdot n_{0,j}}{\sum_{j=3}^m j \cdot n_{0,j}}, \quad \dots$$

$$Q_{pt} = \frac{1}{mN_D} \sum_{i=1}^m \sum_{j=0}^m (i+j) \cdot n_{i,j} = Q_{1v} + \dots + Q_{mv}$$

$$Q_{pv} = \frac{Q_{pt}}{C_{p-1}^{m-1} C_v^m} (1 - \rho_{p+1}) \left( \prod_{i=1}^p \rho_i \right) (1 - \omega_{p,v+1}) \left( \prod_{j=0}^v \omega_{p,j} \right),$$

$$\text{where, } p = 1, 2, \dots, m, \quad v = 0, 1, 2, \dots, m$$

$$\rho_1 = 1, \quad \rho_2 = \beta, \quad \rho_3 = \gamma, \quad \dots, \quad \rho_{m+1} = 0,$$

$$\beta = \frac{\sum_{i=2}^m \sum_{j=0}^m (i+j) \cdot n_{i,j}}{\sum_{i=1}^m \sum_{j=0}^m (i+j) \cdot n_{i,j}}, \quad \gamma = \frac{\sum_{i=3}^m \sum_{j=0}^m (i+j) \cdot n_{i,j}}{\sum_{i=2}^m \sum_{j=0}^m (i+j) \cdot n_{i,j}},$$

$$\delta = \frac{\sum_{i=4}^m \sum_{j=0}^m (i+j) \cdot n_{i,j}}{\sum_{i=3}^m \sum_{j=0}^m (i+j) \cdot n_{i,j}}, \quad \dots$$

$$\omega_{p,0} = 1, \dots, \omega_{p,v} = \frac{\sum_{j=v}^m (j+p) \bullet n_{p,j}}{\sum_{j=v-1}^m (j+p) \bullet n_{p,j}}, \dots, \omega_{p,m+1} = 0,$$

where,  $p = 1, 2, \dots, m$ ,  $v = 1, 2, \dots, m$

Where consider only the dependent failures for a kind of component such as pump, that is,  $v = 0$ , the above equation is reduced to the equation of MGL model.

$$Q_k = \frac{1}{mN_D} \sum_{k=1}^m kn_k = \sum_{k=1}^m Q_k$$

$$Q_k = \frac{1}{m-1 C_{k-1}} \left( \prod_{i=1}^k \rho_i \right) (1 - \rho_k) Q_t$$

$$\rho_1 = 1, \rho_2 = \beta, \rho_3 = \gamma, \dots, \rho_{m+1} = 0,$$

$$Q_t = \frac{1}{mN_D} \sum_{k=1}^m kn_k, \quad \beta = \left( \sum_{k=2}^m kn_k \right) / \left( \sum_{k=1}^m kn_k \right)$$

$$\gamma = \left( \sum_{k=3}^m kn_k \right) / \left( \sum_{k=2}^m kn_k \right), \quad \delta = \left( \sum_{k=4}^m kn_k \right) / \left( \sum_{k=3}^m kn_k \right)$$

Next, the following is the proposed dependent failure model described with the notation of alpha factor model.

$$Q_t = \frac{1}{mN_D} \sum_{i=0}^m \sum_{j=0}^m (i+j) \bullet n_{i,j} = Q_{0t} + Q_{pt}$$

$$Q_{0t} = \frac{1}{mN_D} \sum_{j=0}^m j \bullet n_{0,j}, = Q_{00} + Q_{01} \dots + Q_{0m}, \quad n_{00} = 0$$

$$\alpha_{0v} = \frac{n_{0,v}}{\sum_{j=0}^m n_{0,j}}, \quad \alpha_{ot} = \sum_{j=0}^m j \bullet n_{0,j}$$

$$Q_{0v} = \frac{v}{m-1 C_{v-1}} \bullet \frac{\alpha_{0,v}}{\alpha_t} \bullet Q_{0t}, \quad \text{where } v = 1, \dots, m$$

$$Q_{pt} = \frac{1}{mN_D} \sum_{i=1}^m \sum_{j=0}^m (i+j) \bullet n_{i,j} = Q_{1v} + \dots + Q_{mv}$$

$$Q_{pv} = \frac{(p+v)}{m-1 C_{p-1} C_v} \bullet \frac{\alpha_{p,v}}{\alpha_t} \bullet Q_{pt}$$

where,  $p = 1, 2, \dots, m$ ,  $v = 0, 1, 2, \dots, m$

$$\alpha_{p,v} = \frac{n_{p,v}}{\sum_{i=1}^m \sum_{j=0}^m n_{i,j}}$$

$$\alpha_t = \sum_{i=1}^m \sum_{j=0}^m (i+j) \bullet \alpha_{i,j} = \frac{\sum_{i=1}^m \sum_{j=0}^m (i+j) \bullet n_{i,j}}{\sum_{i=1}^m \sum_{j=0}^m n_{i,j}}$$

The equation is also reduced to alpha factor model where considering only CCF for a kind of component.

$$Q_k = \frac{1}{mN_D} \sum_{k=1}^m kn_k = \sum_{k=1}^m Q_k, \quad Q_k = \frac{k}{m-1 C_{k-1}} \bullet \frac{\alpha_k}{\alpha_t} \bullet Q_t$$

$$\alpha_t = \sum_{k=1}^m k \alpha_k, \quad Q_t = \frac{1}{mN_D} \sum_{k=1}^m kn_k, \quad \alpha_k = n_k / \left( \sum_{k=1}^m n_k \right)$$

The dependent failure model with the notation of the alpha factor model is simpler than that of MGL model. The relationship between parameters of two dependent failure models is as follows:

$$\frac{1 \bullet \alpha_{1,0}}{\alpha_t} = (1 - \beta)(1 - \omega_{1,1}) \omega_{1,0},$$

$$\frac{2 \bullet \alpha_{1,1}}{\alpha_t} = (1 - \beta)(1 - \omega_{1,2}) \omega_{1,0} \omega_{1,1},$$

$$\vdots$$

$$\frac{2 \bullet \alpha_{2,0}}{\alpha_t} = (1 - \gamma) \beta (1 - \omega_{2,1}) \omega_{2,0},$$

$$\frac{4 \bullet \alpha_{2,1}}{\alpha_t} = (1 - \gamma) \beta (1 - \omega_{2,2}) \omega_{2,0} \omega_{2,1},$$

$$\vdots$$

Where considering CCF among only one group, the above equation is reduced to as follows.

$$\frac{1 \bullet \alpha_1}{\alpha_t} = 1 - \beta, \quad \frac{2 \bullet \alpha_2}{\alpha_t} = \beta (1 - \gamma)$$

$$\frac{3 \bullet \alpha_3}{\alpha_t} = \beta \gamma (1 - \delta), \quad \frac{4 \bullet \alpha_4}{\alpha_t} = \beta \gamma \delta, \quad \dots$$

Typical CCF methods used in probabilistic safety analysis are beta factor method and MGL method. The beta factor method can be mentioned as a simplified and conservative method of MGL method. Dependent failure model with the concept of beta factor is easily obtained with that of MGL notation. It is not described in this paper on account of limited space.

### 3. Conclusion

In this paper, the new dependent failure models including CCF methods are proposed applicable to the redundant system with homogeneous and heterogeneous components. One model is expressed in terms of the notation of multi Greek letter model, the other is expressed in terms of that of alpha factor model. They are a kind of generalized MGL model and alpha factor method. Where for one group with homogenous components, these models are reduced to existing CCF models.

### REFERENCES

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