# Nonlinear FE analysis for RC Panel using Embedded Reinforcement Element

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## 1. Introduction

A model for embedding reinforcements in concrete elements for analysis of reinforced concrete (RC) structures is presented. This procedure alleviates the laborious task of generation the input data for the embedded bar elements in three dimensional (3D) finite element (FE) analysis of reinforced concrete structures such as containment buildings of nuclear power plants, particularly when modifications to the concrete element mesh are made, or reinforcement arrangement is changed.

The formulations of embedded element are explained and implemented in NUCAS codes [1]. A numerical example is used to verify the validity of the model.

## 2. Formulations of the embedded element

In 3D nonlinear FE analysis of RC structures, three methods available for the simulation of reinforcement are smeared, discrete and embedded method. The smeared and discrete formulations are dependent on the concrete element mesh. In 3D applications, this can lead to prohibitive computational costs due to the use of many unnecessarily small elements or inaccuracies caused by elements with undesirable aspect ratios. To remedy these problems, embedded formulation is preferable.

### *2.1 Linear embedded element*

It is important to find the intersection point of reinforcement segment with boundaries of concrete elements. There are two methodologies to fine the intersection point such as linear and curved embedded approaches. In this study, linear embedded method is adapted.

To determine the intersection points of $\mathbf{P_1P_2}$ with face of concrete solid elements in Figure 1, the equation of line $\mathbf{P_1P_2}$ is written in a parametric form as following [2],

$$\begin{Bmatrix} x(S) \\ y(S) \\ z(S) \end{Bmatrix} = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{P_1} - S\left[\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{P_2} - \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{P_1}\right]; \quad 0 \le S \le 1 \tag{1}$$

where $S$ is the ratio between the length of $\mathbf{P_1P_2}$ and the length of $\mathbf{P_1P}_a$ in Figure 1.

The equation of a typical face of the element may by written as $\mathbf{P_0P}_a \cdot \mathbf{R} = 0$ where $\mathbf{P_0}$ is a corner node, $\mathbf{P}_a$ is a generic point on the surface and $\mathbf{R}$ is a unit normal vector to this face. Therefore, the procedure will be exact for 8-node linear solid isoparametric elements while introducing some approximation for higher-order elements with curved surfaces. The coordinates of the intersection point $\mathbf{P}_a$ is determined from (1), with $S$ determined as

$$S = \frac{\mathbf{P_1P_0} \cdot \mathbf{R}}{\mathbf{P_1P_2} \cdot \mathbf{R}} \tag{2}$$

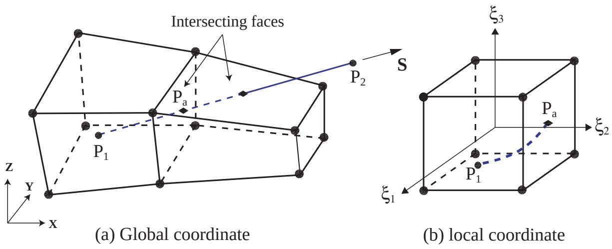


Figure 1. Configures of linear embedded element

### *2.2 Inverse mapping*

The inverse mapping procedure, which is a 3D extension of the method proposed by Elwi and Hrudey[3], is used to find intersection point. A point such as $P_1$ is contained in a given concrete element if its coordinates, $\xi_{1_{P_1}}$ , $\xi_{2_{P_1}}$ and $\xi_{3_{P_1}}$ , in the element local axes satisfy

$$-1 \le \xi_{1_{P_1}},\ \xi_{2_{P_1}},\ \xi_{3_{P_1}} \le 1 \tag{3}$$

In the isoparametric formulation the global coordinates (x,y,z) of a generic point within a solid element are expressed as

$$\mathbf{x} = \sum_{a=1}^{8} N_a(\xi_i)\mathbf{x}_i^a,\ (i = 1,2,3) \tag{4}$$

where $\mathbf{x}_i^a$ is vectors of element nodal coordinates and $N_a$ represents the displacement-shape functions at node *a*. It follows that

$$\begin{Bmatrix} dx \\ dy \\ dz \end{Bmatrix} = [\mathbf{J}]^T \begin{Bmatrix} d\xi_1 \\ d\xi_2 \\ d\xi_3 \end{Bmatrix} \text{ or } \begin{Bmatrix} d\xi_1 \\ d\xi_2 \\ d\xi_3 \end{Bmatrix} = [\mathbf{J}]^{T^{-1}} \begin{Bmatrix} dx \\ dy \\ dz \end{Bmatrix} \tag{5}$$

where $[\mathbf{J}]$ is the Jacobian matrix.

From the equation (2), the coordinates $\left(\xi_1, \xi_2, \xi_3\right)_{P_1}$ are the roots of the following set of equations:

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{P_1} - \begin{bmatrix} N_a & 0 & 0 \\ 0 & N_a & 0 \\ 0 & 0 & N_a \end{bmatrix} \mathbf{x}_i^a = \{0\}, \ \left(i = 1,2,3\right) \tag{6}$$

A Nweton-Raphson iterative procedure has been used for solution. With an initial estimate of $\left[\{\xi_1 \ \xi_2 \ \xi_3\}_{P_1}^0\right]^T = \{0\}$, the solution after *n+1* iterations is determined as

$$\begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix}_{P_1}^{n+1} = \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix}_{P_1}^{n} + \begin{Bmatrix} \Delta\xi_1 \\ \Delta\xi_2 \\ \Delta\xi_3 \end{Bmatrix}_{P_1}^{n+1} \tag{7}$$

where

$$\begin{Bmatrix} \Delta\xi_1 \\ \Delta\xi_2 \\ \Delta\xi_3 \end{Bmatrix}_{P_1}^{n=1} = \left[J^n\right]^{T^{-1}} \left[ \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}_{P_1} - \begin{bmatrix} N_a & 0 & 0 \\ 0 & N_a & 0 \\ 0 & 0 & N_a \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \right] \tag{8}$$

The preceding solution method has been found to converge rapidly. If the current element does not satisfy the equation (3), the procedure is repeated using the nodal coordinates of the next coordinates of the next element until the element containing $\mathbf{P}_1$ is identified.

## 3. Numerical example

To verify the linear embedded element implemented in NUCAS codes, the nonlinear FE analysis for containment wall RC panel subject to biaxial tensile load. Figure 2 shows an average stress-average strain for reinforcement with experimental and analytical results. Before an initial crack is occurred, the stiffness of the embedded element for concrete is almost closed to experiment result as well as FE result using smeared steel layer.

The first crack of FE analysis occurred at 232MPa in hoop direction and at the same time on the overall gauss points. From the test, the first crack occurred along the right surface above the tendon duct placed line when the tensile stress reached 201MPa. The crack stress of the FE results is higher than experiment result. Because of not including the tendon-duct in the analysis, the stiffness of RC element is overestimated as much as the area of the tendon-duct. But, overall stress-strain curve for RC panel is very similar with experimental data.

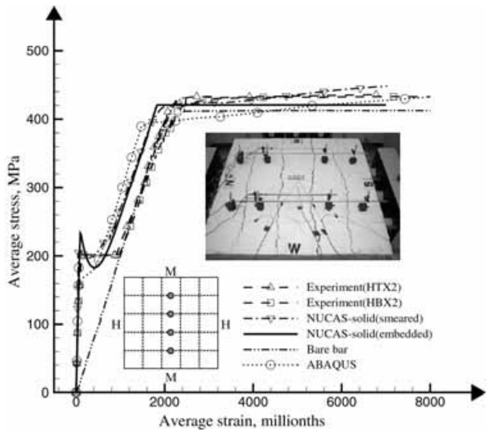


Figure 2. Average stress-strain curve for steel

## 4. Conclusion

In this study, a model of embedding reinforcement with linear embedded formulations for analysis of RC structures is implemented in NUCAS-3D solid element codes. To evaluate the developed model, the nonlinear FE analysis of containment wall panel subjected to biaxial tensile load is employed. From the numerical example, the FE analysis results are good agreement with experimental data.


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## REFERENCES


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