Fatigue Growth of Three Dimensional Cracks in an Elbow

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1. Introduction

In the assessment of structural integrity and the analysis for damage tolerance of structures, the calculation of fracture parameters for three-dimensional cracks remains as an important task because they occupy majority of cracks generated in the industrial field. The finite element alternating method (FEAM) has been presented and used effectively to solve two and three dimensional crack problems [1,2]. The method alternates between the FEM solution for a finite body without a crack and the solution for a crack in an infinite body. Nikishkov, Park and Atluri [3] presented another FEAM procedure, in which the crack solution for an infinite body is obtained by using the symmetric Galerkin boundary element method (SGBEM).

By using the methods, the analysis of three-dimensional planar or non-planar cracks embedded in an infinite or a finite body can be performed more efficiently and more accurately. And the FEAM is particularly efficient for modeling of fatigue crack growth since the finite element mesh for the uncracked body and the boundary mesh for the crack are completely independent. In this paper, with the use of the proposed procedure the stress intensity factors are calculated about some of three dimensional cracks embedded in infinite and finite body. And the stress intensity factors for arbitrarily shaped three-dimensional cracks are calculated to simulate the crack growth.

2. Theory and Simulation

2.1 Governing Equation

Consider an infinite three-dimensional body containing arbitrarily three-dimensional cracks of arbitrary geometry. A distributed load is applied at the crack surface. The crack can be described by a distribution of displacement discontinuity with components [3,4].

$$
\begin{aligned}
&-\int_S\int_S D_\alpha u_i^*(\mathbf{z}) C_{\alpha i\beta j}(\boldsymbol{\xi}-\mathbf{z}) D_\beta u_j(\boldsymbol{\xi}) dS(\boldsymbol{\xi}) dS(\mathbf{z})\, u_{jb} \\
&= \int_S u_k^*(\mathbf{z}) t_k\, dS(\mathbf{z})
\end{aligned} \tag{1}
$$

Here $S = S_+$. is one of crack surfaces, are displacement discontinuities for the crack surface, u_i^* are the components of a continuous test function and t_k are crack face tractions

Assume the crack that is partitioned into boundary elements. Displacement discontinuities and tractions are defined at element nodes, and are interpolated inside the elements with the use of shape functions N_a . With the use of a parametric representation of displacement discontinuities and tractions, we can rewrite the integral equation (1) in the following discretized form.

$$
\begin{aligned}
&-\int_S\int_S C_{\alpha i\beta j} D_\alpha N_a(z) D_\beta N_b(\xi) dS(\xi) dS(z)\, u_{jb} \\
&= \int_S N_a N_q dS(z)\, t_{iq}
\end{aligned} \tag{2}
$$

Using the integral equation (2), displacement discontinuities at element nodes of the crack are defined, and then the stress intensity factors can be calculated from their values.

2.2 Alternating Method

In fracture mechanics problems, combining the SGBEM for modeling an arbitrary non-planar crack in an infinite body, and the finite element method for an uncracked finite body, The independence of the crack model of the finite element model allows us to change the crack model easily when simulating crack growth under monotonic or cyclic loading. The finite element alternating method alternates between the finite element solution for an uncracked body and the displacement discontinuity method solution for a crack in an infinite body. The steps of the alternating iteration procedure are explained in detail by Park et al. [2].

2.3 Surface Crack Growth in an Elbow

For the application of the method, an elbow including a surface crack is used. The length of the elbow is 4000 mm, the diameter 400 mm and the thickness 20 mm. And the curvature of the elbow is 6 times to diameter of the pipe. The material type is the carbon steel pipe, which is generally used as the piping material of non-safety classes in PWR power plants.

ABQUS, the commercial finite element program, is used to obtain the loading condition of the crack [5].

In order to calculate stress intensity factors for surface cracks a semi-elliptical surface crack is considered as shown in Figure 1. The crack is located in the weld zone which has many problems related to the damage of pipes. We calculated the stress intensity factors of the crack when both the length of major axis a and the miner axis b are increased case by case and the results is shown in Figure 2 and 3 with respect to the ratio of crack length to crack thickness.

Next, a semi-elliptical surface crack, located to the axial direction of the pipe, is considered to simulate the fatigue crack growth of the surface crack in the weld zone. Paris equation is applied to calculate fatigue crack growth rate, which explains typical fatigue crack growth behavior in metals. Material constants composing the equation are same to the above values. Based on the stress intensity factor obtained by Paris equation, the crack tip is transformed according to increments in each node. The results are presented in Figure 4 in respect of the angle of crack nodes. As shown in the results, stress intensity factors are maximized in the 90° location and are minimized in the both sides of the crack.

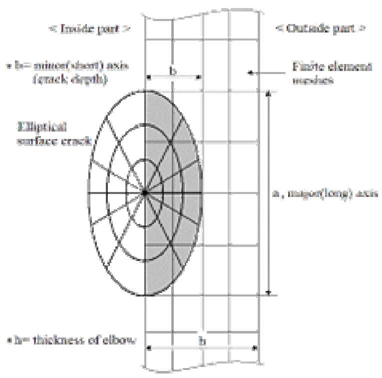

Figure 1. Mesh configuration for a semi-elliptical surface crack in an elbow.

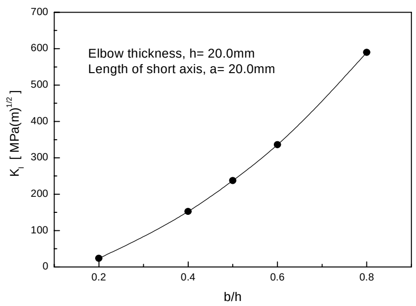

Figure 2. SIF’s according to b/h for a semi-elliptical crack in an elbow.

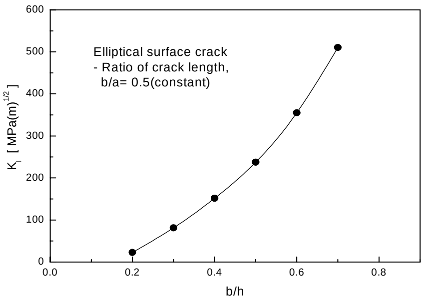

Figure 3. SIF’s according to b/h for a semi-elliptical surface under cyclic loading condition.

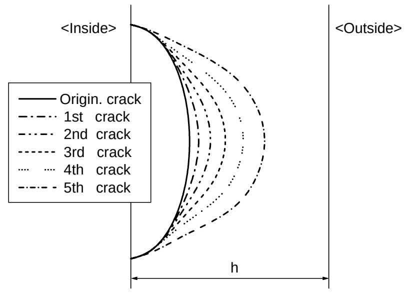

Figure 4. Growth of an elliptical crack in an elbow under cyclic loading condition.

3. Conclusion

The finite element alternating method has been used for the analysis of three dimensional cracks in infinite bodies. Especially, since the finite element mesh for the uncracked body and the boundary mesh for the crack are completely independent, the method is particularly efficient for modeling of fatigue crack growth. As the result of the application, it is demonstrated that the finite element alternating method can be used effectively to analyze the three dimensional crack growth of the structure of industrial facilities.

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