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# A Comparison of Dose Distributions Computed with Three Schemes for Uncollided Flux Calculation on the First Collision Source Method

Jong Woon Kim * and Young-Ouk Lee
*Korea Atomic Energy Research Institute, 1045 Daedeokdaero, Yuseong-gu, Daejeon, Korea, 305-353*
**Corresponding author: jwkim@kaeri.re.kr*

## 1. Introduction

As with any discrete-ordinates code, ray-effects are an inherent problem, especially for shielding type problems with optically thin regions and localized (point) sources. Although ray-effects may be mitigated by increasing the quadrature order, this is often computationally prohibitive.

To mitigate ray-effects, many discrete-ordinates codes use first collision source methods. Such methods are characterized by a decomposition of the flux into its uncollided and collided components. The uncollided flux is calculated analytically and the collided flux is calculated with the discrete ordinates method.

Currently, the MUST (***M***ulti-group ***U***nstructured geometry $\boldsymbol{S}_N$ ***T***ransport) [1] code uses node base continuous scheme for uncollided flux calculation and discontinuous finite element method (DFEM) for collided flux calculation [2].

In this paper, we applied three different schemes for uncollided flux calculation and compared the results.

## 2. Method and results

### *2.1 Schemes for uncollided flux calculation*

In the first collision source method with point source, angular flux is decomposed into two components as uncollided flux, $\psi^{(u)}(\vec{r},\vec{\Omega})$ , and collided flux, $\psi^{(c)}(\vec{r},\vec{\Omega})$, as

$$\vec{\Omega}\cdot\nabla\psi^{(u)}(\vec{r},\vec{\Omega})+\sigma_t(\vec{r})\psi^{(u)}(\vec{r},\vec{\Omega})=\frac{q_p}{4\pi}\delta(\vec{r}-\vec{r}_p), \quad (1)$$

$$\vec{\Omega}\cdot\nabla\psi^{(c)}(\vec{r},\vec{\Omega})+\sigma_t(\vec{r})\psi^{(c)}(\vec{r},\vec{\Omega}) = \sum_{\ell=0}^{L}\sigma_{s,\ell}(\vec{r})\sum_{m=-\ell}^{\ell}Y_{\ell m}^{*}(\vec{\Omega})\phi_\ell^{m,(c)}(\vec{r})+q_s^{(u)}(\vec{r}). \quad (2)$$

The Eq. (2) is calculated by DFEM which is discontinuous method with third order accuracy [3]. However, the Eq. (1) is calculated by ray-tracing technique tracking from the point source to the point(s) where we want to have uncollided flux analytically.

In this comparison, three schemes for calculating uncollided flux are used. The main differences between three schemes are how to choose the point(s). For easy understanding, diagrams for three schemes are illustrated with the one-dimensional element not with the tetrahedral element. The dotted lines in the Figs. 1-3 are the boundaries of elements. From the point source, the flux is getting decreased with $1/4\pi r^2$. Let's assume that the flux is drastically changing within the element like red line in the Figs. 1-3.

For the node base scheme, the uncollided flux is calculated on the node positions, so that uncollided flux is continuous on the boundaries of elements. As we can see in the Fig. 1, if flux is changing drastically within the element, this scheme may overestimate the real flux distribution.

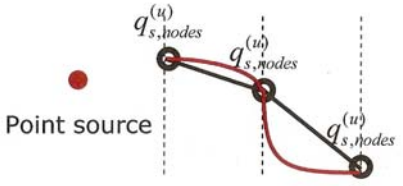



**Fig. 1.** Node base scheme (continuous scheme).

For the 1pt-linear scheme, the uncollided flux is calculated on the center of the tetrahedral element and uses it as uncollided flux on the four vertexes. This is just like step difference spatial difference scheme and might give better flux distribution than node base scheme. However, if element size is not small enough, it will give stepwise flux distribution as shown in Fig. 5.

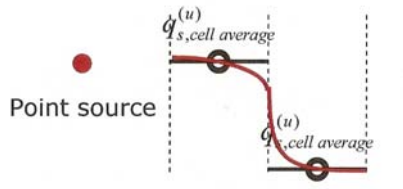



**Fig. 2.** 1-pt linear scheme (discontinuous scheme).

For the 4pt-quadratic scheme, the uncollided fluxes are calculated on the tetrahedral coordinates listed in the Table I and use them as uncollided fluxes on the four vertexes. This scheme allows discontinuity on the boundary of the tetrahedral elements and its error is third order accuracy which is same as DFEM.

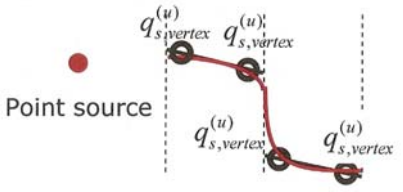
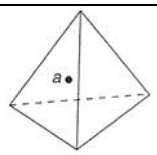
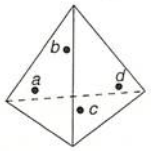



**Fig. 3.** 4-pt quadratic scheme (discontinuous scheme).

**Table I**: Positions for the uncollided flux calculation [4]

| Figures | Error | Points | Tetrahedral Coordinates | Weights |
|---|---|---|---|---|
|  | $R=O(h^2)$ | $a$ | $\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}$ | 1 |
|  | $R=O(h^3)$ | $a$ | $\alpha,\ \beta,\ \beta,\ \beta$ | 1/4 |
| | | $b$ | $\beta,\ \alpha,\ \beta,\ \beta$ | 1/4 |
| | | $c$ | $\beta,\ \beta,\ \alpha,\ \beta$ | 1/4 |
| | | $d$ | $\beta,\ \beta,\ \beta,\ \alpha$ | 1/4 |
| | | | $\alpha$=0.58541020 | |
| | | | $\beta$=0.13819660 | |

*2.2 Tests and Results*

Test Problem consists with $^{252}$Cf source at the origin and 10 cm thick SST plate located at the 10 cm from the source. The detail parameters are listed in the Table II.

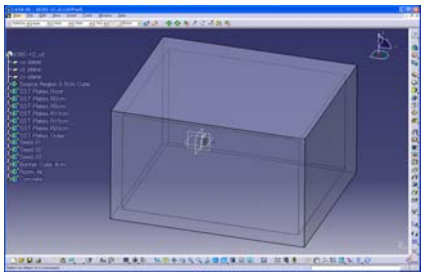


**Fig. 4.** The CAD drawing for the Test Problem.

**Table II**: Parameters for Test Problem

| | |
|---|---|
| Source | |
| $^{252}$Cf / number of source particle: 9.34982e+6 | |
| Geometry | |
| SST Plates (cm) | 50×50×10 (10cm from the source) |
| Room (cm) | 660×560×460 |
| Concrete Wall | 30cm thickness |
| Materials | |
| SST Plates Density (7.9g/cm$^3$) | 20 isotopes (cnat, cr50, cr52, cr53, cr54, fe54, fe56, fe57, fe58, ni58, ni60, ni61, ni62, ni64) |
| Room Air Density (0.001293g/cm$^3$) | Density (0.001293g/cm3) 2 isotopes (n14, o16) |
| Concrete Density (2.3g/cm$^3$) | Density (2.3g/cm3) 14 isotopes (h1, cnat, o16, mg24, mg25, mg26, al27, si28, si29, si30, ca40, ca42, ca43, ca44, ca46, ca48, fe54, fe56, fe57, fe58) |

The dose profiles along the center line are compared in the Fig. 5. The node base scheme gives higher dose profile than those with other two schemes as we expected. The 1pt-linear and 4pt-quadratic schemes give similar results. However, when it comes to the large elements size compared to the real flux distribution (just before the SST plates and inside of SST plates in the Fig. 5), the 1pt-linear scheme shows stepwise dose distribution. Among the three schemes, the 4pt-quadratic scheme gives best results than others.

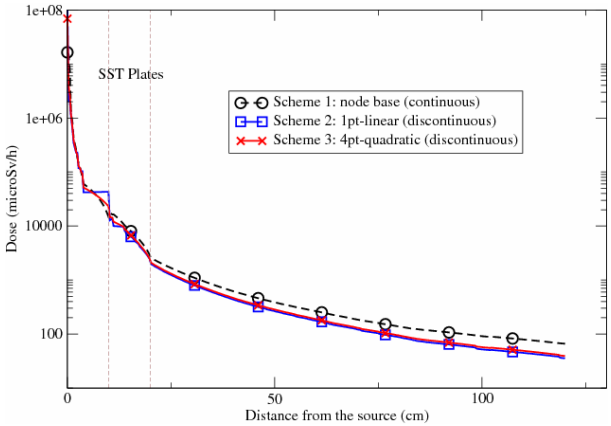



**Fig. 5.** The comparison of the dose profiles along the center line.

## 3. Conclusions

We applied three different schemes for uncollided flux calculation and compared the dose profiles along the center line. The node base continuous scheme gives overestimated dose profile than those with two other schemes. Between the two discontinuous schemes, 1pt-linear scheme might produce stepwise dose distribution when the size of element is large. Among the three schemes, 4pt-quadratic scheme gives best results regardless of element size.

## Acknowledgement

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## REFERENCES

[1] Ser Gi Hong, Jong Woon Kim, and Young Ouk Lee, "Development of MUST (*M*ulti-group *U*nstructured geometry $S_N$ *T*ransport) Code," *Transaction of the Korean Nuclear Society Autumn Meeting*, Gyeongju, Korea, October 2009.
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[4] O.C. Zienkiewicz, R.L. Taylor, and J.Z. Zhu, The Finite Element Method: Its Basic and Fundamentals, Sixth edition, ELSEVIER, pp.164-167, 2005.