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Stiffness Confinement Method with Pseudo Absorption for Spatial Kinetics

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1. Introduction

The stiffness confinement method (SCM) introduced by Chao[1] originally for point kinetics solution was also extended to spatial kinetics[2]. Recently, Chao proposed a refinement of SCM which is to provide a systematic way to update the amplitude frequency in spatial kinetics solution.[3] The primary advantage of the SCM is that it is possible to use larger time step sizes. This advantage comes from the fact because the SCM involves the solution of an eigenvalue problem instead of the ordinary form of a fixed source problem. Since using a large time step size is strongly desired in the direct whole core transport calculation for transient problems, we investigate here the SCM for spatial kinetics first with a simple one-dimensional, one-group diffusion equation and propose an improved formulation. The performance of the improved SCM for spatial kinetics is assessed by comparing the SCM solutions with the standard method solutions employing the Crank-Nicholsen method with exponential transform.[4]

2. SCM for Spatial Kinetics

In order to derive the SCM for spatial kinetics, the dynamic frequency should be defined first and the splitting of the dynamic frequency into the amplitude and the shape frequencies need to be made. Then the method to determine the two frequencies can be derived systematically.

2.1 Amplitude and Shape Frequencies

The dynamic frequency is interpreted as the instantaneous relative change rate of flux and is defined as:

$$\omega(x,t) = \frac{1}{\phi(x,t)}\frac{d\phi(x,t)}{dt} . \qquad (1)$$

The solution of Eq. (1) is obtained as:

$$\phi(x,t) = \phi(x,t_{n-1})e^{\int_{t_{n-1}}^{t}\omega(x,t')dt'} \qquad (2)$$

where t_{n-1} is a time point which would be the beginning time of the *n*-th time step when the time domain is discretized. By approximating the integral in the exponent as:

$$\int_{t_{n-1}}^{t}\omega(x,t')dt' = \bar{\omega}^n(x)(t-t_{n-1}) \qquad (3)$$

in terms of the average dynamic frequency for the *n*-th time interval, Eq. (2) can be expressed as:

$$\phi(x,t) = \phi(x,t_{n-1}+\tau) = \phi(x,t_{n-1})e^{\bar{\omega}^n(x)\tau} \qquad (4)$$

where $\tau = t - t_{n-1}$. Noting that it is possible to factorize $\phi(x,t_{n-1})$ into the amplitude and shape parts as:

$$\phi(x,t_{n-1}) = p(t_{n-1})\hat{\phi}(x,t_{n-1}) \qquad (5)$$

in terms of the flux shape function normalized such that:

$$\int_V \Sigma_f \hat{\phi}(x,t_{n-1})dx = V , \qquad (6)$$

it is also possible to split the dynamic frequency into the amplitude part and the space-dependent part as:

$$\phi(x,t) = p(t_{n-1})\hat{\phi}(x,t_{n-1})e^{\omega_T^n\tau}e^{\omega_S^n(x)\tau} = p(t)\hat{\phi}(x,t) \qquad (7)$$

with the overbar sign omitted for brevity. In order to make the splitting unique, the constraint of normalization is needed for the shape flux at *t* as:

$$\int_V \Sigma_f(x,t)\hat{\phi}(x,t)dx \equiv \int_V \Sigma_f(x,t)\hat{\phi}(x,t_{n-1})e^{\omega_S^n(x)\tau}dx = V . \qquad (8)$$

2.2 Derivation of Static Eigenvalue Equation

With the flux approximation by Eq. (7) and the corresponding definition of the precursor frequency that would lead the following exponential variation:

$$C_i(x,t) = C_i(x,t_{n-1})e^{u_i^n(x)\tau} \qquad (9)$$

where i is the precursor group index, the following equation can be obtained from the time-dependent diffusion equation in which the time derivative can be easily obtained by the use of the exponential function for the time variation:

$$\frac{\omega_T^n+\omega_S^n(x)}{v}\phi - D\nabla^2\phi + \Sigma_a\phi = \frac{1}{k_D}\Big(1-\sum_{i=1}^{6}\frac{u_i^n\beta_i}{u_i^n+\lambda_i}\Big)\nu\Sigma_f\phi \qquad (10)$$

where the dynamic eigenvalue k_D is introduced in order to mitigate the imbalance between the LHS and RHS terms to appear when using inexact values of the amplitude and shape frequencies. The $\frac{\omega^n(x)}{v}$ term is considered as a pseudo absorption cross section that is added in the transient problem. The use of this pseudo absorption cross section on the LHS is different from Chao's approach[3] where it is merged with the fission term on the RHS. Since there are nonfissile regions in the core, however, using the pseudo absorption cross section which would appear everywhere would be more physical. The two components of the dynamic frequency, namely, the single value of the amplitude frequency and the space dependent values of the shape frequency can be obtained in a systematic way iteratively to make $k_D = 1$. This is similar to the critical boron search process except that the shape frequency can be obtained as follows using the eigenfunction of the eigenvalue problem satisfying the normalization condition of Eq. (8) as:

$$\omega_S^{n,l}(x) = \frac{1}{\Delta t_n} \ln \frac{\hat{\phi}^{(l)}(x,t_n)}{\hat{\phi}(x,t_{n-1})} \quad . \qquad (11)$$

where l is the iteration index for forming the eigenvalue problem of Eq. (10).

The precursor density at the end of the time step is updated as follows in terms of the total frequency consisting of the l-th amplitude and shape frequencies:

$$C_i^l(x,t) = e^{-\lambda_i \tau}\left(C_i(x,t_{n-1}) + \beta_i p(t_{n-1})\hat{\phi}(x,t_{n-1})\int_0^{\tau} \nu\Sigma_f e^{(\omega^{n,l}(x)+\lambda_i)t'} dt'\right) \qquad (12)$$

and the precursor frequency, $u_i^{n,l}(x)$, can be updated by using the inverse relation of Eq. (9).

3. Performance Examination

The test problem is a one-dimensional, one-group rod ejection problem in a 400cm high core in which the control rod having worth of 1.5$ is ejected in 0.1 sec. The reference solution was obtained with the Crank-Nichosen method with exponential transform (CNET) using 0.1 msec. The spatial discretization was done with the finite difference method employing the mesh size of 1cm.

First of all, the amplitude frequency and the shape frequencies were obtained from the solution to examine dependence of the dynamic frequencies on the time step sizes used to evaluate those frequencies by using the inverse relation of Eq. (4). It is noted that in Fig. 1 that the amplitude frequency and the shape frequencies at three axial positions are very well measured with even a large time step size of 10 msec.

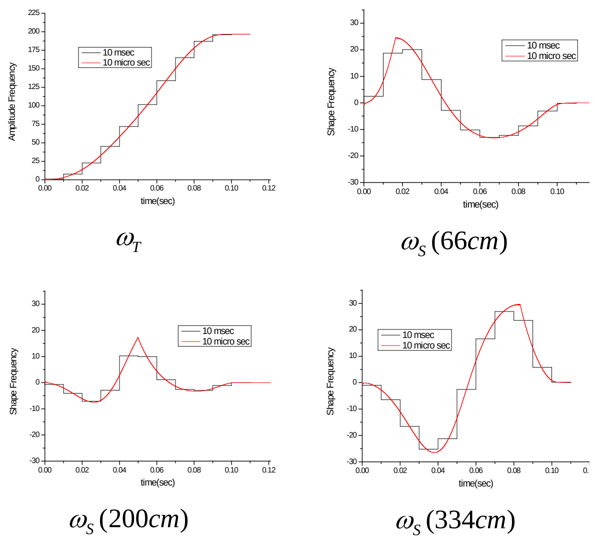

Fig. 1. Dynamic frequencies obtained from reference.

The result of the SCM calculation is now compared with those of the CNET solutions obtained with various time step sizes. As shown in Fig. 2 which shows the behavior of the relative core power, the CNET method performs well with the time step sizes upto 10 msec, but the accuracy deteriorates with 20 msec. The SCM results, however, are good with 20 msec and even the 50 msec case looks reasonable. This demonstrates the capability of the SCM in using large time step sizes. The normalized flux shape obtained when the control rod is ejected by 40% is now compared in Fig. 3. It is shown that the shape obtained with SCM with 10 msec is slightly worse than that of the CNET with 10 msec, but still the agreement is good.

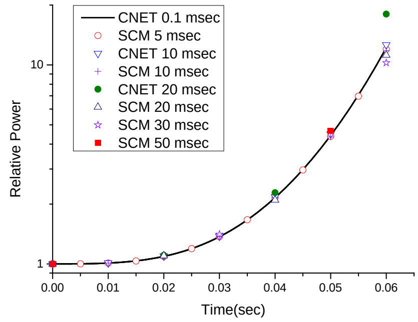

Fig. 2. Comparison of relative core power changes

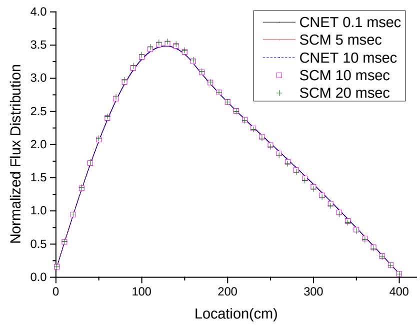

Fig. 3. Comparison of the normalized flux shapes

4. Conclusions

The stiffness confinement method for spatial kinetics was refined with the pseudo absorption term representing the dynamic frequencies. It was verified that the proposed SCM works much better than the Crank-Nicholsen method with exponential transform in that time step sizes larger than 20 msec can be using in a super prompt-critical transient involving 1.5$ reactivity insertion. Since this method uses the steady-state eigenvalue solution framework, this method can be effectively used in transient direct whole core transport problems in which sufficiently large time step sizes are required to save excessive computing time.

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