Performance Evaluation of Power Spectral Density Method for Subcriticality Monitoring of Model Reactor Problem

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1. Introduction

Authors presented a feasibility study of pulsed neutron source, Rossi-alpha, and Feynman-alpha methods for subcriticality monitoring of the fast spectrum Godiva model [1-3]. The observations from that study are as follows: First, the accuracy of those noise analysis methods increases as the core approaches a less critical state. Second, when the detected fission count signals are contaminated with random noise, it is still possible to achieve reasonable accuracy of k_{eff} prediction. This paper presents the results of the power spectral density (PSD) method applied to a thermal spectrum Godiva model for two cases. In the first case, the McCARD-generated pure fission count signals are used for the noise analysis, and in the second case the detector signals are contaminated with random noise whose magnitude is one-fourth of the average of the detector signals.

2. Power Spectral Density Method

The PSD method is theoretically based on the Rossi-alpha method which uses the probability representing auto-correlation of the detector counts in time as follows [3,4]:

$$P(\tau) = F^2\varepsilon^2 + \frac{F\varepsilon^2 D_v(1-\beta)^2}{2(\beta-\rho)\Lambda}\exp(-\alpha\tau) \qquad (1)$$

where F is the average fission rate, τ is the time difference between two detecting time points ($=t_2-t_1$), ε is the detector efficiency, and D_v is the Diven's factor, which is 0.7966 for U-235. The PSD method uses the Fourier transform of the fitted curve in Rossi-alpha analysis, which can be expressed as a function of angular frequency, ω, with unknown coefficients A and B as follows [5].

$$P(\omega) = \int_{-\infty}^{\infty} e^{-i\omega\tau}P(\tau)d\tau \approx A + \frac{B}{\alpha^2+\omega^2} \qquad (2)$$

The decay constant, α, can be calculated by fitting the approximated curve of Eq. (2) to the Fourier function from the Rossi-alpha analysis. Once the value of α is obtained, then the multiplication factor of a system can be calculated as follows:

$$k = \frac{1}{1-\$\cdot\beta} \qquad (3)$$

$$\text{where } \alpha = \alpha_c(1-\$) \text{ and } \alpha_c = \frac{\beta}{\Lambda}$$

and where β is the delayed neutron fraction, Λ is the generation time, and \$ is reactivity in units of β.

3. Application Results

The Godiva model, consisting of a spherical core with isotopes of H-1, U-235 and U-238 and H_2O reflector, was designed as a model problem to test the feasibility of the PSD method. The radii of the core and reflector are 8.741cm and 30cm, respectively, and the density of the core is 18.74g/cc. Table I shows the kinetic parameters of the reactor model.

Table I. Kinetic Parameters of Thermal Godiva Problem

Case	U-235 Enrichment (wt%)	β	Λ (s)	α_c (s^{-1})
A	0.97	6.81E-3	1.02E-5	665
B	1.10	6.89E-3	9.18E-6	751
C	1.25	6.84E-3	8.24E-6	830
D	1.37	6.88E-3	7.60E-6	905

Monte Carlo simulations with the McCARD code [6] were performed for the four cases in Table I. The first set of simulations was performed in a McCARD criticality mode to calculate the reference k_{eff} values for each case. In the second set of simulations, the neutron sources were injected into the core randomly over the simulation time for analysis with the PSD method.

3.1 Analysis with Pure Signals

The auto-correlation in the frequency domain was plotted in Figure 1 as blue circles from the McCARD-generated data. From the red fitted line, k_{eff} was estimated to be 0.90217, whereas the McCARD reference k_{eff} is 0.90428±0.00014. The result of each case is shown in Table II.

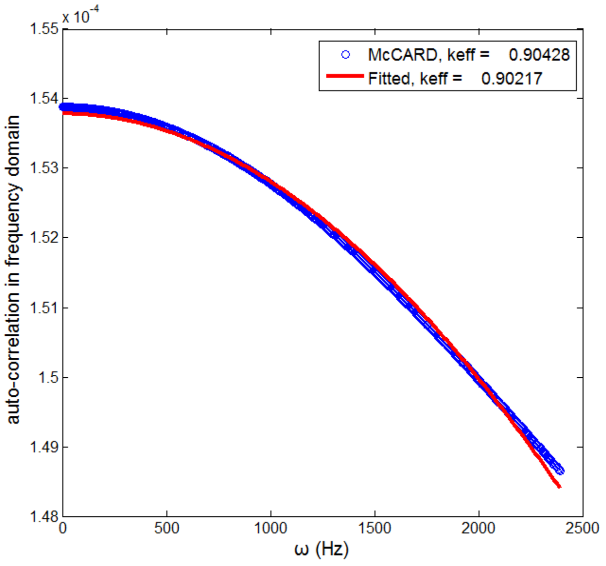

Fig. 1. Analysis Result of PSD Method for the Case B

Table II. Summary of k_{eff} values of thermal Godiva model

Case	Reference	PSD
A	0.85269±0.00013	0.85414
B	0.90428±0.00014	0.90217
C	0.95566±0.00015	0.95791
D	0.99269±0.00016	0.99009

3.2 Analysis with Noise Contamination

The McCARD-generated signals are contaminated with random noise whose magnitude is one-fourth of the average of the detector signals as follows:

$$\tilde{C}(i) = C(i) + R \times \xi_i \qquad (5)$$
$$where \;\; R = \frac{1}{2N}\sum_{i=1}^{N} C(i)$$

where ξ_i is a random number between 0 and 1, N is the total number of time bins, $\tilde{C}\,(i)$ is the fission count detector signal in the *i-th* time bin used for the analyses, and *C(i)* is the pure detector signal before the contaminations. Analyses were repeated twenty times in order to obtain an average k_{eff} and standard deviations for more stable results. Other conditions, except the noise contamination, were identical with the previous analysis. The result of each case is shown in Table III.

Table III. Summary of k_{eff} values of thermal Godiva model with contaminated detector signals

Case	Reference	PSD
A	0.85269±0.00013	0.83321 (3845)*
B	0.90428±0.00014	0.92167 (3020)
C	0.95566±0.00015	0.95829 (391)
D	0.99269±0.00016	0.98349 (50)

* : Standard Deviation (pcm) of the twenty times of analyses with noise contamination

4. Conclusions

In this paper, the thermal spectrum Godiva model was used for analysis with the power spectral density (PSD) method to evaluate the feasibility of subcriticality monitoring. It is observed that the PSD method shows high accuracy with the pure fission count signals. Also, reasonable accuracy is shown for each case when the signals are contaminated with arbitrary noise. However, for one of the twenty repetitions of case B, it is noticed that the results of case A and B are unstable because their standard deviations of twenty repetitions are too big. In comparison with Rossi-alpha analysis [3], the standard deviation of each case shows higher values, which seems to be caused by the fact that the PSD reuses the results of the Rossi-alpha method. Further study is needed to reduce such instability of the PSD method due to the large standard deviations of repeated analyses with the contaminated signals.

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