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Uncertainty Quantification of Physical Model for Best Estimate Safety Analysis

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1. Introduction

Models of complex physical systems usually involve epistemic uncertainty which arises from the inability to specify an exact value for a parameter that is assumed to have a constant value in the reference calculation. Epistemic uncertainties characterize a degree of belief regarding the location of the appropriate value of each parameter, which leads to the uncertainties on the responses reflecting a corresponding degree of belief regarding the location of the appropriate response values.

The uncertainties on the input parameters/physical models can be reduced if experimental data are properly used for a Bayesian based model calibration [1]. In this work, the model calibration was done to reduce the simulation code's input parameters' uncertainties, and subsequently simulation code's prediction uncertainties of design constraining responses. Each parameter/physical Model's fidelity was identified as well to determine major sources of the modeling uncertainty. This analysis is important in deciding where additional efforts should be given to improve our simulation model.

2. Description of the actual work

Following the Bayesian approach, a posteriori distribution for the parameter vector **p** can be derived using the a priori distribution of the parameters and the likelihood function, i.e., probability distribution of the observables [2]. If the distributions of the parameters and observables are Gaussian and the sensitivity equations are mildly nonlinear, deterministic approach, based upon a first order truncated Taylor series for the responses, was utilized to determine the a posteriori mean values and standard deviations of the parameters. This was done using sensitivity coefficients, i.e. assuming the sensor responses to input parameters perturbations were linear. To address mild nonlinearity, sensitivity coefficient values were redetermined linearizing about the a posteriori input parameter values and inverse theory once again used to obtain updated a posteriori input parameters values. These linearization iterations were continued until convergence. Utilizing a posteriori input parameters' values and uncertainties, a posteriori uncertainties of the limiting system responses can be determined by propagating the parameter uncertainties through the simulation model. This was done by using the Safety and Performance Analysis Code (SPACE) [3] developed at multiple research institutes to predict thermal hydraulic system responses of nuclear power plants.

Note that for certain transients, nonlinear and discontinuous behaviors can be observed. The deterministic approach is inappropriate to treat this behavior due to the nonlinear relationship between the system responses and the parameters, hence the potential for a non-Gaussian nature of the a posteriori distributions. This provides motivation that the transients that generate nonlinear system responses be differentiated from those that behave relatively linearly. To address the nonlinear responses in determining the a posteriori distributions of the parameters, Markov Chain Monte Carlo (MCMC) simulation was conducted, which seeks to determine the steady state Markov distribution by generating Markov chains which coincide with the target distribution, i.e. the a posteriori distribution of the parameters [4]. MCMC has proven effective for nonlinear response problems with multiple parameters to adjust. However this method is not applicable if the simulation model requires substantial CPU time to execute due to the computational burden.

3. Result

Employing Bennett's heated tube test results [5] and Becker's post Critical Heat Flux (CHF) experimental data [6], including the introduction of sensor errors consistent with the sensor signals known uncertainties, a posteriori distributions of the parameters were determined for the linear system. It was shown as expected that a larger reduction in uncertainty can be achieved for the parameters if data from multiple experiments are properly utilized for the analysis. The best estimated mean value and standard deviation, for example, for the critical heat flux are 0.846 and 0.0158, respectively, while its a priori mean value and standard deviation are 1.0 and 0.35, respectively.

The MCMC simulation was also completed for the Bennett and Becker experimental data. Figures 1 and 2 present the a posteriori distributions of the selected parameters computed using about 3,500 MCMC samples. Uncertainties are observed to be reduced, but non-Gaussian distributions occur due to the nonlinearity of the system. Figure 1 shows the uncertainty on the interfacial heat transfer coefficient in inverted slug flow was not reduced very much by the model calibration since the parameter does not affect very much the system. In this case the experiments performed do not provide enough information to determine the degree of belief regarding the location of the parameter. Figure 2 however, shows the uncertainty on the critical heat flux model was reduced substantially since the parameter plays important role in achieving better agreement between measured and predicted sensor response values.

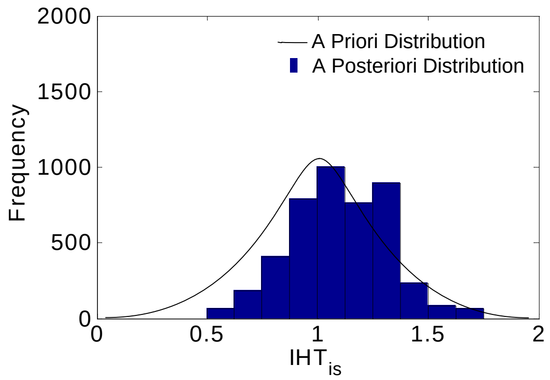

Fig 1. A priori and a posteriori distributions of the interfacial heat transfer coefficient in inverted slug flow

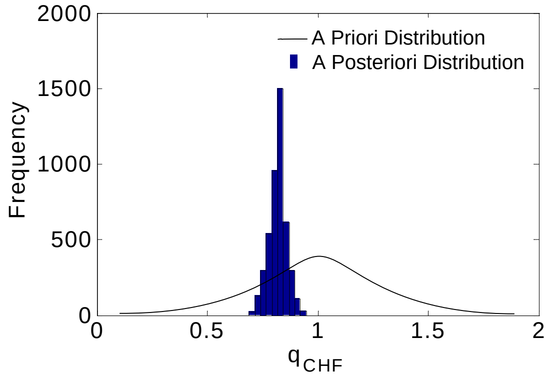

Fig 2. A priori and a posteriori distributions of the critical heat flux value

4. Conclusion

The goal of this work is to develop higher fidelity model by completing experiments and doing uncertainty quantification. Thermal hydraulic parameters were adjusted for both mildly nonlinear and highly nonlinear systems, and their a posteriori parameter uncertainties were propagated through the simulation model to predict a posteriori uncertainties of the key system attributes. To solve both highly nonlinear as well as mildly nonlinear problem, both deterministic and probabilistic methods were used to complete uncertainty quantification. To accomplish this, the Bayesian approach modified by regularization is used for the mildly nonlinear problem to incorporate available information in quantifying uncertainties. The a priori information considered are the parameters and the experimental data together with their uncertainties. The results indicate that substantial reductions in uncertainties on the system responses can be achieved using experimental data to obtain a posteriori input parameters' uncertainty distributions. The MCMC method was used for the highly nonlinear transient. Due to the computational burden, this method would not be applicable if there are many parameters, but it can provide the best solution since the algorithm does not approximate the responses while the deterministic approach assumes linearity of the responses with regard to dependencies on the parameters. Using MCMC non-Gaussian a posteriori distributions of the parameters with reduced uncertainties were obtained due to the nonlinearity of the system sensitivity equations.

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