Volume-Averaged Momentum Equations and Wall Drag for Disperse Phase

Byoung Jae Kim [a], Jungwoo Kim [b], Kyung Doo Kim [a*]
[a]*Korea Atomic Energy Research Institute, Dukjing-dong, Yuseong-gu, Daejeon*
[b]*Seoul National University of Science & Technology, 232 Gongneung-ro, Nowon-gu, Seoul*
[*]*Corresponding author: kdkim@kaeri.re.kr*

1. Introduction

The state-of-the-art CHATHARE, TRACE, and COBRA codes do not impose any wall drag on the disperse phase based on observation that most droplet/bubbles do not touch the wall. RELAP5 code considers the wall drag for the dispersed phase based on the wetted fraction concept. The hydrodynamic behavior of the disperse phase is of importance in the nuclear safety analysis. In particular, the droplet flow rate in horizontal hot legs is closely associated with steam binding and stream generator u-tubes. Despite its importance, it is still questionable how to impose the wall drag on the disperse phase.

The two-fluid equations are formulated based on the interpenetrating continua concept that two phase fluids occupy simultaneously the same space, which are obtained by applying the time and/or space averaging process to the local instantaneous conservation equations (Drew 1983, Ransom 1994, Ishii 2011). Each term in the averaged equations has a physical meaning or contribution, but it must be interpreted with care. The averaged equations sometimes fail to explain physical phenomena. For example, Podowski (2009) indicated that ignoring the wall drag on bubbles causes the bubble velocity to be faster than the water velocity for steady and horizontal bubbly flow. He stated that the total wall friction should be apportioned in proportion to the each phase volume fraction without explanation of any physical reasons. Moreover, the discussion was made centering around fully-developed flow in a straight pipe.

In this paper, volume-average momentum equations are newly derived for disperse flow. The wall drag for the disperse phase is discussed.

2. Equation of Particle Motion

To gain insight into hydrodynamics for the disperse phase, we start with the equation of a single particle motion. Maxey and Riley (1983) derived the equation of a small particle motion under non-uniform and unsteady flows at low Reynolds numbers,

$$\rho_d \frac{d\vec{v}_d}{dt} = \rho_p \vec{g} + \rho_c \left(\frac{D\vec{v}_c}{Dt} - \vec{g} \right) + \vec{f}_d + \vec{f}_a + \vec{f}_h, \tag{1}$$

where the subscript *d* and *c* stand for the disperse phase and the continuous phase, respectively. The second term on RHS (right hand side) accounts for the force by stresses due to undisturbed (or ambient) continuous phase. In Eq. 1, $\vec{v}_c$ is undisturbed continuous phase velocity. In other words, it is the continuous phase velocity evaluated at the particle center in the hypothetical situation in which the particle were absent. The last three terms on RHS are the hydrodynamic forces (drag, added mass, and history forces) by the disturbed flow stress, which are functions of the particle velocity and disturbed continuous phase velocity. From Eq. 1, it can be readily shown that a bubble is faster than water in a contraction and viceversa in an expansion. It can also be explained that a droplet is slower than gas in a contraction and viceversa in an expansion. Although the equation was developed for a solid particle, the basic concept can be utilized for a fluid particle.

3. Volume-Averaged Momentum Equation

3.1 Multi-Dimensional Equation

In this study, a “particle” means a fluid particle such as droplet and bubble. Detailed averaging steps are not described in this paper. Important procedures and assumptions are summarized below.

- The surface tension effect is not considered, nor is the phase change. In this case, the interface jump condition is greatly simplified and the two phasic interface drags sum to zero.
- The continuous phase stress ($\boldsymbol{\sigma}_c$) are divided into the undisturbed stress ($\boldsymbol{\sigma}_c^0$) and the disturbed stress ($\boldsymbol{\sigma}'_c$) by particles.

$$\boldsymbol{\sigma}_c = \boldsymbol{\sigma}_c^0 + \boldsymbol{\sigma}'_c = (-p_c^0 \mathbf{I} + \boldsymbol{\tau}_c^0) + (-p'_c \mathbf{I} + \boldsymbol{\tau}'_c) \tag{2}$$

- In averaging context, $\nabla \cdot \langle \sigma_c^0 \rangle \approx \nabla \cdot \langle \sigma \rangle$ is reasonably assumed. Here, σ may be interpreted as the stress of the mixture. As a result, the following equations are formulated.

$$RHS_d = -\alpha_d \nabla \langle p \rangle + \alpha_d \nabla \cdot \langle \boldsymbol{\tau} \rangle + F_i + \alpha_d \rho_d \vec{g} \tag{3}$$

$$RHS_c = -\alpha_c \nabla \langle p \rangle + \alpha_c \nabla \cdot \langle \boldsymbol{\tau} \rangle - F_i + \alpha_c \rho_c \vec{g} \tag{4}$$

where F_i accounts for the hydrodynamic forces. Note the second terms in Eqs. 3 and 4. The volume fractions are outside of the divergence operator. This feature differs from the standard multi-fluid equation in which the volume fractions are inside the divergence operator. Equations 3 and 4, however, are consistent with the equations developed for disperse flow by Sirignano (2005), Moraga et al. (2006), Prosperetti (2009), and Crowe et al. (2011). The first terms on RHSs in Eqs. 3 and 4 correspond to the second term in RHS of Eq.1.

Let us look at the standard two-fluid equation for phase k without gravity.

$$RHS_k = -\nabla\left(\alpha_k \langle p_k \rangle_k\right) + \nabla \cdot \left(\alpha_k \langle \boldsymbol{\tau}_k \rangle_k\right) + M_k \tag{5}$$

$$M_k = M_k^d - \left(-\langle p_k \rangle_i + \langle \tau_k \rangle_i\right)\nabla \alpha_k \tag{6}$$

M_k is the momentum transfer on phase k. M_k^d is refered to as the generalized interface force, which corresponds to F_i in Eqs. 3and 4. Equation 5 can be rewritten by

$$\begin{aligned} RHS_k = &-\alpha_k \nabla \langle p_k \rangle_k + \nabla \cdot \left(\alpha_k \langle \boldsymbol{\tau}_k \rangle_k\right) + M_k^d \\ &+ (\langle p_k \rangle_i - \langle p_k \rangle_i)\nabla \alpha_k - \langle \tau_k \rangle_i \nabla \alpha_k \end{aligned} \tag{7}$$

Ishii and Hibiki (2011) and Enwald (1996) neglected the last term for disperse flow. Also, Drew (1983) does not consider the last term. Consequently, if both the average pressure and viscous stress in the bulk fluid and at the interface are approximately the same, the standard momentum equation is given by

$$RHS_k = -\alpha_k \nabla \langle p_k \rangle_k + \nabla \cdot \left(\alpha_k \langle \boldsymbol{\tau}_k \rangle_k\right) + M_k^d \tag{8}$$

3.2 One-Dimensional Equation

One can obtain the one-dimensional equations in a similar manner.

$$RHS_d = -\alpha_d \partial \langle p \rangle / \partial x + \alpha_d wall_{fric} + F_i + \alpha_d \rho_d \vec{g} \tag{9}$$

$$RHS_c = -\alpha_c \partial \langle p \rangle \partial x + \alpha_c wall_{fric} - F_i + \alpha_c \rho_c \vec{g} \tag{10}$$

In the above, $wall_{fric}$ is the total pressure drop by wall friction. This result indicates that the phasic wall drag is proportional to the phasic volume fraction.

4. Application to SPACE code

Horizontal disperse flows were simulated in a straight pipe, contraction, and expansion, respectively. The test was performed at the 10 bar saturation condition. The pipe diameter is 2cm and the length is 5.94m.

Figures 1 and 2 show the velocity variations along downstream for bubbly flow in the expansion. In Fig.1, no wall drag is imposed on the bubble, however in Fig.2, wall drag is imposed following Eqs. 8 and 10. The flow area is increased by 25% at 0.5m. At the inlet, the bubble velocity and water velocity are set to the same value, and the void fraction is 0.05. Ideally, in the section x=0~0.5m, the bubble and water velocities must be the same. As seen, this behavior is observed in Fig. 2. However, the bubble velocity is shown to be considerably faster than the water velocity in Fig. 1. Moreover, the bubble is slower than water in some distance from x=0.5m, after that, it becomes faster than water again in far downstream. On the other hand, in Fig. 2, water is faster in the expansion region and two velocities become close as going downstream, which is physically correct. This behavior can be explained by Eqs. 9 and 10. We performed direct numerical simulations for a single fluid particle in a pipe, contraction, and expansion, respectively. Though the results are provided in this paper, the velocity behaviors are qualitatively similar to those in Fig. 2.

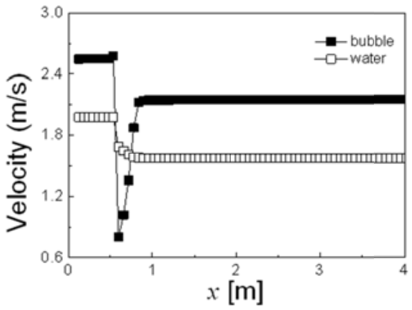

Fig. 1. No wall drag is imposed on the bubble phase

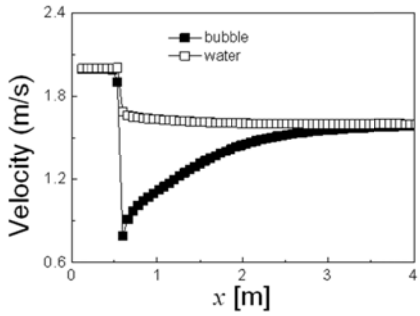

Fig. 2. Wall drag is imposed on bubble by Eqs. 8 and 9

3. Conclusions

Volume-averaged momentum equations have been formulated for disperse flow. The proposed equations show more physical results.

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