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Mathematical Methodology for New Modeling of Water Hammer in Emergency Core Cooling System

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1. Introduction

It is a matter of general knowledge to analyze the water-hammer. In engineering insight, the water-hammer study has carried out through the experimental work and the fluid mechanics. In this study, a new access methodology is introduced by Newton mechanics and a mathematical method. Also, NRC Generic Letter 2008-01 requires nuclear power plant operators to evaluate the effect of water-hammer for the protection of pipes of the Emergency Core Cooling System, which is related to the Residual Heat Removal System and the Containment Spray System [1]. This paper includes modeling, the processes of derivation of the mathematical equations and the comparison with other experimental work [2].

2. Modeling

In any case, the change of pressure is transmitted equally to every point of the pipe line. When such a change causes a sudden rise of pressure, this is known as water-hammer. Its magnitude may be large with even comparatively low velocities of flow, and it is very important not only in the evaluation of the safety of a pipe line but also in the view point of regulation.

A water-hammer is similar to the state of any sudden close of valve between the opening pipes and the water-filled pipes.

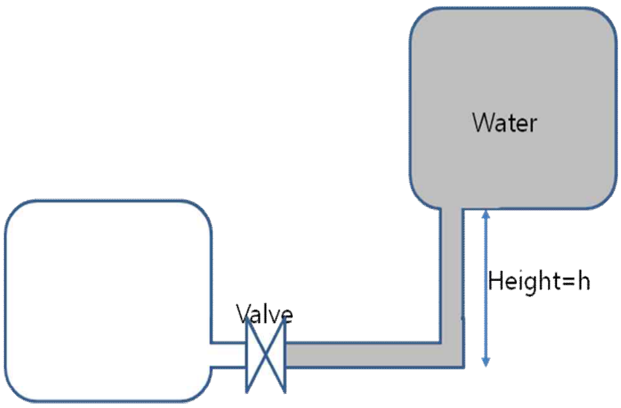

Fig.1 Initial Condition of the Conceptual Frame

If a rigid body of weight W lbs. has its velocity in any given direction changed by an amount dV ft/sec., in a time dt seconds, its acceleration is dV/dt, and the change of momentum per seconds in this direction is W/g•dV/dt in gravitational units. This characteristic is used for mathematical derivation.

3. Mathematical Process

A acceleration and a change in momentum are used to express a force in the Newton mechanics, so the force F must be applied to the body with the change of the behavior direction, its magnitude being given by the equation (1):

$$F = \left(\frac{W}{g}\right) \bullet \left(\frac{dV}{dt}\right) \tag{1}$$

If water be flowing along a rigid pipe whose cross-sectional area at any particular point is α, equation (1) to accelerate a very small element of the pipe line of length dx will be equivalent to equation(2) as below:

$$F = \left(\frac{W}{g}\right) \bullet \alpha \bullet dx \bullet \left(\frac{dV}{dt}\right) \tag{2}$$

Also equation (2) is changed to equation (3) as below:

$$dP = \left(\frac{W}{g}\right) \bullet dx \bullet \left(\frac{dV}{dt}\right) \tag{3}$$

In a pipe of length L ft. the difference of pressure at the two ends, equation (3) is mutated to equation (4).

$$P = \left(\frac{W}{g}\right) \bullet \int_0^L dx \bullet \left(\frac{dV}{dt}\right) \tag{4}$$

Here, P is the summation of the segments of pressures in branch pipes.

Then the loss of pressure from entrance to exit, as statistic flow of velocity V ft. per sec., is written by equation (5) as below:

$$P = \left(\frac{WV^2}{2g}\right) \bullet \left(1 + \frac{fL}{m}\right) \tag{5}$$

Here, the first term is the head loss in feet, with changing from pressure to kinetic energy, the second term is the head loss of pipe friction.

Under the condition of no flow, the equation (5) can be changed to equation (6).

$$P = \frac{W}{g}\left(L\frac{dV}{dt} + \frac{V^2}{2} \bullet \left(1 + \frac{fL}{m}\right)\right) \tag{6}$$

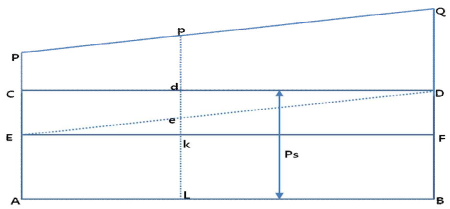

Fig.2 Pressure change (AB=time axis, AP=Pressure axis)

Fig.2 shows a state of water-hammer when dv/dt is constant. AC(Ps) represents the static pressure of the full-open valve and the statistic flow of velocity v1.

From next process, we get the method to deal with the previous considerations. For Fig. 2, we consider following parameters

$$EC=\left(\frac{Wv1^2}{2g}\right)\bullet\left(1+\frac{fL}{m}\right) \quad (7)$$

$$ed=\left(\frac{WV^2}{2g}\right)\bullet\left(1+\frac{fL}{m}\right) \quad (8)$$

$$EP=ep=DQ=\left(\frac{WL}{g}\right)\bullet\left(\frac{dV}{dt}\right) \quad (9)$$

$$pd=-\frac{W}{g}\left(L\frac{dV}{dt}+\frac{V^2}{2g}\left(1+\frac{fL}{m}\right)\right) \quad (10)$$

$$p=Ps-\frac{WL}{g}\left(\frac{dV}{dt}\right) \quad (11)$$

Here, the flow of water-hammer is expressed as below:

$$\frac{d}{dx}\left(\frac{P}{W}+\frac{V^2}{2g}+z\right)=-\left(\frac{1}{g}\frac{dV}{dt}+f\frac{V^2}{2gm}\right) \quad (12)$$

This is Bernoulli's modified equation to explain the accelerated motion of a viscous fluid.

Continuously, integrating both sides of equation (12) with respect to L, equation (12) is changed to equation (13):

$$\frac{P}{W}+\frac{V^2}{2g}+z=-\frac{1}{g}\int_0^L\frac{dV}{dt}dL-\frac{f}{2gm}\int_0^L V^2dL+C \quad (13)$$

Here, initial conditions are below:

P=p1, z=z1, $\int_0^L\frac{dV}{dt}dL=0$, V=Va, $\int_0^L V^2dL=0$

Again, final conditions are below:

P=p0, z=z0, $\int_0^L\frac{dV}{dt}dL=\mathrm{L}\frac{dV}{dt}$, $\int_0^L V^2dL=\mathrm{Va}^2\mathrm{L}$

Therefore, inserting these conditions, final equation is follows:

$$a_0\frac{dV_0}{dt}+V_0\frac{da_0}{dt}=\frac{a}{L}\left(\frac{V_0^2}{2}\left(1+\frac{a_0^2}{a^2}\frac{f}{2gm}\right)-gh\right) \quad (14)$$

The solution of equation (14) is follows:

$$ka_0\frac{dV_0}{dt}=V_0^2-bV_0-c \quad (15)$$

Equation (15) is calculated by integral.

$$\int\frac{dV_0}{V_0^2-bV_0-c}+D=\frac{1}{k}\int\frac{dt}{a_0}=\frac{1}{ka1}\int\frac{dt}{t} \quad (16)$$

From (16), we get equation (17).

$$\frac{1}{2c}\log\frac{c+V_0}{c-V_0}+D=\frac{t}{k} \quad (17)$$

V_0=0, t=0, we have D=0 and the velocity of the face of pipes at an instant t seconds after the valve opens is below:

$$V_0=c\frac{1-e^{-\frac{2c}{k}}\bullet t}{1+e^{-\frac{2c}{k}}\bullet t} \quad (18)$$

4. Results and Discussion

Table 1. Condition in Comparing with Experiment Study

No.	Experiment Conditions in University of Wisconsin[2]	
1	Drive-pipe	85.35ft. long
2	diameter	2.05 inch
3	Friction factor	0.09
4	Constant K	3.97

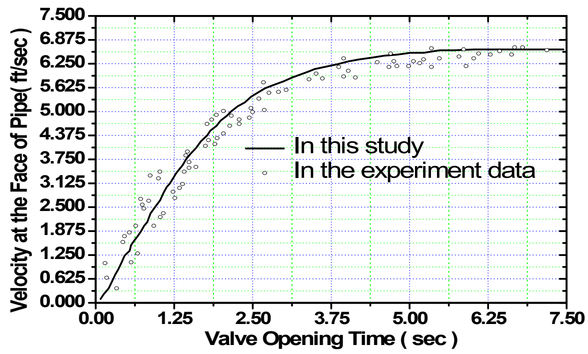

Fig.3 Comparison of water-hammer effects between this calculation and other experiment study

To prove this study, the mathematical methodology is compared with experiment results of the University of Wisconsin. Comparison is carried out in the condition of Table 1.

Fig.3 shows that the velocity at the face of pipe in water-hammer is in good agreement with experiment data.

5. Conclusions

To analyze the effect of water-hammer, this mathematical methodology is carried out. This study is in good agreement with other experiment results as above (Fig.3). This method is very efficient to explain the water-hammer phenomena.

REFERENCES

[1] USNRC, Managing Gas Accumulation in Emergency Core Cooling, Decay Heat Removal, and Containment Spray Systems, Generic Letter 2008-01, 2008.

[2] Harza, L. F., Bulletin University of Wisconsin. Pp. 152, 157(1908).