Advanced Model of Spray Droplet Shape in Containment

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1. Introduction

A containment spray system is to remove fission products in atmosphere in containment. The shape of spray droplets has the pattern of behavior like ellipse objects or rain droplets. In this field, Lee of KHNP (Korea Hydro Nuclear Power) have carried out the shape modeling of spray droplets in 2012[1]. In this study, to promote Lee's model, the new advanced methodology is introduced. This study is based on the mathematical equation [2,3]. Here, the induced results are used to calculate a volume and a surface area of spray droplet. Also, these are compared with Clift's experimental study in condition of non-sphere in falling mechanics [2]. The surface of spray droplets is efficiently calculated using new methodology. The mathematical technique in this work is based on ellipse integral and ellipse geometry.

2. Methodology

In this section, to improve Lee's methodology, a ellipsoid at the centered coordinate is applied. Directly, three-dimensional spray droplets shape is simulated.

2.1 Surface area of spray droplet in three dimensions

Spray droplet shape is similar to flat-ellipsoid and also its shape on the eccentricity e.
The form and the surface strongly affect on the function of removing fission products in containments.
Generally, for the case in which two axes are equal to b=c, the surface is generated by rotation around the x-axis of the half-ellipse of equation (1) with Y>0.

$$\frac{X^2}{a^2}+\frac{Y^2}{b^2}=1 \tag{1}$$

On that half-ellipse, $dy/dx = -b^2x/(a^2y)$, and hence the surface area of the spheroid is written as below:

$$A=2\int_0^a 2\pi y\sqrt{1+\frac{b^4x^2}{a^4y^2}}dx=4\pi\int_0^a y\sqrt{y^2+\frac{b^4x^2}{a^4}}dx \tag{2}$$

$$A=4\pi b\int_0^a\sqrt{1-\frac{x^2}{a^2}+\frac{b^2}{a^2}\frac{x^2}{a^2}}dx \tag{3}$$

$$A=4\pi ab\int_0^1\sqrt{1-\left(1-\frac{b^2}{a^2}\right)u^2}\,du \tag{4}$$

$$A=4\pi ab\int_0^1\sqrt{1-\delta u^2}\,du \tag{5}$$

Where, u=x/a and $\delta=1-\frac{b^2}{a^2}$, which is used for replace integral.
Where, this equation can be selected by three options as below:
Option1: a>b

$$A=2\pi b\left(a\times\frac{\arcsin\sqrt{\delta}}{\sqrt{\delta}}+b\right) \tag{6}$$

Option2: a=b

$$A=2\pi b(a+b)=4\pi a^2 \tag{7}$$

Option3: a<b

$$A=2\pi b\left(a\times\frac{\operatorname{arcsinh}\sqrt{-\delta}}{\sqrt{-\delta}}+b\right) \tag{8}$$

Here, due to the falling spray droplet is crashed so the option1 is selected.
Continuously, option 1 is going on calculating the surface area of ellipse spray droplets.
Applying Power series into equation (6), it is changed as below:

$$A=\pi\left[2a^2+b^2\frac{1}{\sqrt{-\delta}}\log\left(\frac{1+\sqrt{-\delta}}{1-\sqrt{-\delta}}\right)\right] \tag{9}$$

$$A=2\pi b\left(a\left[1+\frac{1}{6}\delta+\frac{3}{40}\delta^2+\frac{5}{112}\delta^3+\cdots\right]+b\right) \tag{10}$$

$$A=4\pi ab\int_0^1(1-\delta u^2)^{1/2}du \tag{11}$$

$$A=4\pi ab\int_0^1\left(1-\frac{1}{2}\delta u^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\delta^2u^2-\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}\delta^3u^6+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{4!}\delta^4u^8+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)}{5!}\delta^5u^{10}+\cdots\right)du \tag{12}$$

Integrating equation (12), the results is written as equation (13).

$$A=\left(1-\frac{1}{2}\frac{\delta}{3}-\frac{1}{2}\frac{\delta^2}{3}-\frac{1}{16}\frac{\delta^3}{7}-\frac{5}{128}\frac{\delta^4}{9}-\frac{7}{256}\frac{\delta^5}{11}\cdots\cdots\right) \tag{13}$$

Here, equation (13) is resulted from Power series of $\frac{\arcsin\sqrt{\delta}}{\sqrt{\delta}}$.

2.2 Condition of Ellipse

In previous section, the calculation methodology of the surface area of spray droplet is introduced. But, in the surface area of ellipse, the discriminant is required to select the general form of ellipse. The discriminant have been introduced by Seung Chan Lee of KHNP in 2012.

Therefore, the condition discriminant of ellipse is achieved using Seung Chan Lee's methodology. The equation of the discriminant is written as below [1]:

$$a\,x^2 + 2b\,xy + c\,y^2 + 2\,dx + 2\,fy + g = 0 \qquad (14)$$

Here, the shape of ellipse must be satisfied in condition of equation (15) and equation (16)[1].

$$\Delta = \begin{vmatrix} a & b & d \\ d & c & f \\ d & f & g \end{vmatrix},\ J = \begin{vmatrix} a & b \\ b & c \end{vmatrix},\ I = a + c \qquad (15)$$

$$\Delta \neq 0,\ \ J > 0,\ \frac{\Delta}{I} < 0,\ a \neq c,\ J = ac - b^2 \neq 0 \quad (16)$$

The center of the ellipse (x_0, y_0) is given by

$$x_0 = \frac{cd - bf}{b^2 - ac} \qquad y_0 = \frac{af - bd}{b^2 - ac} \qquad (17)$$

The semi-axes lengths are below [1]:

$$a' = \sqrt{\frac{2(af^2 + cd^2 + gb^2 - 2bdf - acg)}{(b^2 - ac)\left[\sqrt{(a-c)^2 + 4b^2} - (a+c)\right]}} \qquad (18)$$

$$b' = \sqrt{\frac{2(af^2 + cd^2 + gb^2 - 2bdf - acg)}{(b^2 - ac)\left[-\sqrt{(a-c)^2 + 4b^2} - (a+c)\right]}} \qquad (19)$$

Where a, b, c, d, e, f and g are random variables ranged from 0 to 1.

2.3 Determination of Surface area

Section 2.1 and section 2.2 are very important to determine the surface area of spray droplets.

In the section 2.1, using the method of calculating the surface area of ellipse, an arbitrary surface area is calculated. Using section2.1 and section 2.2, the refined methodology is generated.

If (x, y, z) coordinate system is changed to (φ, θ) coordinate system, $\cos\theta = z/c$.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \sin^2\theta \qquad (20)$$

$$\frac{x^2}{(a\sin\theta)^2} + \frac{y^2}{(b\sin\theta)^2} = 1 \qquad (21)$$

Letting $\cos\varphi = y/(b\sin\theta)$, so that $\sin\varphi = x/(a\sin\theta)$.

That is written as below:

$$X = a\sin\theta\sin\varphi \qquad , y = b\sin\theta\cos\varphi \qquad (22)$$

Using the differential factor of (22), (22) is changed into (23).

$$(dxdy) = ab\sin\theta\cos\theta\,d\theta\,d\varphi \qquad (23)$$

$$S = ab\int_{\varphi=0}^{\pi/2}\int_{\theta=0}^{\pi/2}\sin\theta\sqrt{1 - p^2\sin\theta}\,d\theta\,d\varphi \qquad (24)$$

Here, equation (24) is modified to reflect the previous section 2.2 using equation(5) and equation(10).

Using the Power series of parameter p, we can integrate for θ from 0 to $\pi/2$.

$$\int_{\theta=0}^{\pi/2}\sin\theta\sqrt{1 - p^2\sin\theta}\,d\theta \qquad (24)$$

Power series for term (24) is generated as below:

$$\int_0^{\pi/2}\sin\theta\left(1 - \frac{1}{2}p\sin^2\theta + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}p^2\sin^4\theta - \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}p^3\sin^6\theta + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{4!}p^4\sin^8\theta + \cdots\right)du \qquad (25)$$

Using equation (25), surface area of spray droplet shape can be expressed into simple form as below:

$$A = \left(1 - \frac{1}{2}\frac{p}{3}\pi - \frac{1}{2}\frac{p^2}{3}\frac{\pi\cdot1\cdot3}{2\cdot2\cdot4} - \frac{1}{16}\frac{p^3}{7}\frac{\pi\cdot1\cdot3\cdot5}{2\cdot2\cdot4\cdot6} - \frac{5}{128}\frac{p^4}{9}\frac{\pi\cdot1\cdot3\cdot5\cdot7}{2\cdot2\cdot4\cdot6\cdot8} - \frac{7}{256}\frac{p^5}{11}\frac{\pi\cdot1\cdot3\cdot5\cdot7\cdot9}{2\cdot2\cdot4\cdot6\cdot8\cdot10}\cdots\cdots\right) \qquad (26)$$

3. Result and Discussion

From equations (5), (10), (11), (12), (18), (19), (24) and equation (25), the surface area of spray droplet is generated such as simple form of equation (26).

Fig.1 shows the spherical volume ratio for the surface area of spray droplets using the equation (26). In Fig. 1, this study is compared with Clift's experimental results of spray droplet surface. The difference between Clift's experimental results and this calculation is within 0.3%.

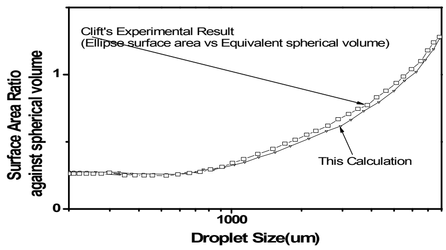

Fig. 1 Surface area ratio comparing with other study

4. Conclusions

The advanced model of the shape of spray droplet is introduced in this study.

Spray droplets are efficiently simulated using a tri-axial ellipsoid. The difference between the experimental results and advanced methodology results is within 0.3%. These results are promoted comparing with Seung Chan Lee's results of KHNP in 2012. Clift's experimental results are in good agreement with this study.

REFERENCES

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