# **On the Generation of Response Spectrum**

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### 1. Introduction

A response spectrum is simply a plot of the peak or steady-state response (displacement, velocity or acceleration) of structures with varying natural frequency that are forced into motion by the same base vibration or shock. The resulting plot can then be used to pick up (or out) the response of any linear structural system, given its natural frequency. Such a use is in assessing the peak response of buildings to earthquakes [1].

Response spectra can also be used in assessing the response of linear systems with multiple modes of oscillation (multi-degree of freedom systems), although they are accurate only for low levels of damping. Modal analysis is performed to identify relevant modes and the response in those modes. The peak response of each mode can be obtained from the response spectrum. This peak response is then combined to estimate a total response. A typical modal combination method is the square root of the sum of the squares (SRSS) provided that the modal frequencies are not closely spaced. When the modal frequencies are closely spaced, the SRSS is known to be significantly unconservative. The result of response spectrum method is typically different from that of time history method since phase information is lost in the process of generating the response spectrum.

Response spectra are very useful tools of earthquake engineering for analyzing the performance of structures and equipment in earthquakes, since most of them behave principally as simple oscillators (also known as single degree of freedom systems). Thus, if you can find out the natural frequencies of a structure considered, then the peak response of the structure can be estimated by reading the value from the ground response spectrum for the appropriate frequency. Therefore it is necessary to obtain response spectra from the time history files from seismographs for purposes mentioned above

There are many methods to calculate response spectra but they are not available. In this study, therefore, the program to compute a response spectrum from a given acceleration time history data is developed. And also the method to generate response spectra using commercial program, ANSYS [2], is suggested, which is very simple and easy to make an input deck for any format of acceleration time history file. By comparing response spectra between the program generated here and ANSYS, the two methods developed in this study are verified to be effective in generating relevant response spectra.

### 2. Theory

A response spectrum is generated by imposing the motion of the point of interest on a series of single-mass oscillator over a period of time and calculating the maximum displacement, velocity, or acceleration. Considering a single mass oscillator as shown in Figure 1 with  $M_i$  = mass of oscillator *i*,  $C_i$  = damping of oscillator *i*,  $K_i$  = stiffness of oscillator *i*,  $u_i$  = motion of oscillator *i* and  $u_b$  = motion of base, the natural frequency in the absence of damping is:

$$\omega_i = \sqrt{\frac{K_i}{M_i}} \tag{1}$$

The basic equation of motion of the oscillator can be given as:

$$M_{i}\ddot{u}_{i} + C_{i}\dot{u}_{i}^{r} + K_{i}u_{i}^{r} = 0$$
 (2)

where the relative motion of oscillator *i* is defined by  $u_i^r = u_i - u_b$ . The ratio of viscous damping is given by:

$$\xi_i = \frac{C_i}{C_{cr,i}} \tag{3}$$

where  $C_{cr,i} (= 2\sqrt{K_iM_i})$  is a critical damping coefficient. Combing Equations (1) through (3) gives the following equation which will be solved essentially as a linear transient dynamic analysis.

$$\ddot{u}_i^r + 2\xi_i \omega_i \dot{u}_i^r + \omega_i^2 u_i^r = u_b \tag{4}$$

The solution of Equation (4) for the maximum response,  $u_{i_{\text{max}}}^r$ , at various frequencies results in the spectral response curve.



Figure 1. Single mass oscillator

#### 3. Method to Generate Spectrum

The RSFTH (Response Spectrum from Time History) program is developed to compute a response spectrum from a given acceleration time history file. The time history is input from the output file of the GENTH program.

Also, the response spectra can be generated in the ANSYS run as follows;

- 1st step Build the very simple model
- 2nd step Perform transient time history analysis with base excitation of acceleration
- 3rd step Generate spectra in post-processing

## 4. Results and Discussion

The 1940 El Centro earthquake (or 1940 Imperial Valley earthquake) occurred at 21:35 Pacific Standard Time on May 18 (05:35 UTC on May 19) in the Imperial Valley in southeastern Southern California near the international border of the United States and Mexico [3]. It had a magnitude of 6.9 and a maximum perceived intensity of X (*Intense*) on the Mercalli intensity scale. It was the first major earthquake to be recorded by a strong-motion seismograph located next to a fault rupture. The earthquake was characterized as a typical moderate-sized destructive event with a complex energy release signature. It was the strongest recorded earthquake to hit the Imperial Valley, and caused widespread damage to irrigation systems and led to the deaths of nine people.

Acceleration time histories in east-west direction are shown in Figure 2 [4]. The instrument that recorded the accelerogram was attached to the El Centro Terminal Substation Building's concrete floor, and not in a freefield location. The record may have under-represented the high frequency motions of the ground because of soil-structure interaction of the massive foundation with the surrounding soft soil.



Figure 2. Acceleration time history of El Centro earthquake in east-west direction

Using the acceleration time history file, the response spectra are generated as shown in Figure 3 using the program RSFTH. Also from the ANSYS runs, the response spectra are generated as shown in Figure 3. In the ANSYS run pseudo response spectra can be generated.

The response spectra generated by RSFTH is compared with that by the commercial program ANSYS and there is no difference between them as shown in Figure 3 verifying the validity of the program RSFTH developed and the method of ANSYS suggested in this study.



Figure 3. Comparison of response spectrum generated

## 5. Conclusions

Response spectra are very useful tools in the earthquake engineering for analyzing the performance of structures and equipment under earthquakes. If natural frequencies of a structure considered are searched out, then a peak response of the structure can be estimated from the ground response spectrum corresponding to an appropriate frequency. Therefore, it is necessary to obtain response spectra using a time history files obtained from seismographs considered.

In this study, two methods to generate response spectra are presented; one is to use the RSFTH program developed in this study and the other is to use the commercial program, ANSYS. These two methods are very simple and easy to use for any format of acceleration time histories. The validity of the two methods is shown by comparing the response spectra generated by RSFTH and ANSYS.

### REFERENCES

- [1] http://en.wikipedia.org/wiki/Response\_spectrum
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- [3] http://en.wikipedia.org/wiki/1940\_El\_Centro\_earthquake
- [4] http://www.vibrationdata.com/elcentro.htm