Mode Analysis of Wake Flow from a Cylinder using Double Proper Orthogonal Decomposition

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1. Introduction

Proper Orthogonal Decomposition (POD) is a useful method that can systematically analyze a higher order system including large amounts of data set of turbulent flow fields or oscillatory motions. It can be used to determine dominant modes of complicated natural phenomena [1, 2] so that one can easily understand their basic physics. While POD can be applied to stationary phenomena, for transient cases where the main modes may evolve over time, more additional POD is helpful to obtain the information on the variance of the main modes. This is referred to the double POD (DPOD) method [3]. In this study DPOD is applied to the 2D cylinder wake flow data obtained during the start-up transient, and the physical meaning of the eigenvalues, main and shift modes, and time coefficients are examined.

2. Double Proper Orthogonal Decomposition and Its Application to Cylinder Wake

2.1 Double Proper Orthogonal Decomposition(DPOD)

The proper orthogonal decomposition (POD) decomposes the flow field data using spatial modes and temporal modes [1, 2]. For a periodically time-evolving flow field, POD can be written as shown in equation (1), and this is called SPOD (Short time POD) [3].

$$u^{(i)}(x, y, t) = \sum_{j=1}^{J} a_j^{(i)}(t)\phi_j^{(i)}(x, y) \qquad (1)$$

Here, $u^{(i)}(x, y, t)$ are flow field parameters for i^{th} cycle, $a_j^{(i)}(t)$ are the mode amplitudes of the spatial modes $\phi_j^{(i)}(x, y)$. For the case of transient flow field, the spatial modes $\phi_j^{(i)}(x, y)$ may vary as time progresses. If K bins of data set are available DPOD can be performed as the following equation (2) [3].

$$u^{(i)}(x, y, t) = \sum_{j=1}^{J}\sum_{k=1}^{K} a_{jk}^{(i)}(t)\Phi_{jk}(x, y) \qquad (2)$$

In the limiting case of $K = 1$ Eq. (2) recovers the original POD or SPOD. The different modes designated by the index j represent the physical modes as Eq. (1). On the other hand, the index k identifies the transient changes of these physical modes. For example, Φ_{11} is the mean flow mode and $\Phi_{12}, \Phi_{13}, \Phi_{14}, \cdots$ are the shift modes, which allow the POD mode ensemble to adjust for changes in the spatial modes. The shift modes modify a given physical mode to match a new flow state due to transient condition or change of flow speed, etc.

2.2 Application of DPOD to Cylinder Wake Flow

Computational fluid dynamic analysis was performed to obtain the velocity field data from two-dimensional cylinder wake flow during start-up transient. The analysis conditions are as follows: incompressible air, $Re = 10,000$, $\Delta t = 0.001s$ and MP $k - \varepsilon$ turbulence model. Flow velocity data were acquired for about 1,000 locations as shown in Fig. 1.

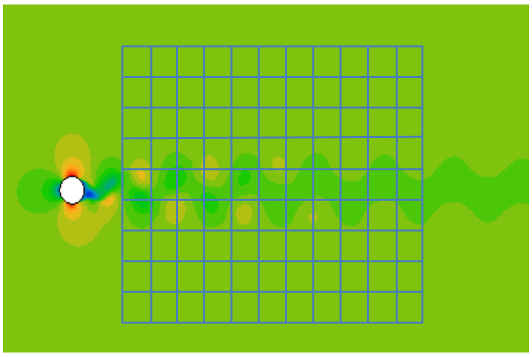

Fig. 1.CFD analysis of wake flow from a 2D cylinder and grid points for data acquisition

As an example of POD modes, Fig. 2 compares the second POD mode from bins 2 and 10. The change of spatial mode in Fig. 2 is clearly shown to raise the necessity of shift modes in order to more deeply understand the basic physics of the transient flow. Therefore DPOD was applied to the same flow field.

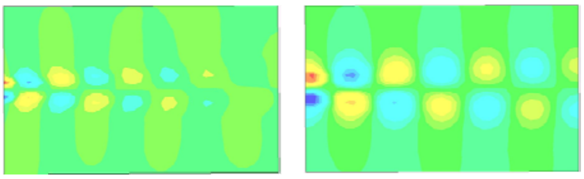

Fig. 2.SPOD mode 2 from Bin 2 and Bin 10
(left and right, respectively)

Fig. 3 shows the DPOD modes Φ_{jk} obtained by using Eq. (2). Fig. 4 tells the eigenvalues of the DPOD modes. The magnitudes correspond to the energy contents of each mode, so Φ_{11} seems the most governing mode. However, it is just the mean flow, therefore, Φ_{21} and Φ_{31} are actually the important modes, *i.e.*, Karman vortex shedding, of the fluctuating components of the flow. Shift mode $\Phi_{2,j}$ and shift mode $\Phi_{3,j}$ are paired and this is due to the symmetry of the geometry and flow conditions.

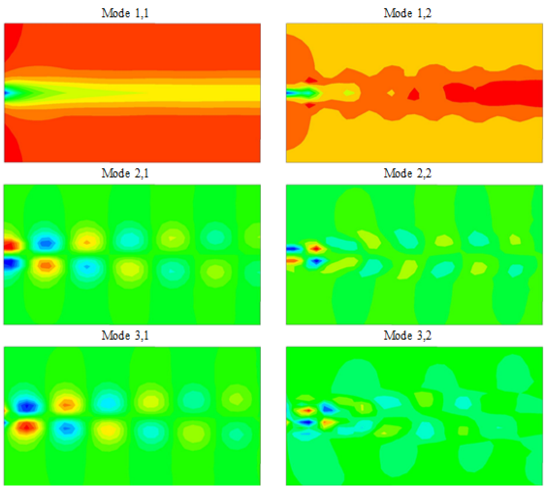

Fig. 2. Spatial DPOD modes of the start-up transient flow

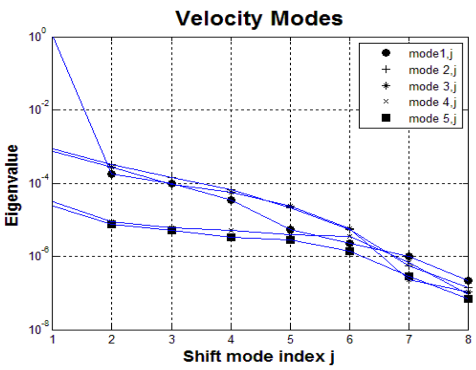

Fig. 4. Eigenvalues of the DPOD modes

Time coefficients of DPOD modes are shown in Fig. 5, which reveals that the influence of modes (1,2), (2,2) and (3,2) significantly decreases after about 1 second after start-up. It is clearly shown in Fig. 5 that the main contribution to the vortex shedding is due to the Mode (2,1) and (2,2) at steady state flow condition.

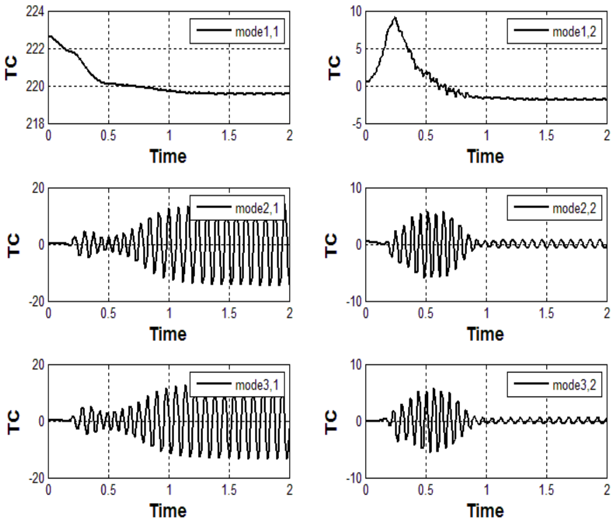

Fig. 5. Time Coefficient of spatial modes

3. Conclusions

A computer program for double proper orthogonal decomposition is developed in this research. It systematically analyzes complex natural phenomena such as transient flow field or oscillatory motions so that the mainly-contributing modes can be identified. By this way one can understand their fundamental physics and construct the low-dimensional models. As an example, two-dimensional circular cylinder wake flow field was successfully analyzed using DPOD. The eigenvalues, spatial mode shapes and time coefficients of physical modes and their shift modes were obtained and their physical meaning was reviewed to prove the reasonability. The results of study can be applied to determine the governing modes of the complicated natural phenomena such as turbulent flow or fluid-structure interaction problems, and has various potential applications.

ACKNOWLEDGEMENT

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