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NEM and FDM Hybrid Method for 3-Dimensional Reactor Core Analysis

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ABSTRACT

A new method for 3-dimensional reactor core analysis is proposed in this paper. The solution procedure of this method is composed of two step calculation. The first one is the fine mesh FDM calculation in each radial plane, and the second one is the NEM calculation in the z direction. For consistent coupling of those two different methods, the fission source of the NEM calculation is fixed in inner iteration and z-directional partial currents is used as source in the radial FDM calculation. The accuracy and stability of this new method are shown by the successful application to the IAEA3D PWR benchmark problem and a modified 3-dminesional EPRI-9 benchmark problem.

1. INTRODUCTION

Currently nodal methods such as NEM (Nodal Expansion Method)[1] and ANM (Analytic Nodal Method)[2] are exclusively favored in practical reactor core analysis because of the fast calculation speed and accuracy comparable to fine-mesh finite difference calculations, with the aid of the assembly homogenization[3] techniques. But, with nodal method, one cannot obtain directly detail informations such as pin power distributions, so the so-called dehomogenization procedures are inevitable in the nodal methods. On the one hand, in the fine mesh FDM calculation, it is possible to get the detail pin power information, however, it demands too much calculation time.

The objective of the present work is to develop a nodal and FDM (Finite Difference Method) hybrid algorithm to calculate three-dimensional power distribution without the assembly homogenization by exploiting the advantage of the two methods. Generally, material properties are fairly homogeneous in the axial direction. This implies that a coarse mesh nodal method can be used for the z-direction approximation. In this paper, we developed an NEM/FDM hybrid algorithm, where the 4^{th}-order NEM approximation is used for z-directional coarse meshes and the neutron diffusion equations are approximated by the standard FDM in each radial plane.

2. NEM/FDM HYBRID METHODOLOGY

In the NEM/FDM hybrid method, the calculation procedure is divided into two steps in every inner iteration. The first step is 2-dimensional FDM calculation for each radial plane and the second step is z-directional NEM calculation. Consequently, the inner iteration is composed of two step calculations, one is the radial FDM calculation and the other one is the z-directional NEM sweeping.

Eq. (1) is the neutron balance equation to be solved in 3-dimensional geometry.

$$\nabla \cdot j_g(r) + \Sigma_{rg} f_g(r) = \sum_{g'} \left(\frac{c_g}{k_{eff}} n\Sigma_{fg'}(r) + \Sigma_{sgg'}(r) \right) f_{g'}(r) \qquad (1)$$

where all notations are standard.

For the 2-dimensional radial FDM calculation, incoming neutron currents from upper and lower planes should be determined at each plane. In the NEM/FDM hybrid method, the incoming currents are directly available from the z-directional NEM solutions. For NEM calculation, the z-directional transverse leakage of each node is obtained from the linear approximation of flux in radial direction.

In the z-directional NEM calculation, the 1-dimensional flux is expanded with 4^{th}-order polynomials and transverse leakage is expanded with 2^{nd}-order polynomials as in Eq. (2).

$$\phi_{gz}^{m}(z) = \phi_{g}^{m} + \sum_{i=1}^{4} a_{ig}^{m} h_{i}(z') \,,\; L_{gz}^{m}(z) = L_{g}^{m} + \sum_{i=1}^{2} K_{ig}^{m} h_{i}(z') \;;\; g = 1,2, z' = z / h_{z}^{m} \,, \qquad (2)$$

where expansion functions are the Legendre polynomials. Then, the NEM approximation for z-direction can be represented in the form of Eq. (3).

$$\left(\Sigma_{rg}^{m} + \frac{2C_{1g}^{m}}{h_{z}^{m}} \right) \phi_{g}^{m} = \left(\frac{\chi_{g}}{k_{eff}} \nu\Sigma_{fg'}^{m} + \Sigma_{sgg'}^{m} \right) \phi_{g'}^{m} + \frac{2C_{1g}^{m}}{h_{z}^{m}} \left(2(j_{gzL}^{+m} + j_{gzR}^{-m}) - a_{4g}^{m} \right) - L_{gz}^{m} \,, \qquad (3)$$

where

$$L_{gz}^{m} = \frac{1}{h_{x}^{m}} (j_{gxr}^{m} - j_{gxl}^{m}) + \frac{1}{h_{y}^{m}} (j_{gyr}^{m} - j_{gyl}^{m}) \,,$$

$$C_{1g}^{m} = \frac{6\beta_{g}^{m}}{1 + 12\beta_{g}^{m}} \,, \quad \beta_{gu}^{m} = \frac{D_{g}^{m}}{h_{u}^{m}} \,,$$

a_{ig}^{m}, i=1,2,3,4 : flux expansion coefficients in z direction,

j_{gus}^{m} : net current at nodal interface,

$j_{gus}^{\pm m}$, u=x,y,z, s=l,r : partial currents at nodal interface.

Eq. (3) is solved by the conventional NEM solution procedure except the fact that x, y-directional net currents at nodal interfaces are given by Eq. (4) which is directly derived from FDM linear flux approximation and that fission sources are fixed at each inner iteration just like FDM inner iteration.

$$j_{gus}^{m} = - \frac{2\beta_{gu}^{l} \beta_{gu}^{r}}{(\beta_{gu}^{l} + \beta_{gu}^{r})} \left(\phi_{g}^{r} - \phi_{g}^{l} \right), s = l, r \,, u = x, y \,. \qquad (4)$$

From Eq. (4), the z-directional average transeverse leakage of a node m can be represented by Eq. (5) using the node average flux of node m and neighboring nodes, which are given from radial fine mesh FDM solutions.

$$L_{gz}^{m} = \sum_{n=W,E,S,N} \left[p_{g}^{n} \phi_{g}^{n} \right] + p_{g}^{m} \phi_{g}^{m} \,, \qquad (5)$$

where

$$p_{g}^{n} = - \frac{1}{h_{u}^{m}} \frac{2\beta_{gu}^{m} \beta_{gu}^{n}}{(\beta_{gu}^{m} + \beta_{gu}^{n})} \,, p_{g}^{m} = -p_{g}^{W} - p_{g}^{E} - p_{g}^{S} - p_{g}^{N} \,,$$

W = left node in x direction, E = right node in x direction,

S = left node in y direction, N = right node in y direction.

Eq. (3) is solved for the fixed fission source terms at each inner iteration to be consistent with the radial FDM calculation. And when determining the 3rd and 4th flux expansion coefficients by solving Eq. (6) and Eq. (7), the flux expandsion coefficients of fission source terms are also fixed, which means that the shape of fission source is fixed during an inner iteration.

$$(\Sigma_{rg}^{m}+42\frac{b_{gu}^{m}}{a_{u}^{m}})a_{3gu}^{m}-\sum_{g'}(\frac{c_{g}}{k_{eff}}n\Sigma_{fg'}^{m}+\Sigma_{sgg}^{m})a_{3g'u}^{m}=\frac{7}{6}\left\{\sum_{g'}(\frac{c_{g}}{k_{eff}}n\Sigma_{fg'}^{m}+\Sigma_{sgg}^{m})a_{1g'u}^{m}-K_{1gu}^{m}-\Sigma_{rg}^{m}a_{1gu}^{m}\right\} \quad (6)$$

$$(\Sigma_{rg}^{m}+90\frac{b_{gu}^{m}}{a_{u}^{m}})a_{4gu}^{m}-\sum_{g'}(\frac{c_{g}}{k_{eff}}n\Sigma_{fg'}^{m}+\Sigma_{sgg}^{m})a_{4g'u}^{m}=\frac{3}{2}\left\{K_{2gu}^{m}+\Sigma_{rg}^{m}a_{2gu}^{m}-\sum_{g'}(\frac{c_{g}}{k_{eff}}n\Sigma_{fg'}^{m}+\Sigma_{sgg}^{m})a_{2g'u}^{m}\right\} \quad (7)$$

Eq. (1) is approximated by FDM in a radial plane and they can be written as Eq. (8).

$$b_{g}^{W}f_{g}^{W}+b_{g}^{E}f_{g}^{E}+b_{g}^{S}f_{g}^{S}+b_{g}^{N}f_{g}^{N}+b_{g}^{m}f_{g}^{m}=s_{g}^{m}, \quad (8)$$

where

$$b_{g}^{n}=-\frac{2}{h_{u}^{m}}\frac{b_{gu}^{m}b_{gu}^{n}}{b_{gu}^{m}+b_{gu}^{n}},$$

$$b_{g}^{m}=-\sum_{n=W,E,S,N}b_{g}^{n}+\Sigma_{rg}^{m}+\frac{2C_{1gz}^{m}}{h_{z}^{m}},$$

$$s_{g}^{m}=\left(\frac{c_{g}}{k_{eff}}n\Sigma_{fg'}^{m}+\Sigma_{sgg'}^{m}\right)f_{g'}^{m}+\frac{2C_{1gz}^{m}}{h_{z}^{m}}\left(2(j_{gzL}^{+m}+j_{gzR}^{-m})-a_{4gz}^{m}\right).$$

In Eq. (8), the axial flux shpae is expanded by 4th-order polynomials as Eq. (2) and source of the right hand side is the sum of fission source and incoming currents from lower and upper plane.

To find the solution to Eq. (1), Eq. (3) and Eq. (8) are repeatedly solved in the whole reactor core until the convergence criteria are satisfied.

3. NUMERICAL TESTS

To demonstrate the accuracy and computational efficiency of the NEM/FDM hybrid algorithm, the IAEA3D PWR benchmark problem[2] and a modified 3-dimensional EPRI-9 problem[2] were analyzed with a computer code based on the new method.

First, the solutions of the IAEA3D PWR benchmark problem obtained by the new method were compared with those of the VENTURE code, which uses the standard FDM, in Table I. Basically, the two codes utilize the same FDM for x- and y-direction. Therefore, the numerical tests were focused on the effects of the axial mesh size for a fixed x,y mesh system ($\Delta x = \Delta y = 10$ cm). In Table I, one can note that the NEM/FDM hybrid algorithm provides consistent solutions and is very insensitive to the axial mesh size. As shown in Table I, difference in the eigenvalues between $\Delta z=5$ and $\Delta z=30$ is less than 2pcm. In other words, a very large axial mesh sized, e.g., 20 cm, can be used with little compromize of the accuracy in the NEM/FDM hybrid method. Comparing the results of the new method and VENTURE, it is observed that the accuracy of the new algorithm with Δz=20cm is comparable to the VENTURE solution of Δz=2cm. Also, it is worthwhile to note that NEM/FDM hybrid algorithm provides a comparable accuracy several times faster than VENTURE. Meanwhile, the computer code using the new method is not fully optimized. Therefore, discrepancy in the computing speed would be much larger if the new algorithm were accelerated by more efficient acceleration schemes.

Secondly, a modified EPRI-9 benchmark problem is solved with $\Delta x = \Delta y$=1.4cm and Δz=20cm. The original EPRI-9 benchmark problem is a 2-dimensional problem with cell wise cross section, but in this study, this problem is modified to 3-dimension problem that comprises of 120cm active core and top/bottom axial reflectors of 20cm. And the incoming current zero boundary condition was imposed in z direction boundaries. By the new method, the modified 3-dimensional EPRI-9 benchmark problem was successfully solved using the cell wise cross sections without the assembly homogenization. The eigenvalue is 0.90128 and radial power shape is showed in Figure I.

4. CONCLUSIONS

The nodal expansion method is consistently coupled with the finite difference method for efficient analysis of heterogeneous 3-dimensional reactor core. In the NEM/FDM hybrid method, each axial plane is solved by FDM and the consecutive planes are coupled via NEM using 4th-order polynomial expansion for the flux distribution. The newly developed algorithm was compared with VENTURE in terms of accuracy and speed over the IAEA3D benchmark problem. Also, a modified 3-dimensional EPRI-9 benchmark problem was solved successfully. Numerical tests confirm that the NEM/FDM hybrid method is stable and provides consistent solutions. Comparison with VENTURE shows that about 10-times larger axial mesh size can be used in the new algorithm to obtain the comparable accuracy. In addition, we found that the new algorithm is several times faster than the VENTURE code, despite that the new algorithm is not fully optimized.

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Table I. Comparison of NEM/FDM hybrid and VENTURE for the IAEA3D problem (Δx=Δy=10cm)

METHOD		k-eff	Computing Time(sec)
NEM/FDM HYBRID	Δz = 5 cm	1.029123	626
	Δz = 10 cm	1.029120	219
	Δz = 20 cm	1.029103	114
	Δz = 30 cm	1.029102	78
VENTURE	Δz = 1 cm	1.029122	1575
	Δz = 2 cm	1.029115	330
	Δz = 4 cm	1.029096	228
	Δz = 0*)	1.029124	-

*) Extrapolated

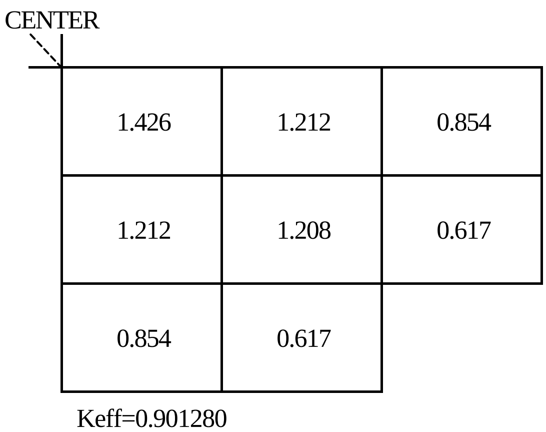

Figure I. Radial Power Distribution of the modified 3-dimensional EPRI-9 benchmark problem

CENTER

1.433	1.214	0.851	1410(77)
1.433	1.214	0.851	852(45)
1.433	1.214	0.851	869(47)
1.426	1.212	0.854	657(29)
1.214	1.211	0.613	
1.214	1.211	0.613	
1.214	1.210	0.613	
1.212	1.208	0.617	
0.851	0.613	0.903818	4th
0.851	0.613	0.903733	3rd
0.851	0.613	0.903502	2nd
0.854	0.617	0.901280	FDM