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A Nonlinear Combination of CMFD (Coarse-Mesh Finite Difference) and FMFD (Fine-Mesh Finite Difference) Methods

Ho Cheol Shin, Yong Hee Kim, Yong Bae Kim, and Sung Man Bae

Korea Electric Power Research Institute
Nuclear Power Laboratory
103-16, Munji-dong, Yusung-gu Taejon, 305-380, Korea

Abstract

An efficient acceleration scheme is introduced for the fine-mesh finite difference method, where the coarse-mesh finite difference method is nonlinearly coupled with the high-order finite difference representation of the neutron diffusion operator. The coarse-mesh operator is iteratively corrected such that its solution is equivalent to that of the fine-mesh operator. The correction factors are updated by using the one-node-based high order solution, not the two-node solution as in the conventional nonlinear nodal methods. The efficiency and accuracy of the new method is demonstrated over a benchmark problem (IAEA-2D), relative to a production code, VENTURE. Numerical results show that the computational speed of the new algorithm is 10 ~15 times faster than that of VENTURE, without compromising the accuracy of the solution. In addition to the fast convergence, the new algorithm is easy to implement and also is highly parallel.

I. Introduction

This paper is concerned with speeding up the convergence of the fine-mesh finite difference (FMFD) method for the neutron diffusion problem. Solving the reactor eigenvalue problems involves two-level iterations: inner and outer iterations. In general, inner iteration takes much longer time than outer iteration. Therefore, an efficient inner iteration algorithm is required in reducing the computing time. The line successive overrelaxation (LSOR) scheme and the Chebyshev acceleration method are popularly used in solving the reactor eigenvalue problem in inner and outer iterations, respectively. However, the convergence rate with those acceleration techniques is fairly slow when the problem size is large as in the FMFD calculations[1, 2].

The objective of this paper is to develop a nonlinear acceleration scheme which has a high parallelism. The basic idea of the newly developed algorithm originates from the nonlinear coarse-mesh finite difference (CMFD) scheme for the nodal methods, where the low-order CMFD operator is iteratively corrected through a global-local iteration so that the final solution of the CMFD problem is equivalent to the high-order nodal solution[3, 4, 5]. In the present application, the high-order solution is determined by using the FMFD operators. This algorithm is similar to the two-grid algorithm such as the

coarse-mesh rebalancing (CMR) method[1]. However, unlike the CMR, the local high-order problem can be solved independently and the low-order CMFD operator is nonlinearly updated through a simple arithmetic operations in the new nonlinear CMFD algorithm..

II. The One-Node CMFD Nonlinear Algorithm

After Smith[3] introduced the fundamental idea of the nonlinear nodal methods based on the CMFD acceleration, Joo and Downar[4] demonstrated that it was very efficient within the framework of the analytic nodal method (ANM) and the nodal expansion method (NEM). Recently, Moon et al.[5] developed a nonlinear CMFD method for the analytic function expansion method (AFEN), where two correction factors are introduced at every interface, instead of one factor in the conventional scheme. Adoption of the two correction factors is attributed to the fact that the AFEN method does not utilize the transverse integration procedure as in ANM and NEM.

In spite of the fundamental differences between the one- and two-factor approaches, a common feature is that the correction factors for the CMFD operators are updated by solving a two-node problem in both methods. On the one hand, in the case of the two-factor approach, it can be noticed that the correction factors can be determined by solving only a one-node problem with a high-order method. This is because both interface current and flux are preserved in the two-factor approach. By virtue of this concept, we developed a new nonlinear CMFD algorithm which can significantly reduce the computing time of the FMFD problem. In the new algorithm, the CMFD and FMFD method are nonlinearly coupled as in the conventional nonlinear nodal methods. However, the two correction factors are iteratively improved by solving a one-node problem with the FMFD method, not the two-node problem.

The fixed-source one-node problem can be well defined by imposing the boundary conditions of the incoming partial currents on the interfaces between the coarse mesh nodes. In defining the one-node problem, it is very important to evaluate accurate boundary conditions as well as the fixed source distribution. However, the CMFD problem provides only average quantities for the node-averaged fluxes and surface-averaged partial currents. This difficulty is overcome by noting that the FMFD solutions have the distributions for the boundary partial currents and fixed source. In other words, profiles of the partial currents and group fluxes from the FMFD solutions are used as the shape functions and they are combined with the average values from the CMFD solutions.

In Fig. 1, the overall flow chart of the new CMFD algorithm is depicted schematically. The CMFD solution provides the global eigenvalue, average fluxes, and the partial current to be used in one-node problems as the boundary conditions. It is possible to calculate two correctional coupling coefficients per node surface using the equality condition between the currents from the CMFD solution and the new net currents from one-node problems. The correction factors for the CMFD operator are iteratively updated until the global convergence is achieved.

A definite advantage of the one-node-based CMFD algorithm over the two-node-based approach is the reduction of the number of local problems. The number of local problems in the new algorithm, to be solved at each outer iteration, is almost one half and one third of the two-node-based schemes in 2-dimensional and 3-dimensional rectangular geometries, respectively. Furthermore, the local problems can be solved independently. Therefore, the new CMFD algorithm could be very effectively implemented on the parallel computing platforms.

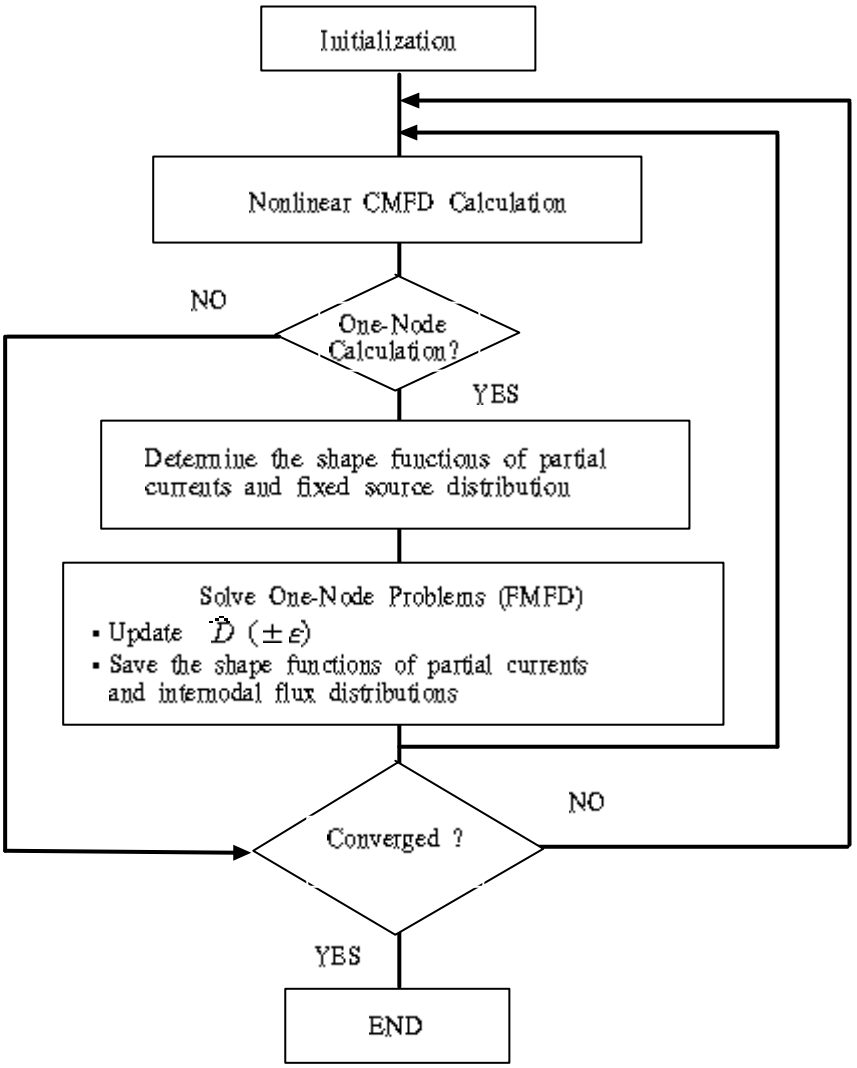

Fig. 1. Calculational flow diagram for the CMFD algorithm

II.1 Formulation of CMFD Module

The CMFD module where all coarse meshes are elliptically coupled is responsible for the global neutron balance over the whole domain. Additionally, the CMFD module provides the global eigenvalue and the partial currents across all interface boundaries of the coarse-mesh nodes, which are used in one-node local problems to calculate the

correctional coupling coefficients. It should be noted that the partial currents from the CMFD module are used as the boundary conditions in the one-node calculations.

The 2-group, 2-dimensional neutron diffusion equation can be written in the following form:

$$\nabla \cdot J_g(x,y) + \Sigma_{rg}\Phi_g(x,y) = \frac{\chi_g}{k_{eff}} \sum_{g'=1}^{2} \nu\Sigma_{fg'}(x,y)\Phi_{g'}(x,y) + \sum_{g' \neq g}^{2} \Sigma_{sg'g}(x,y)\Phi_{g'}(x,y), \quad (1)$$

where all notations are standard.

In Eq. (1), all group constants are assumed to be homogeneous within a coarse rectangular mesh. The mesh configuration used in the development of the method is illustrated in Fig. 2.

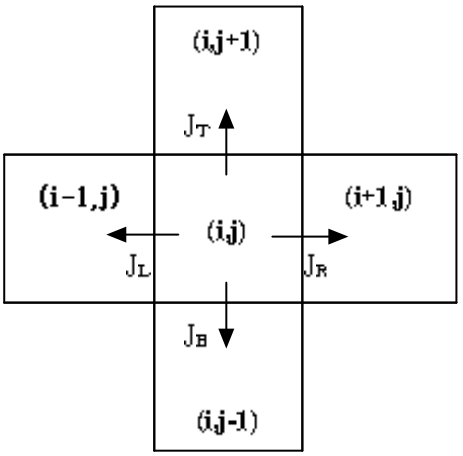

Fig. 2. Mesh configuration in the CMFD module

Integrating Eq. (1) over the spatial volume of a mesh yields the following neutron balance equation.

$$\bar{J}_{gR} + \bar{J}_{gL} + \bar{J}_{gT} + \bar{J}_{gB} + \Sigma_{rg}\bar{\Phi}_g = \frac{\chi_g}{k_{eff}} \sum_{g'=1}^{2} \nu\Sigma_{fg'}\bar{\Phi}_{g'} + \sum_{g' \neq g}^{2} \Sigma_{sg'g}\bar{\Phi}_{g'}, \quad (2)$$

where $\bar{J}_g$ denotes the surface-averaged gth group net current along the boundary.

Generally, the finite difference method is inaccurate when the mesh size is large. To improve the accuracy of the CMFD operator, the neutron currents in Eq. (2) are approximated by introducing two correctional factors, as in Ref. 5. In the two-factor CMFD algorithm, neutron current across the right side of the (i,j) mesh is defined in the following way:

$$J(x_i+\varepsilon) = -\frac{2D_{i+1,j}\Delta y}{\Delta x}(\Phi_s - \bar{\Phi}_{i+1,j}) - \frac{2\hat{D}(x_i+\varepsilon)\Delta y}{\Delta x}(\Phi_s + \bar{\Phi}_{i+1,j}) \quad (3)$$

$$J(x_i-\varepsilon) = -\frac{2D_{i,j}\Delta y}{\Delta x}(\Phi_s - \bar{\Phi}_{i,j}) - \frac{2\hat{D}(x_i-\varepsilon)\Delta y}{\Delta x}(\Phi_s + \bar{\Phi}_{i,j}) \quad (4)$$

where $D_{i,j}$ and $D_{i+1,j}$ are the diffusion coefficients and ε is an infinitesimally small value. In Eqs. (3) and (4), $\hat{D}(x_i+\varepsilon)$ and $\hat{D}(x_i-\varepsilon)$ denote the two correction factor which should be determined by solving one-node problems. These factors are unknown

values until global calculation is completed and are updated by solving one-node problems during a nonlinear iteration procedure.

Equating Eq. (3) and Eq. (4) for continuity of current at the interface, we obtain the surface flux:

$$\overline{\Phi}_s = \frac{(D_{i,j} - \hat{D}(x_i - \varepsilon))\,\overline{\Phi}_{i,j} + (D_{i+1,j} - \hat{D}(x_i + \varepsilon))\,\overline{\Phi}_{i+1,j}}{(D_{i,j} + D_{i+1,j} + \hat{D}(x_i - \varepsilon) + \hat{D}(x_i + \varepsilon))} \tag{5}$$

Substituting Eq. (5) into Eq. (3), $\overline{J}_R \cong J_R(x_i \pm \varepsilon)$ becomes

$$\overline{J}_R = -\frac{2(D_{i,j}D_{i+1,j} - \hat{D}(x_i - \varepsilon)\hat{D}(x_i + \varepsilon))}{(D_{i,j} + D_{i+1,j} + \hat{D}(x_i - \varepsilon) + \hat{D}(x_i + \varepsilon))}(\overline{\Phi}_{i+1,j} - \overline{\Phi}_{i,j}) \tag{6}$$
$$-\frac{2(D_{i+1,j}\hat{D}(x_i - \varepsilon) - D_{i,j}\hat{D}(x_i + \varepsilon))}{(D_{i,j} + D_{i+1,j} + \hat{D}(x_i - \varepsilon) + \hat{D}(x_i + \varepsilon))}(\overline{\Phi}_{i+1,j} + \overline{\Phi}_{i,j})$$

where R denotes the right boundary of the coarse-mesh (i, j) .

Introducing Eq. (6) into Eq. (2), the following linear matrix equation for the neutron diffusion equation of Eq. (1),

$$a^R_{i,j}\overline{\Phi}_{i+1,j} + a^L_{i,j}\overline{\Phi}_{i-1,j} + a^T_{i,j}\overline{\Phi}_{i,j+1} + a^B_{i,j}\overline{\Phi}_{i,j-1} + a^C_{i,j}\overline{\Phi}_{i,j} = S_{i,j} \tag{7}$$

The iterative procedure for solving Eq. (7) includes two levels of iterative scheme : inner and outer iteration. In the CMFD module, two acceleration methods were used to speed up the convergence rate; the line successive over-relaxation (LSOR) method for the inner iteration and the one-parameter asymptotic source extrapolation (ASE) method for the outer iteration.

The CMFD module calculates the partial current across the coarse-mesh surface by using the Robin natural boundary condition, that is,

$$D_g\frac{\delta\Phi_g}{\delta n} + \frac{\overline{\Phi}_g(x, y)}{2} = 2J^{IN}_g(x, y) \tag{8}$$

where $\delta/\delta n$ denotes the outward normal derivative. Rearranging Eq. (8) for the incoming current yields

$$\overline{J}_{IN} = \frac{\overline{\Phi}_S}{4} - \frac{\overline{J}_{NET}}{2} \tag{9}$$

Substituting Eq. (5) and Eq. (6) into Eq. (9), $\overline{J}_{IN}(x_i)\,|_{y_j}$,the incoming partial current from the right-hand side node, can be represented as follows.

$$\overline{J}_{IN}\,|_{y_j} = \frac{(D_{i+1,j} - \hat{D}^{+x} + 4D_{i,j}D_{i+1,j} - 4\hat{D}^{-x}\hat{D}^{+x} + 4D_{i+1,j}\hat{D}^{-x} - 4D_{i,j}\hat{D}^{+x})}{4(D_{i,j} + D_{i+1,j} + \hat{D}^{-x} + \hat{D}^{+x})}\overline{\Phi}_{i+1,j} \tag{10}$$
$$+\frac{(D_{i,j} - \hat{D}^{-x} - 4D_{i,j}D_{i+1,j} + 4\hat{D}^{-x}\hat{D}^{+x} + 4D_{i+1,j}\hat{D}^{-x} - 4D_{i,j}\hat{D}^{+x})}{4(D_{i,j} + D_{i+1,j} + \hat{D}^{-x} + \hat{D}^{+x})}\overline{\Phi}_{i,j}$$

When determining the boundary condition of the one-node problem, partial currents of Eq. (10) is used as the amplitude of the partial currents along the interface.

II.2 Formulation of FMFD Module

In previous works for the nonlinear CMFD acceleration, the correction factors are updated by solving two-node problems defined on the neighboring two nodes. This is

because the net currents are used as the boundary conditions in the local high-order calculation. However, in this study, the high-order solution to calculate the correction factors is obtained by solving a one-node local problems represented by the fine-mesh finite difference method.

The one-node problem is defined in a domain corresponding to a single node of the CMFD problem. The one-node problem consists of many fine meshes, as shown in Fig.3.

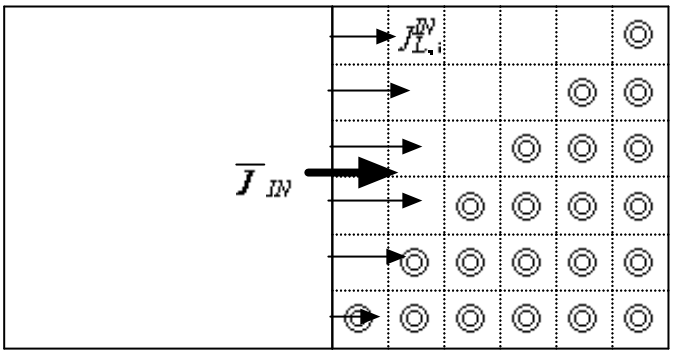

Fig. 3. Interfaces coupling between CMFD and FMFD

The one-node problem is a fixed source problem and the solution is obtained using the conventional finite difference method, i.e., without any correction factor. The major concern of the one-node problem is how to treat the boundary condition and how to incorporate the coupling between the local domains. With respect to the boundary condition, the known value is only the $\overline{J}^{IN}$ in Eq. (10), the surface-averaged incoming current of the coarse-mesh problem. However, as shown in Fig.3, this current cannot represent the actual incoming currents of the fine-meshes along the surface of the local domain. It is worthwhile to note that the profile of partial currents is required to solve the one-node problem.

In this paper, we define the boundary conditions of the one-node problem in a in the following form:

$$J^{IN}_{L,i} \mid_{\Gamma_F} = \overline{J}^{IN} \mid_{\Gamma_L} \times \phi_i \qquad (11)$$

where ϕ_i is the shape function representing the distribution of the normalized incoming current, and Γ_F and Γ_L denote the boundaries of fine-mesh and coarse-mesh systems, respectively. Unfortunately, the incoming current distribution along the interfaces are usually unknown. This difficulty can be overcome by using the relation between incoming and outgoing currents. The relation means that the incoming current distribution is equal to the outgoing current distribution from its neighboring node.

Basically, the one-node problems can be solved independently. However, for better convergence, one-node problems is solved in a sweeping sequence in this work. The sweeping starts with the left-bottom local problem and ends with the right-top problem. Therefore, the incoming currents of left and bottom sides of a one-node problem are the

outgoing currents calculated in adjacent domain at the current step and those of right and top sides are the outgoing currents calculated at the previous sweeping. That is,

$$J^{IN}_{L,i} \mid^{(q)}_{\Delta y} = \overline{J}^{OUT} \mid^{(q)}_{\Gamma_L} \times \phi^{(p)}_{L,i} \tag{12.a}$$

$$J^{IN}_{R,i} \mid^{(q)}_{\Delta x} = \overline{J}^{OUT} \mid^{(q)}_{\Gamma_R} \times \phi^{(p)}_{R,i} \tag{12.b}$$

$$J^{IN}_{R,i} \mid^{(q)}_{\Delta y} = \overline{J}^{OUT} \mid^{(q)}_{\Gamma_R} \times \phi^{(p-1)}_{R,i} \tag{12.c}$$

$$J^{IN}_{T,i} \mid^{(q)}_{\Delta x} = \overline{J}^{OUT} \mid^{(q)}_{\Gamma_T} \times \phi^{(p-1)}_{T,i} \tag{12.d}$$

where q and p denote the q-th CMFD iteration and p-th FMFD iteration, respectively.

To calculate the fixed source in the one-node problem, the flux distribution should be given. As in the modulation of partial currents, the flux distribution within the coarse mesh (i, j) is defined as

$$\Phi_{i,j}(x, y) = \overline{\Phi}_{i,j} \times \phi_{i,j}(x, y) \tag{13}$$

where $\overline{\Phi}_{i,j}$, the magnitude function, is the volume-averaged flux from the CMFD calculation, and $\phi_{i,j}(x, y)$, the shape function, is the normalized flux distribution from the previous one-node calculations.

After completion of the one-node calculations, the two correction factor is updated, which is subsequently used in the next CMFD iterations. In order to calculate the correctional coupling coefficients ($\hat{D}$), we assume that the surface-averaged current of CMFD is equal to the summation of fine mesh currents:

$$\overline{J}^{NET} \mid_{\Gamma} = \sum_{k=1}^{N} J_k^{NET} \tag{14}$$

where N is the number of fine meshes along a boundary of the one-node problem. Using Eq. (3), Eq. (4) and Eq. (14), the two correction factors can be improved. For example, at the left surface of the (i+1,j) node and the right surface of the (i,j) node, the correction factors can be expressed by

$$\hat{D}(x_i + \varepsilon) = -\frac{\sum_{k=1}^{N} J_k^{i+1\to i} + 2D_{i+1,j}(\Phi_s - \overline{\Phi}_{i+1,j})}{2(\Phi_s + \overline{\Phi}_{i+1,j})} \tag{15.a}$$

$$\hat{D}(x_i - \varepsilon) = -\frac{\sum_{k=1}^{N} J_k^{i\to i+1} + 2D_{i,j}(\Phi_s - \overline{\Phi}_{i,j})}{2(\Phi_s + \overline{\Phi}_{i,j})} \tag{15.b}$$

where $J^{i+1\to i}$ means the net current from the node $(i+1, j)$ to the node (i, j) and $J^{i\to i+1}$ from the node *(i,j)* to *(i+1,j)*. In Eq. (15), Φ_s is the integrated value of the coarse mesh surface fluxes over the node boundary.

III. Numerical Testing and Results

The applicability and accuracy of the new nonlinear CMFD algorithm were tested against an well-established benchmark problem: two-dimensional International Atomic Energy Agency benchmark problem (2D-IAEA)[6]. The IAEA PWR test problem is a simplified two dimensional and two-group benchmark problem. The reactor consists of a

two-zone core containing 177 fuel assemblies each having a width of 20 cm. The core is reflected by 20 cm of water. Each of nine fully-inserted control rods are represented as smeared absorbers in a single fuel assembly.

Based on the new algorithm, a computer code, named CMFM-2D, was developed. First, the results of CMFM-2D were compared with those of the VENTURE code to verify the consistency of the one-node CMFD algorithm. Basically, both codes use the same finite difference method as the neutronic solver. As shown in Fig. 4, the multiplication factor and the normalized assembly power from CMFM-2D and VENTURE are exactly identical for the same spatial mesh configuration (20×20 grid per assembly). This result indicates that the nonlinear iteration scheme of CMFM-2D is consistent and stable. Table I summarizes the execution time of both codes. CMFM-2D appears to provide the same accuracy with approximately 10 percents computation effort of the VENTURE code.

Secondly, the result of CMFM-2D was compared with the reference solution[6] obtained with a fine-mesh nodal calculation. In the case of 20×20 nodes per assembly, the maximum error in the assembly power is less than one percent at one of the low power assemblies adjacent to the reflector. It should be noted that the errors in assembly powers were reduced with refined mesh system, i.e., 32x32 mesh configuration per assembly. These results indicate that a spatial mesh of 1.0 cm is sufficient to achieve high accuracy as well as the reasonable execution time.

Reference CMFM-2D(20x20) VENTURE(20x20) CMFM-2D Error(%, 20x20) CMFM-2D Error(%, 32x32)					0.5849 0.5794 0.5794 -0.91 -0.32		
				0.4706 0.4689 0.4689 -0.32 -0.13	0.6856 0.6838 0.6838 -0.22 -0.07	0.5972 0.5913 0.5913 -0.95 -0.35	
			1.1929 1.1948 1.1948 0.23 0.08	0.9670 0.9680 0.9680 0.17 0.06	0.9064 0.9048 0.9048 -0.12 -0.04	0.8461 0.8406 0.8406 -0.61 -0.21	
		1.4694 1.4735 1.4735 0.34 0.12	1.3451 1.3479 1.3479 0.28 0.10	1.1792 1.1803 1.1803 0.15 0.06	1.0705 1.0694 1.0694 -0.05 -0.02	0.9752 0.9719 0.9719 -0.29 -0.10	0.6921 0.6853 0.6853 -0.95 -0.33
	1.4351 1.4404 1.4404 0.43 0.15	1.4799 1.4846 1.4846 0.38 0.13	1.3149 1.3185 1.3185 0.33 0.11	1.0697 1.0719 1.0719 0.26 0.10	1.0361 1.0357 1.0357 0.02 0.01	0.9504 0.9478 0.9478 -0.22 -0.07	0.7358 0.7307 0.7307 -0.65 -0.22
0.7456 0.7465 0.7465 0.19 0.04	1.3097 1.3155 1.3155 0.50 0.18	1.4537 1.4586 1.4586 0.40 0.14	1.2107 1.2150 1.2150 0.41 0.15	0.6100 0.6094 0.6094 -0.03 -0.02	0.9351 0.9354 0.9354 0.09 0.04	0.9343 0.9321 0.9321 -0.18 -0.05	0.7549 0.7499 0.7499 -0.61 -0.20

Fig. 4. Error in normalized assembly power for the IAEA 2D benchmark problem

Table Ⅰ. Summary of results for the IAEA-2D PWR benchmark problem

	VENTURE 20×20	CMFM-2D 20×20	CMFM-2D 32×32
Eigenvalue Error(%) [Reference : 1.02959]	0.003	0.003	0.001
Max. Error in Power	-0.95	-0.95	-0.35
Avg. Error in Power	0.35	0.35	0.12
Execution Time (sec.)	467	31	70

In the nonlinear coupling of CMFD and FMFD, two factors mainly affects the computational performance. One is the update frequency of the correction factors for the CMFD operator. Numerical tests show that the best efficiency can be obtained when the FMFD problems are solved every 4 ~6 CMFD iterations. The other important factor is the size of each local problem. Generally, too many local problems result in low numerical performance, while too large local problems may cause degradation of the convergence rate. We observed that the one-node problem of 4x4 ~ 10x10 fine-mesh grid provided fast convergence.

Ⅳ. Conclusions and Recommendations

In this paper, a new two-factor nonlinear CMFD scheme, distinguished from the previous two-node approaches, is developed to accelerate the fine-mesh finite difference method for the neutron diffusion equation. The bottomline of the new algorithm is that both fine-mesh FDM and coarse-mesh FDM is coupled in a manner of global-local interactions. The nonlinear correction factors are updated by solving only a local problem defined in a single node of the coarse-mesh problem, not the two-node problem as in the conventional nonlinear nodal algorithms. The one-node problem can be well defined by using the partial currents at the boundaries, which are inferred from the coarse mesh solutions.

Numerical tests for a 2-dimensional benchmark PWR problem demonstrate that the new algorithm is much faster, 10 ~15 times, than the VENTURE code, which utilizes the standard acceleration scheme. Also it is confirmed that the new algorithm is stable and provides almost the same solutions as the fine-mesh operator. In the new algorithm, most computing time is expended for solving the local problems since the CMFD problem requires negligible computing time relative to the FMFD problem. Furthermore, the time-consuming one-node problems are solved independently. Therefore, the performance of the new algorithm would be maximized on the parallel platforms. Besides, the new nonlinear CMFD algorithm can easily be applied to the hexagonal geometry as in the LMFBR core.

The nonlinear CMFD acceleration, based on the two-node local problem, turned out to be very efficient for various nodal methods. However, the present work shows that both CMFD and very high order operator, i.e., FMFD, can be coupled via the one-node

local problem. This means that the newly developed algorithm can be an alternative nonlinear CMFD acceleration scheme for the nodal methods which does not utilize the so-called transverse integration procedure.

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