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Free Vibration of a Rectangular Plate in Contact with Unbounded Fluid

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Abstract

A theoretical study on the free vibration of a rectangular plate in contact with an unbounded fluid is presented. In the theoretical approach, the simply supported boundary condition along the plate edges and the ideal fluid are assumed. On the contacting surface between the plate and fluid, the compatibility requirement is considered for the fluid-structure interaction. It is found that the normalized natural frequencies increase according to increase of the mode numbers. The effect of fluid depth on the coupled natural frequencies is also theoretically observed.

1. Introduction

This paper deals with the free vibration analysis of a rectangular plate in contact with laterally unbounded fluid. The work arises as a part of a project investigating free vibration of displacers in an integral reactor, SMART (System-integrated Modular Advanced ReacTor) which is under development by KAERI (Korea Atomic Energy Research Institute). The displacers, as an assembly of arbitrary shaped thin plates in contact with coolant, is installed into the reactor in order to moderate irradiation from the core. Therefore, the purpose of this paper is to develop a theoretical method for free vibration of a rectangular plate in contact with water in conjunction with the displacers. However, there are only a few available theoretical results on the free vibration of rectangular plate in contact with water. Montero de Espinosa et al. [1] studied the vibration of plates submerged in water mainly to the lower modes by the approximate analytical method and experiments. Hagedorn [2] dealt with the theoretical free vibrations of an infinite elastic plate in the presence of water. Kim et al. [3, 4] studied the effect of the supporting boundary conditions of plates and presented experimental results which verified their theoretical results. Fu and Price [5] investigated the vibration of a cantilever plate partially or fully immersed in water. Muthuverrappan et al. [6] also studied the free vibration of a cantilever rectangular plate immersed in water, and they investigated an effect of boundary conditions of the plate on added mass of fluid. Robinson and Palmer [7] carried out a modal analysis of a plate floating on a liquid. Rao and Ganesan [8] carried out a theoretical study on vibration of plates immersed in hot fluid. Kwak [9] obtained the natural frequencies of a rectangular plate floating on a fluid. The previous studies were focused on the plate immersed in fluid and these are different from when only the one side of the plate is in contact with fluid.

This paper is concerned with the hydrodynamic coupling effect on the free vibration characteristics of a simply supported rectangular plate in contact with water. The natural frequencies of the fluid-coupled system can be obtained using theoretical calculation. The normalized natural frequencies are obtained in order to estimate the relative added mass effect of fluid on each vibration mode of the rectangular plate.

2. Theoretical Background

2.1 Formulation for Rectangular Plate

Figure 1 represents a rectangular plate contacting with fluid, where *a*, *b* and *h* represent the widths of the edge and the thickness of the rectangular plate, respectively. For the simple formulation of the theory, the following assumptions are made: (a) the fluid motion is small; (b) the fluid motion is incompressible, non-viscous and irrotational; (c) the hydrodynamic loading on the rectangular plate has an insignificant effect on the deflection curve; (d) the rectangular plate is made of linearly elastic, homogeneous, and isotropic material; (e) the shear deformation and rotary inertia are negligible. The equation of motion for transverse displacement, *w*, of the rectangular plate is:

$$\nabla^4 w + \frac{\rho\, h}{D}\frac{\partial^2 w}{\partial t^2} = \frac{p}{D} \tag{1}$$

where $D = E\, h^3 / 12\,(1 - \mu^2)$ is the flexural rigidity of the rectangular plate; ρ, μ, p and E are the density, Poisson's ratio, hydrodynamic pressure on the plate and Young's modulus of the plate, respectively. In addition,

$$\nabla^4 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)^2 \tag{2}$$

is the biharmonic operator in the Cartesian coordinates *x* and *y*. As the hydrodynamic loading on the rectangular plate has an insignificant effect on the mode shape, the solution takes the form of

$$w(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{mn}(x, y) \exp(i\omega t) \tag{3}$$

where, $i = \sqrt{-1}$, ω is the coupled circular natural frequency of the fluid-coupled system. The mode shapes of the plate can be assumed for the simply supported edges:

$$W_{mn}(x, y) = A_{mn} \sin\left(\frac{m\pi x}{a} \right) \sin\left(\frac{n\pi y}{b} \right), \tag{4}$$

in which *m* and *n* are the number of modes.

2.2 Velocity Potential

Next consider the fluid region with which the rectangular plate and the rigid bottom wall are surrounded. However, the fluid is unbounded in the lateral direction. The three-dimensional oscillatory fluid flow can be described with the velocity potential. The bottom surface of the rectangular plate is in contact with an inviscid and incompressible fluid and the top surface of the plate is exposed to air. The fluid movement due to vibration of the plate can be described using the spatial velocity potential that satisfies Laplace's equation:

$$\nabla^2 \boldsymbol{F}(x, y, z, t) = 0 \tag{5}$$

It is possible to separate the velocity potential function $\boldsymbol{F}$ with respect to x and y by observing that the container supporting the edges of the plates are assumed to be rigid, as in the case of the completely contact rectangular plate. Thus the general solution of equation (5) is;

$$\boldsymbol{F}(x,y,z,t) = i\boldsymbol{w}\boldsymbol{f}(x,y,z)\exp(i\boldsymbol{w}t) \tag{6}$$

The boundary conditions along the rigid bottom wall assures the zero fluid velocity, are given by:

$$\left(\frac{\partial \boldsymbol{f}}{\partial z}\right)_{z=0} = 0 \quad \text{at } z = 0 \text{ of the rigid bottom} \tag{7}$$

When we consider the boundary conditions described by equation (7), the spatial velocity potential $\boldsymbol{f}(x,y,z)$ can be written as followings;

$$\boldsymbol{f}(x, y, z) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} B_{mn} \sin\left(\frac{m\boldsymbol{p}x}{a}\right)\sin\left(\frac{n\boldsymbol{p}y}{b}\right)\cosh(\boldsymbol{b}_{mn}z), \tag{8}$$

where,

$$\boldsymbol{b}_{mn} = \sqrt{\left(\frac{m\boldsymbol{p}}{a}\right)^2 + \left(\frac{n\boldsymbol{p}}{b}\right)^2} \tag{9}$$

Equations (8) and (9) automatically satisfy the Laplace's equation (5) and the above boundary conditions described by equation (7).

2.3 Method of Solution

In order to determine the constant B_{mn} of fluid motion, a compatibility condition at the fluid interfacing surface in contact with the plate is used. The compatibility condition along the fluid interface with the rectangular plate, yields;

$$w = -\frac{\partial \boldsymbol{f}}{\partial z} \quad \text{at} \quad z = d, \tag{10}$$

Substitution of equations (5) and (11) into equation (13) gives

$$A_{mn} \sin\left(\frac{m\boldsymbol{p}\,x}{a}\right)\sin\left(\frac{n\boldsymbol{p}\,y}{b}\right) = -\boldsymbol{b}_{mn}\, B_{mn} \sin\left(\frac{m\boldsymbol{p}\,x}{a}\right)\sin\left(\frac{n\boldsymbol{p}\,y}{b}\right)\sinh\,(\boldsymbol{b}_{mn}\, d) \tag{11}$$

Now, The constant B_{mn} can be written in terms of the constant A_{mn};

$$B_{mn} = -\frac{1}{\boldsymbol{b}_{mn}\sinh\,(\boldsymbol{b}_{mn}\, d)}\, A_{mn} \tag{12}$$

The hydrodynamic pressure along the wetted rectangular plate surface can be given by

$$p\left(x,\, y,\, t\right) = \boldsymbol{r}_o\, \boldsymbol{w}^2 \boldsymbol{f}\left(x,\, y,\, d\right) \exp(i\boldsymbol{w}t)\,. \tag{13}$$

Finally, the hydrodynamic force on the plate can be written as

$$\frac{p\left(x,\, y,\, d,t\right)}{D} = \frac{\boldsymbol{r}_o\,\boldsymbol{w}^2}{D\,\boldsymbol{b}_{mn}} A_{mn} \sin\left(\frac{m\boldsymbol{p}\,x}{a}\right)\sin\left(\frac{n\boldsymbol{p}\,y}{b}\right)\left[\frac{\cosh\left(\boldsymbol{b}_{mn}\,d\right)}{\sinh\left(\boldsymbol{b}_{mn}\,d\right)}\right]\exp(i\boldsymbol{w}t) \tag{14}$$

Substitution of equations (4) and (14) into the governing equation (1) leads to the frequency equation.

$$\boldsymbol{w}_f = \boldsymbol{p}^2 \sqrt{\frac{D}{\left\{\boldsymbol{r}\,h + \dfrac{\boldsymbol{r}_o}{\boldsymbol{b}_{mn}\tanh\left(\boldsymbol{b}_{mn} d\right)}\right\}}\left[\left(\frac{m}{a}\right)^4 + 2\left(\frac{m\,n}{a\,b}\right)^2 + \left(\frac{n}{b}\right)^4\right]} \tag{15}$$

The normalized coupled natural frequencies of the rectangular plate in contact with fluid can be related to the natural frequencies in a vacuum, w :

$$\boldsymbol{z} = \frac{\boldsymbol{w}_f}{\boldsymbol{w}_o} == \sqrt{\frac{1}{1 + \left[\dfrac{\boldsymbol{r}_o}{\boldsymbol{r}\,h\,\boldsymbol{b}_{mn}\tanh\left(\boldsymbol{b}_{mn}\,d\right)}\right]}} \tag{16}$$

3. Examples and Discussion

On the basis of the preceding analysis, equation (15) is solved in order to find the coupled natural frequencies of a rectangular plate in contact with the unbounded fluid. In order to investigate the characteristics of the natural frequencies of the fluid-coupled system, an example is examined. The rectangular plate is made of aluminum having an area of 300 mm$\times$400 mm and a thickness of 3 mm. The physical properties of the material are as follows: Young's modulus = 69.0 GPa, Poisson's ratio = 0.3, and mass density = 2700 kg/m^3. Water is used as a fluid in contact with the plate, having a density of 1000 kg/m^3. The viscosity and the compressibility of water are neglected in the calculation. Table 1 demonstrates the result from the theoretical analysis of the fluid-coupled rectangular plate for Case 1 (d = 60 mm), Case 2 (d = 120 mm) and Case 3 (d = 240 mm). The mode number n = 1 and m = 1 does not appeared in the case of bounded fluid because of volume conservation of fluid. However, the first mode (n = 1 and m = 1 mode) exist for the case of unbounded fluid. The coupled natural frequencies of the fluid contacting plate are always less than the corresponding natural frequencies of the plate in vacuum or in air due to the added mass of fluid. For the first mode (n = 1 and m = 1 mode) of Case 2 (d = 120 mm) the frequency is reduced by about 70% as shown in Table 1. As the fluid depth is increased, the natural frequencies also slightly increase in the lower modes. However, the differences between them are negligible for the higher modes. Figures 2 and 3 illustrates the normalized natural frequencies of the plate as defined in equation (16). The normalized natural frequencies always have values between one and zero due to the added mass effect of fluid. Generally speaking, an increase in the number of diametrical and rectangular modes causes a monotonic increase in the normalized natural frequencies since the complex mode shapes of rectangular plates will produce separations of fluid flow during vibration which reduces the hydrodynamic added mass. That is, the separations of fluid flow will make the stroke of fluid motion shorten, and eventually, reduce the relative added mass. This phenomenon appears similarly in a cylindrical tank filled with fluid, as depicted by Jeong [10]. As the number of nodal lines of the rectangular plates increase, the normalized natural frequencies increase by the gradual reduction of the relative added mass effect. Therefore, an increase of nodal lines causes an increase in the normalized natural frequencies. As we compare the natural frequencies in Figure 3, the decrease of the water depth, d, effects the coupled natural frequencies. The decrease of water depth will enlarge the hydraulic coupling effect for the lower modes. As the distance d changes from 120 mm to 60 mm, the normalized natural frequency with n = 1 and m = 1, decreases from 0.298 to 0.255 due to the hydrodynamic coupling effect. However the difference is reduced according to increase of mode numbers as shown in Figure 3.

4. Conclusions

An analytical method to estimate the natural frequencies of a rectangular plate in contact with a fluid is developed. In the theoretical approach, the simply supported boundary condition along the plate edges and the ideal fluid are assumed. It is found that the fluid depth reduction results in a decrease in the coupled natural frequencies. An increase in the number of diametrical and rectangular modes shows a monotonic increase in the normalized natural frequencies due to the separation effect. This study can be used in an estimation of hydrodynamic effect on the plate in contact with fluid.

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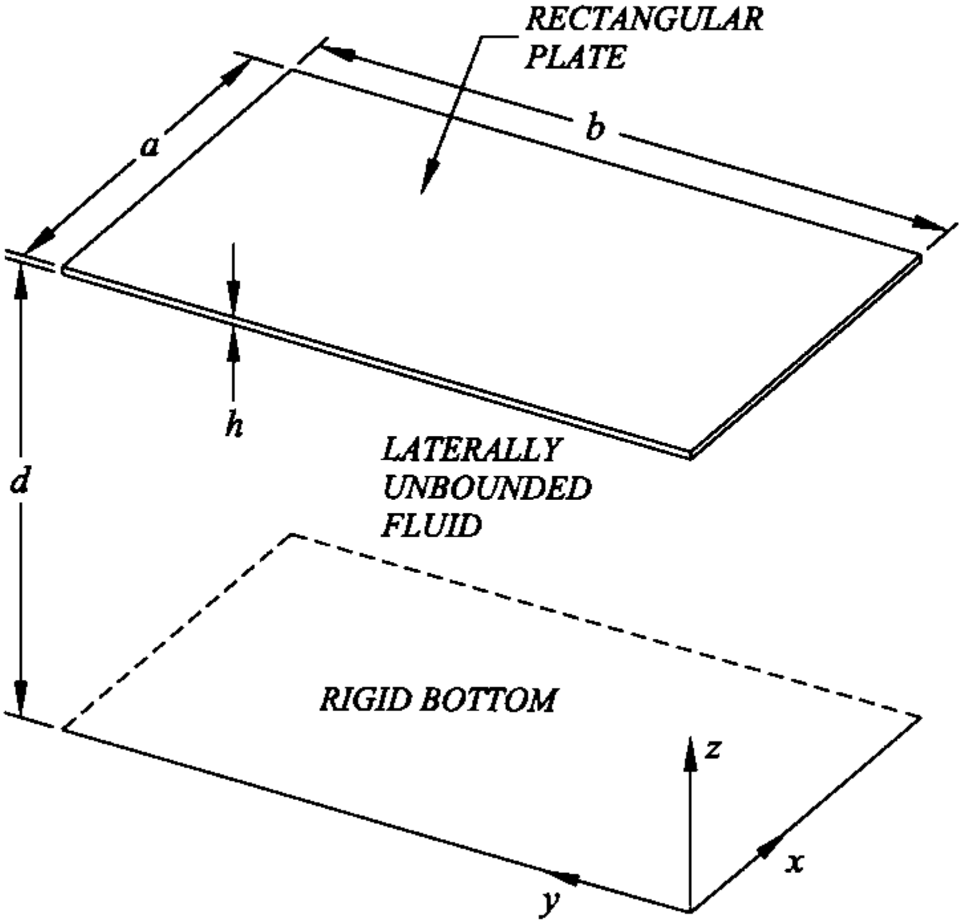
Table 1 Natural Frequencies (Hz) of a Rectangular Plate

Mode		Natural Frequency (Hz)			
n	*m*	In vacuum	*d* = 60 mm	*d* = 120 mm	*d* = 240 mm
1	1	125.2	31.9	37.3	38.7
	2	260.3	86.5	93.9	94.8
	3	485.6	194.4	201.5	201.8
	4	801.0	362.9	368.3	368.4
2	1	365.5	135.0	142.6	143.1
	2	500.6	202.0	209.1	209.4
	3	725.9	321.5	327.3	327.4
	4	1041.3	499.8	503.9	503.9
3	1	766.0	343.5	349.0	349.2
	2	901.1	419.2	424.0	424.1
	3	1126.4	549.6	553.3	553.3
	4	1441.8	739.1	741.7	741.7
4	1	1326.6	669.0	672.0	672.0
	2	1461.8	751.3	753.9	753.9
	3	1687.1	891.0	893.0	893.1
	4	2002.5	1091.4	1092.8	1092.8

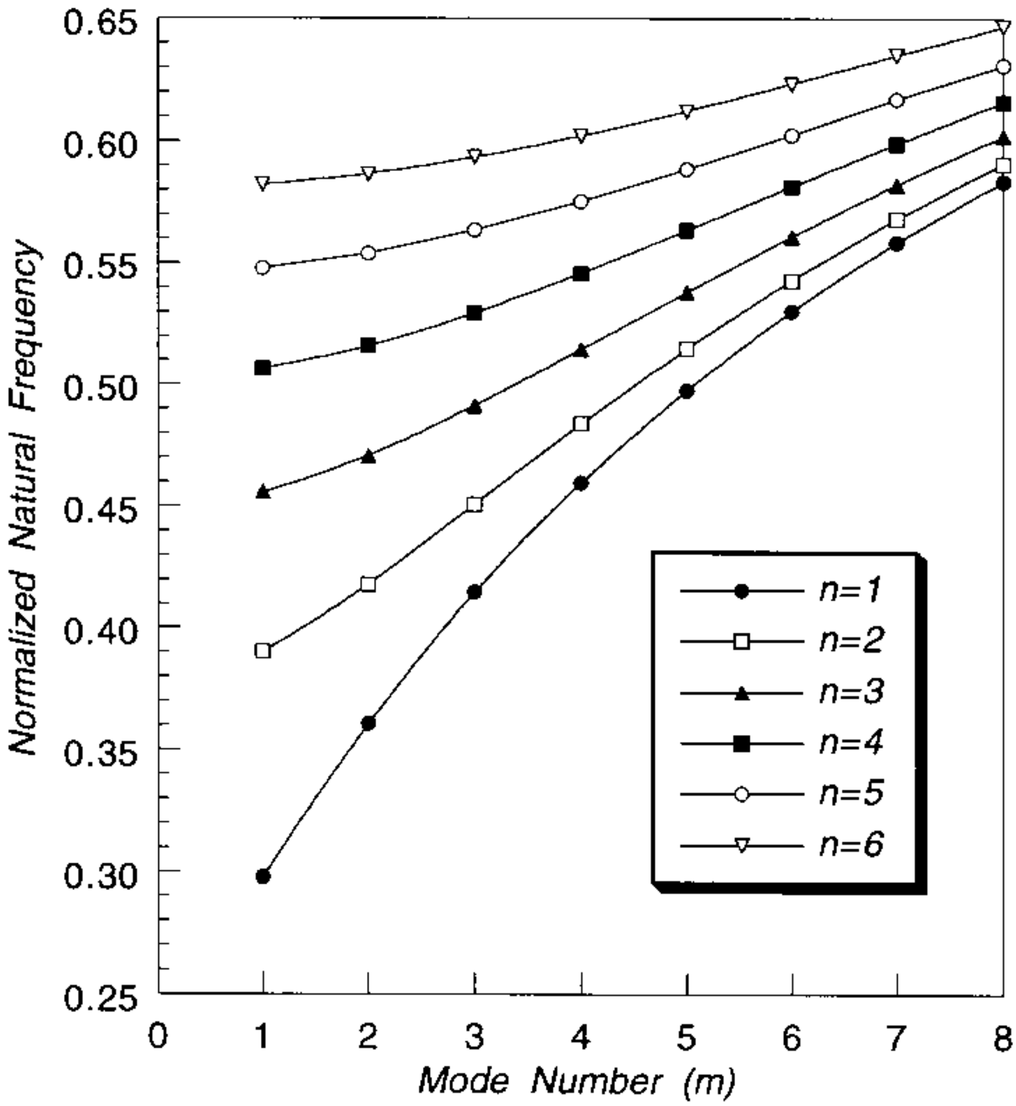
Figure 1 A Rectangular Plate in Contact with Unbounded Fluid

Figure 2 Normalized Natural Frequencies of a Rectangular Plate in Contact with Fluid (Case 1 : d = 120 mm)

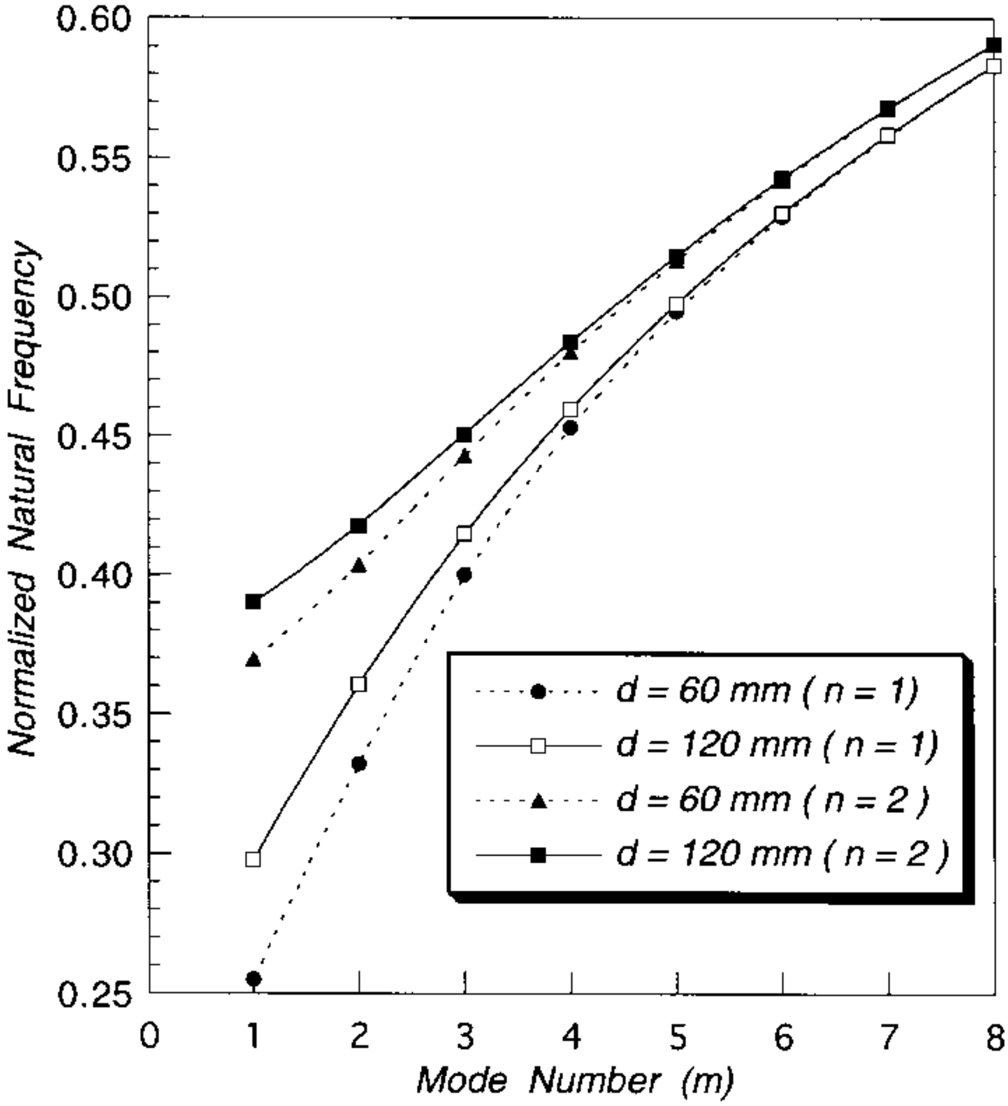
Figure 3 Effect of Fluid Depth on Normalized Natural Frequencies of a Rectangular Plate



RECTANGULAR
PLATE
a
b
h
d
LATERALLY
UNBOUNDED
FLUID
RIGID BOTTOM
z
x
y



Normalized Natural Frequency
Mode Number (m)
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0
1
2
3
4
5
6
7
8
n=1
n=2
n=3
n=4
n=5
n=6



0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
Normalized Natural Frequency
0
1
2
3
4
5
6
7
8
Mode Number (m)
d = 60 mm (n = 1)
d = 120 mm (n = 1)
d = 60 mm (n = 2)
d = 120 mm (n = 2)