The Partial Current-Based CMFD (p-CMFD) Method Revisited

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1. Introduction

The concept and some numerical results of nonlinear acceleration of the nodal diffusion method were first reported in [1, 2]. An explicit formula of the so-called coarse-mesh finite difference (CMFD) method for nodal diffusion equations was given for the first time in the literature (to the author's knowledge) in Sutton [3] (although its idea was referred to Smith). The CMFD method is popularly used in the current reactor design and analysis [4, 5], in which nodal parameters are provided by isolated single-assembly lattice calculation.

The fine-mesh heterogeneous transport (high-order) calculation was accelerated by a balancing equation over a coarse-mesh phase space in a nonlinear iteration scheme, also resulting in a diffusion-like finite difference form of low-order equation [6, 7]. The low-order equation consisted of homogenized parameters based on equivalence theory. A modern form of this procedure (with explicit formula) also turned into CMFD methodology [8, 9].

We proposed a partial current-based CMFD (p-CMFD) method a few years ago [10, 11] as an alternative to the CMFD method, that shows significantly improved convergence performance.

This paper revisits p-CMFD, expounding why it performs better, based on physical plausibility, theoretical argument, and Fourier convergence analysis.

2. Description of p-CMFD Method

Let us first consider the fine-group neutron transport equation with Legendre expansion of the scattering term,

$$\begin{aligned}\vec{\Omega}\cdot\nabla\psi_g(\vec{r},\vec{\Omega})+\sigma_{tg}(\vec{r})\psi_g(\vec{r},\vec{\Omega}) &= \sum_{l=0}^{\infty}\sum_{m=-l}^{l} Y_{lm}(\vec{\Omega})\sum_{g'}\sigma_{lgg'}(\vec{r})\phi_{lg'}^{m}(\vec{r}) \\ &+\frac{1}{k_{eff}}\chi_g\sum_{g'}\nu\sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r}),\end{aligned} \quad (1)$$

where g is the fine-group index. After angle-integration of Eq. (1), we define condensed, homogenized cross sections on coarse-mesh cells by flux-weighted volume and energy "integration" of the resulting equation. It is then converted to the neutron balance equation on coarse meshes without approximation:

$$\frac{1}{V_i}\sum_{is}A_{is}\bar{J}_G(is)+\bar{\sigma}_{tG,i}\bar{\phi}_{G,i}=\sum_{G'}(\bar{\sigma}_{GG',i}+\frac{\chi_G}{k_{eff}}\nu\bar{\sigma}_{fG',i})\bar{\phi}_{G',i}, \quad (2)$$

where i is the index of coarse-mesh cell, is is the index of surface of the corresponding coarse-mesh cell, and G is the coarse-group index.

Now, let us obtain the equation of the p-CMFD acceleration methodology [10, 11]. At the surface (i+1/2) between coarse-mesh cells i and i+1, the outgoing and incoming partial currents are related with the corresponding cell-average scalar fluxes, respectively, as:

$$\begin{aligned}\bar{J}^{+}_{G,i+1/2} &= -(1/2)\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right)+\hat{D}^{+}_{G,i+1/2}\bar{\phi}_{G,i}, \\ \bar{J}^{-}_{G,i+1/2} &= (1/2)\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right)+\hat{D}^{-}_{G,i+1/2}\bar{\phi}_{G,i+1},\end{aligned} \quad (3)$$

where $\tilde{D}_{G,i+1/2}$ is arbitrary; usually chosen as the coupling coefficient determined in ordinary finite difference method. The two correction factors $\hat{D}^{+}_{G,i+1/2}$ and $\hat{D}^{-}_{G,i+1/2}$ are defined to preserve the respective partial currents as:

$$\begin{aligned}\hat{D}^{+}_{G,i+1/2} &= \frac{2\bar{J}^{+}_{G,i+1/2}+\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right)}{2\bar{\phi}_{G,i}}, \\ \hat{D}^{-}_{G,i+1/2} &= \frac{2\bar{J}^{-}_{G,i+1/2}-\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right)}{2\bar{\phi}_{G,i+1}},\end{aligned} \quad (4)$$

where the partial currents and cell-average fluxes are obtained from the high-order transport equation (1). Then, the net current is obtained from Eq. (3) as:

$$\begin{aligned}\bar{J}_{G,i+1/2} &= -\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right) \\ &\quad -\left(\hat{D}^{-}_{G,i+1/2}\bar{\phi}_{G,i+1}-\hat{D}^{+}_{G,i+1/2}\bar{\phi}_{G,i}\right).\end{aligned} \quad (5)$$

The net current relation (Eq. (5)) is then substituted into Eq. (2). The result is a finite difference form of low-order diffusion-type equations called p-CMFD equation that can be easily solved:

$$\begin{aligned}&-(\tilde{D}_{G,i-1/2}+\hat{D}^{+}_{G,i-1/2})\bar{\phi}_{G,i-1}+[\tilde{D}_{G,i+1/2}+\hat{D}^{+}_{G,i+1/2}+\tilde{D}_{G,i-1/2} \\ &+\hat{D}^{-}_{G,i-1/2}+\sigma_{tG,i}h_i]\bar{\phi}_{G,i}-(\tilde{D}_{G,i+1/2}+\hat{D}^{-}_{G,i+1/2})\bar{\phi}_{G,i+1} \\ &=h_i\sum_{G'}\left(\bar{\sigma}_{GG',i}+\frac{\chi_G}{k_{eff}}\nu\bar{\sigma}_{fG',i}\right)\bar{\phi}_{G',i},\end{aligned} \quad (6)$$

in one-dimensional case (two- and three-dimensional cases are similar). The solution is then "modulated" to

be used in the right hand side of Eq. (1) for next iteration.

Note that it does not require to define and provide discontinuity factors explicitly, although surface flux is allowed to be discontinuous. The *two* correction factors (per coarse-mesh cell per direction) play an important role of equivalence parameters; *consistent* with the generalized equivalence theory in homogenization.

However, in the CMFD method, the net current is related with the cell-average fluxes as

$$\bar{J}_{G,i+1/2} = -\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1} - \bar{\phi}_{G,i}\right) - \hat{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1} + \bar{\phi}_{G,i}\right), \quad (7)$$

where $\tilde{D}_{G,i+1/2}$ is arbitrarily chosen but $\hat{D}_{G,i+1/2}$ is defined to preserve the net current. Since $\hat{D}_{G,i+1/2}$ is used for both node i and i+1, *one* correction factor is used per coarse-mesh cell per direction in CMFD. This is incompatible with the generalized equivalence theory, lacking one degree of freedom.

3. Key Results of p-CMFD vs CMFD

We performed Fourier convergence analysis on p-CMFD and CMFD for fixed-source problems [10-12] with diamond difference (DD) scheme for various q's (q= granularity, the number of fine-mesh cells in a coarse-mesh cell). Fig. 1 shows the improved convergence behavior of p-CMFD over CMFD. Numerical test results on realistic core problems are available in [10-12].

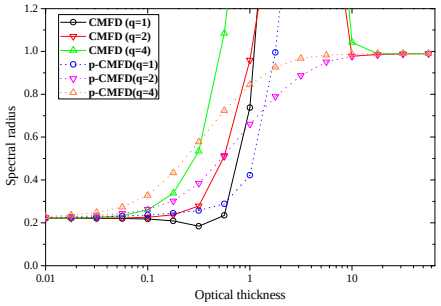

Fig. 1. Results of Fourier analysis of CMFD and p-CMFD with DD, c=0.99, and S_{16}.

A recent study [13] provides Fourier convergence analysis on p-CMFD, CMFD and related methods for eigenvalue problems in the case of q=1. The discretization scheme used is step characteristic (SC). Fig. 2 shows the significantly improved behavior of p-CMFD over CMFD.

4. Concluding Remarks

This paper describes and compares performance of p-CMFD and CMFD as acceleration methods of the transport calculation. p-CMFD provides significantly improved performance due to: i) the use of two correction factors, that is consistent with the equivalence theory in homogenization (this does not require additional computational load), and ii) Eq. (3) for p-CMFD is more physically based than Eq. (7) for CMFD.

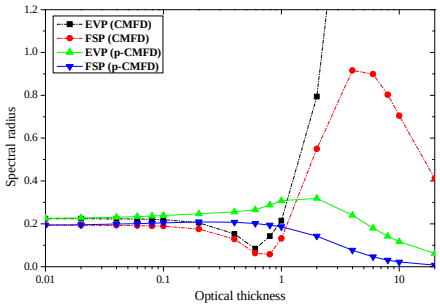

Fig. 2. Results of Fourier analysis of CMFD and p-CMFD with SC, c=0.9, and S_{10}. (EVP = eigenvalue problem, FSP = fixed source problem, reproduced from [13])

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