Transactions of the Korean Nuclear Society Autumn Meeting
Gyeongju, Korea, October 25-26, 2012

History-Based Batch Method Preserving Tally Means

Hyung Jin Shim *, Sung Hoon Choi
Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 151-744, Korea
**Corresponding author: shimhj@snu.ac.kr*

1. Introduction

In the Monte Carlo (MC) eigenvalue calculations, the sample variance of a tally mean calculated from its cycle-wise estimates is biased because of the inter-cycle correlations of the fission source distribution (FSD). Recently, we proposed a new real variance estimation method named the history-based batch method [1,2] in which a MC run is treated as multiple runs with small number of histories per cycle to generate independent tally estimates. In this paper, the history-based batch method based on the weight correction is presented to preserve the tally mean from the original MC run. The effectiveness of the new method is examined for the weakly coupled fissile array problem [3] as a function of the dominance ratio and the batch size, in comparison with other schemes available [4-6].

2. History-Based Batch Method with Weight Correction

Let's consider a MC eigenvalue calculation with N active cycles on M histories per cycle. In the same way as Ueki et al [5] formulated the bias of the apparent variance defined by the expected value of the sample variance, the variance bias of a tally estimate $\bar{Q}$ can be derived as

$$\sigma^2\left[\bar{Q}\right]-\sigma_A^2\left[\bar{Q}\right]=\frac{1}{NM\left(NM-1\right)}\sum_{i,j}\sum_{i',j'\neq i,j}\mathrm{cov}\left[Q_{ij},Q_{i'j'}\right]. \quad (1)$$

$\sigma^2[\bar{Q}]$ and $\sigma_A^2[\bar{Q}]$ denotes the real and apparent variance, respectively. Q_{ij} is the estimate of a tally Q from the j-th neutron history at active cycle i.

The dependency of Q_{ij}'s in the MC power method arises from two reasons; the genealogical dependency and the normalization scheme of fission sources. The genealogical dependency means the correlation of the fission sources having the same ancestor in the MC cycle-by-cycle iterations. The other dependency is induced from the normalization scheme of fission sources to set the number of histories per cycle to a given number of M by adjusting the weight of fission sources for cycle i as

$$w_i = M/M_i\ , \quad (2)$$

where M_i is the number of fission sources generated at cyle i-1 for the i-th cycle simulations.

An easy way to obtain independent estimates free from the genealogical and normalization dependencies of the MC eigenvalue calculations is to repeat MC runs with different random number sequences. In the history-based batch method [1,2], a MC eigenvalue run on N active cycles with M histories per cycle is treated as the N_B repeated runs on N active cycles with M/N_B histories per cycle, where the value of M/N_B is called the batch size. However it should be noted that this strategy cannot produce the same value of $\bar{Q}$ from the original calculations because the fission source normalization are conducted by using the number of M/N_B, not M.

In order to take benefit of the multiple-running strategy in respect of the sample independency and preserve the value of $\bar{Q}$, the history-based batch with the weight correction makes batches by grouping histories having the same ancestors. And the batch tallies Q^k (k=1,2,…,N_B) free from the normalization dependency are estimated in the middle of the source normalization by Eq. (2) with M to preserve $\bar{Q}$. Because the independent Q^k should be calculated with the source weight for the batch size of M/N_B as

$$w_i^k = \left(M/N_B\right)/M_i^k\ , \quad (3)$$

where w_i^k and M_i^k are the weight and number, respectively, of the fission sources in batch k for cycle i, Q^k can be tallied with correcting the particle weights by a factor f:

$$f_i^k = w_i^k/w_i = \left(M_i/N_B\right)/M_i^k\ . \quad (4)$$

f_i^k is the weight correction factor for histories in batch k at cycle i. By using f_i^k, Q^k can be estimated by

$$Q^k = \frac{1}{N\left(M/N_B\right)}\sum_{i=1}^{N}\sum_{j\in k}f_i^k Q_{ij};\ k=1,2,\cdots,N_B\ . \quad (5)$$

From Q^k's, $\sigma^2[\bar{Q}]$ can be obtained by

$$\sigma^2\left[\bar{Q}\right]\cong\sigma^2\left[\bar{Q}_{\mathrm{HB}}\right]=\frac{1}{N_B(N_B-1)}\sum_{k=1}^{N_B}\left(Q^k-\bar{Q}_{\mathrm{HB}}\right)^2; \quad (6)$$

$$\bar{Q}_{\mathrm{HB}}=\frac{1}{N_B}\sum_{k=1}^{N_B}Q^k\ . \quad (7)$$

Supposing Q^k follows the normal distribution, a 100(1-α)% confidence interval of $\sigma^2[\bar{Q}]$ of Eq. (6) can be estimated by

$$\left(\frac{(N_B-1)\cdot\sigma^2\left[\bar{Q}\right]}{\chi^2(N_B-1,\alpha/2)},\ \frac{(N_B-1)\cdot\sigma^2\left[\bar{Q}\right]}{\chi^2(N_B-1,1-\alpha/2)}\right), \quad (8)$$

where $\chi^2(a,b)$ denotes the value ν that satisfies probability$\{Z > \nu\} = \int_{\nu}^{\infty} p_Z(z)dz = \alpha$ in which $p_Z(z)$ is the chi-square distribution of a degrees of freedom.

3. Numerical Results

In order to examine the effectiveness of the history-based batch method with the weight correction, continuous energy MC analyses are performed for the weakly coupled fissile array problem [3] by McCARD [7]. The problem is composed of two uranyl nitrate aqueous solution slabs and a separating concrete slab which are 69cm wide and 50cm high. The MC calculations were conducted for two cases with concrete slab thicknesses of 0 and 30cm to vary the dominance ratio. The real variance of fission powers for seven sub-regions of a fuel slab having the same thickness of 5cm are estimated in the MC eigenvalue calculations using 500 active cycles with 1,000,000 histories per cycle. For the comparisons, the reference relative standard deviations (RSDs) are calculated from 100 replicas with different random number sequences.

Figure 1 shows the comparison of RSD of the outer-most region's fission power estimated by the new method varying the batch size from 1000 to 100,000 with the reference one for the concrete thickness of 30 cm. Table 1 compares RSDs of the regional fission powers estimated from various real variance estimation methods with the reference RSD. In the table, σ_{REF}, σ_S, σ_{CB}, σ_{Ueki}, σ_{FSD}, and σ_{HB} denote the RSD estimated by the reference calculations, the sample SD, the Gelbard's batch method [4], the Ueki's method [5], the FSD's inter-cycle correlation method [6], and the new method with the batch size of 50,000. From the table, we can see that the FSD's inter-cycle method and the new method yield remarkably better estimates than the others for the 30cm concrete slab problem.

4. Conclusions

The history-based batch method with the weight correction is proposed to estimate the real variance in the MC eigenvalue calculations. From the numerical results, it is demonstrated that the new method can predict the real variance more accurately than the other existing methods.

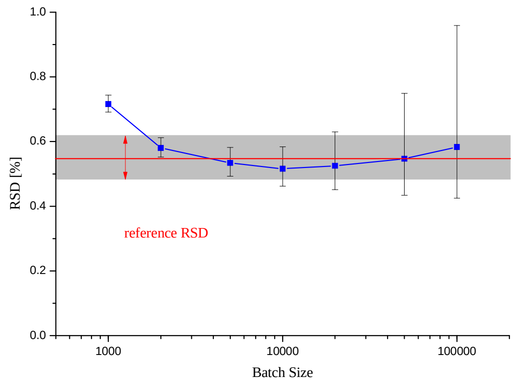

Fig. 1. Comparison of RSD for the weakly coupled fissile array with the concrete thickness of 30cm

REFERENCES

[1] H. J. Shim, Y. H. Kim, C. H. Kim, "History-Based Batch Method for a Real Variance Estimation in Monte Carlo Eigenvalue Calculations," *Trans. Am. Nucl. Soc.*, **100**, 300, 2009.
[2] H. J. Shim, S. H. Choi, C. H. Kim, ''Unbiased-Variance Estimation by Grouping Histories in Monte Carlo Eigenvalue Calculations,'' Joint International Conference on Supercomputing in Nuclear Applications and Monte Carlo 2010 (SNA + MC2010), Tokyo, Japan, October 17-21, 2010.
[3] T. Yamamoto, T. Nakamura, Y. Miyoshi, "Fission Source Convergence of Monte Carlo Criticality Calculations in Weakly Coupled Fissile Arrays," *J. Nucl. Sci. Technol.*, **37**[1], 41, 2000.
[4] E. M. Gelbard and R. Prael, "Computation of Standard Deviations in Eigenvalue Calculations," *Prog. Nucl. Energy*, **24**, 237, 1990.
[5] T. Ueki, T. Mori, and M. Nakagawa, "Error Estimation and their Biases in Monte Carlo Eigenvalue Calculations," *Nucl. Sci. Eng.*, **125**, 1, 1997.
[6] H. J. Shim, C. H. Kim, "Real Variance Estimation Using an Inter-Cycle Fission Source Correlation for Monte Carlo Eigenvalue Calculations," *Nucl. Sci. Eng.*, **162**, 98, 2009.
[7] H. J. Shim, et al., "McCARD: Monte Carlo Code for Advanced Reactor Design and Analysis," *Nucl. Eng. Technol.*, **44**[2], 161~176, 2012.

Table 1. Comparison of RSD's (%) of regional fission power tallies for the weakly coupled fissile array

T_c [cm]	Region	σ_{REF} (90% conf. intv.)	σ_S	σ_{CB}	σ_{Ueki}	σ_{FSD}	σ_{HB} (90% conf. intv.)
0	1, Outer-most	0.090 (0.081 , 0.103)	0.034	0.068	0.076	0.083	0.098 (0.077 , 0.134)
	2	0.079 (0.070 , 0.089)	0.024	0.060	0.068	0.075	0.084 (0.067 , 0.115)
	3	0.068 (0.061 , 0.077)	0.019	0.052	0.065	0.067	0.073 (0.058 , 0.101)
	4	0.063 (0.056 , 0.071)	0.017	0.042	0.052	0.056	0.070 (0.055 , 0.096)
	5	0.050 (0.045 , 0.057)	0.016	0.037	0.044	0.044	0.059 (0.046 , 0.080)
	6	0.037 (0.033 , 0.042)	0.015	0.029	0.032	0.033	0.049 (0.039 , 0.068)
	7, Inner-most	0.032 (0.029 , 0.037)	0.015	0.024	0.027	0.025	0.046 (0.037 , 0.064)
30	1, Outer-most	0.546 (0.490 , 0.619)	0.030	0.106	0.147	0.437	0.547 (0.434 , 0.749)
	2	0.539 (0.483 , 0.611)	0.021	0.095	0.133	0.434	0.534 (0.424 , 0.732)
	3	0.533 (0.477 , 0.604)	0.018	0.092	0.131	0.431	0.527 (0.418 , 0.722)
	4	0.522 (0.468 , 0.591)	0.016	0.098	0.138	0.424	0.521 (0.413 , 0.714)
	5	0.506 (0.453 , 0.573)	0.016	0.105	0.148	0.413	0.493 (0.391 , 0.676)
	6	0.484 (0.434 , 0.549)	0.017	0.100	0.141	0.395	0.475 (0.377 , 0.651)
	7, Inner-most	0.429 (0.385 , 0.487)	0.020	0.090	0.124	0.352	0.417 (0.331 , 0.571)