Efficient Response Spectrum Analysis of a Reactor Using Model Order Reduction

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1. Introduction

A response spectrum analysis (RSA) has been widely used to evaluate the structural integrity of various structural components in the nuclear industry. However, solving the large and complex structural systems numerically using the RSA requires a considerable amount of computational resources and time. To overcome this problem, this paper proposes the RSA based on the model order reduction (MOR) technique achieved by applying a projection from a higher-order to a lower-order space using Krylov subspaces generated by the Arnoldi algorithm [1]. The dynamic characteristics of the final reduced system are almost identical with those of the full system by matching the moments of the reduced system with those of the full system up to the required *n*th order. It is remarkably efficient in terms of computation time and does not require a global system. Numerical examples demonstrate that the proposed method saves computational costs effectively, and provides a reduced system framework that predicts the accurate responses of a global system.

2. Reduction of Model Based on MOR

The Krylov subspace method, which is based on the moment matching technique has been recognized as one of the desirable ways of a model order reduction (MOR) [2,3].

The state-space model can be simplified by appropriately defining the stiffness K and mass M matrices of a global system.

$$\{K-\lambda M\}\bar{\varphi}=\{0\}, \quad (1)$$

where, λ and $\bar{\varphi}$ are the eigenvalue and corresponding eigenvector, respectively.

The general process of a model order reduction is to find an approximated state variable (z) with a small number of degrees of freedom by obtaining the transformation matrix, T, which satisfies the following relation:

$$\bar{\varphi} \cong T\bar{z}, \quad \text{where } \bar{\varphi} \in R^N, \, T \in R^{N\times n}, \; \bar{z} \in R^N \quad (2)$$

Then obtain the following reduced-order system:

$$\{K_r-\lambda M_r\}\bar{z}=\{0\}, \qquad \bar{q}=T\bar{z} \quad (3)$$

where $K_r=T^TKT$ and $M_r=T^TMT$.

The stability and accuracy of model order reduction depend on how well the transformation matrix is obtained while preserving the essential properties of the original system.

The reduction is obtained using Krylove subspaces generated by the Arnoldi algorithm [1]. The *n*th Krylove subspace is defined as

$$\begin{aligned} \text{colspan}(T) &= K_n\left(K^{-1}M,\, K^{-1}\bar{F}\right) \\ &=\text{span}\left(K^{-1}\bar{F},\, K^{-1}MK^{-1}\bar{F},...,(K^{-1}M)^{n-1}K^{-1}\bar{F}\right) \end{aligned} \quad (4)$$

It has already been provn that if the Krylov subspace consists of all linear combinations of the column vectors of the transformation matrix, T, the moments of the reduced system are coincident with those of the original system up to the *n*th order [4].

3. Response Spectrum Analysis

A response spectrum analysis has been performed to evaluate the structural responses under seismic events.

A discrete dynamic system is governed by the following equation

$$M\ddot{\bar{X}} + K\bar{X} = \bar{F}, \quad (5)$$

where, $\bar{F}$ is the base excitation function. $\bar{X}$ is represented into $\phi\bar{q}$ in the general mode coordinate.

Let ϕ be the eigenvector matrix. After taking the eigenvector matrix into both sides of equation (5), it is divided by the generalized mass. Due to the orthogonality to the stiffness and mass matrix, equation (6) is obtained as follows:

$$\ddot{\bar{q}} + \Omega\bar{X} = -\bar{\Gamma}\ddot{u}_g, \quad (6)$$

where, Γ is the modal participation factor.

$$\bar{\Gamma} = \frac{\phi^T M\bar{u}}{\phi^T M\phi}, \quad (7)$$

where, $\bar{u}$ is the influence vector, which represents the displacements of the masses resulting from the static application of a unit ground displacement.

Mode coefficient vector, D_i, which determines the scale of structural response is obtained as follows:

$$D_i = \Gamma_i S_i, \quad (8)$$

where, S is the structural maximum response computed from the base excitation.

Finally, the response of structure U_i is represented as follows:

$$U_i = \phi_i D_i \quad (9)$$

The square root of the sum of the squares (SRSS) method is used to combine the total response in each mode.

3. Numerical Result

To validate the efficiency and accuracy of the finite element formulation based on the proposed framework, the calculated results are compared with those obtained from ANSYS software. The numerical model is shown in Figure 1. The elements used in the analysis model are the solid element. The total number of elements is 7099, and the total number of nodes is 2386. The total weight of the numeric model is about 280 kg. A total 30 modes are considered for the modal response combination to take into account a modal effective mass of 90% of the model. The translational 3 degrees of freedom of nodes near the pipe lines of the coolant are constrained. The material properties are given in Table 1.

Table 2 shows the natural frequencies and response. It can be observed that the first natural frequency and response are 6.04 Hz and 14.3 mm, respectively. The computed results agree very well with those of ANSYS software, and the degrees of freedom are dramatically decreased. It is confirmed that the proposed numerical approach is almost the same as the results for the full model without reduction.

The response spectrum curve is assumed to be a constant type with a spectral displacement of 8 mm in the x-direction axis at the boundary condition of the numerical model. Figure 2 shows the results of the response spectrum analysis using ANSYS software.

Table 1: Material Property of a Reactor

Component	Reactor
Modulus of Elasticity	183,450 (N/mm^2)
Mass Density	7.83e^{-9} (N sec^2/mm^4)
Poisson's Ratio	0.3

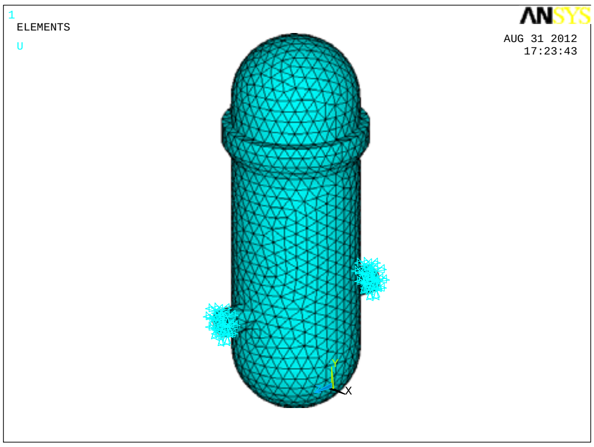

Fig. 1. Finite element model of a reactor

Table 2: Natural Frequencies and Responses

	FEM (proposed)	FEM (ANSYS)
First Natural Freq.	6.04 Hz	6.04 Hz
Second Natural Freq.	19.97 Hz	19.97 Hz
Displacement Response	14.34 mm	14.35 mm
DOF	30	7158

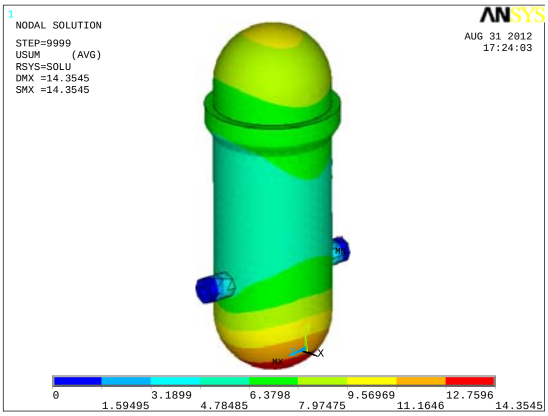

Fig. 2. Response of reactor by ANSYS

4. Conclusion

In this study, a finite element method based on the model order reduction has been developed for a response spectrum analysis. Its performance and accuracy are verified by the solution of the reactor model example. The computed results by the proposed method are compared to those of ANSYS software. The computational time and memory usage without any significant loss of solution accuracy are reduced using the Krylov-based model order reduction method (MOR). In addition, the numerical result shows the applicability to practical problems with complex geometries and boundary conditions.

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