On the Horizontal Counter Current Model for Multi-Fluid Flow

Sang Yong Lee [a*], Chan Eok Park [a], Jong Joo Sohn [a], V. H. Ransom [b]
[a]*KEPCO E&C Company, Inc., 150 Deokjin-dong, Yuseong-gu, Daejeon, 305-353*
[b]*School of Nuclear Engineering, Purdue University, 400 Central Drive, West Lafayette, IN 47907-2017, USA*
[2)]*Tel: +1-208-521-7934, Fax: +1-765 494 9570:ransom@ecn.purdue.edu*
*Corresponding author:sanglee@kepco-enc.com

1. Introduction

Through the time/volume averaging technique [1, 2, 5, 6, 7], one can get the interfacial forces caused by the volume fraction gradients in the horizontal channel or horizontal counter current model (HCC). But, in reality, there are still very diverse differences in modeling the term and in applying the term in the computer programs.

Table I Period and Wave Speed of Circular Cavity

Sources	Period(sec)	Wave speed(m/sec)
SPACE calc.	19.7	1.02
RELAP5 calc.	27.3	0.73
TRACE calc.[9]	15.8	1.26
RELAP5 [5]	7.2	2.78
Ransom [6]	7.8	2.56
Linear Theory	14.7*/15.6**	1.36*/1.27**
* equivalent rectangular cavity, ** pipe theory		

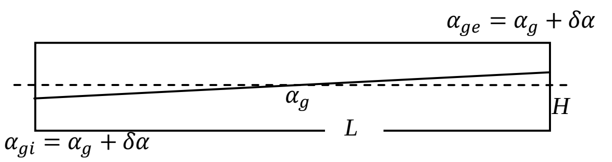

Fig.1. Cavity with Linear Profile Perturbation

Table I shows five different results for the estimation of the period for the circular cavity filled with saturated water of 10Mpa, in which the vapor volume fraction is linearly perturbed with $\delta\alpha = 0.06(x/L - 0.5)$ from base value of 0.5 along the length x, as depicted in the Fig.1. The length (L) and the diameter (H) of the cavity are 10m and 0.5m respectively. It was found that the virtual mass coefficients, different models and mistakes in the documents are three main sources of the differences.

2. Various Models

Wallis's intuitive derivation in his textbook [3], derives the interfacial force term by intuition in the rectangular channel as follows;

$$\Phi_g = \alpha_g \alpha_l g(\rho_l - \rho_g) H \frac{\delta\alpha_l}{\delta x} \quad (1)$$

$$\Phi_l = -\alpha_g \alpha_l g(\rho_l - \rho_g) H \frac{\delta\alpha_l}{\delta x} \quad (2)$$

The most rigorous derivations are found in RELAP5 related documents [5, 6]. The results are the same as Wallis' derivation. But circular pipe model is also derived as follows;

$$\Phi_g = \alpha_l \alpha_g(\rho_l - \rho_g)\left(\frac{\pi g H}{4\sin\theta}\right)\frac{\partial\alpha_l}{\partial x} \quad (3)$$

$$\Phi_l = -\alpha_l \alpha_g(\rho_l - \rho_g)\left(\frac{\pi g H}{4\sin\theta}\right)\frac{\partial\alpha_l}{\partial x} \quad (4)$$

In TRAC [8] a simpler model is derived and the term is added only to liquid phase;

$$\Phi_l = -(\rho_l - \rho_g) g H \frac{\partial\alpha_l}{\partial x} \quad (5)$$

3. Effects of the virtual mass coefficients

The difference momentum equation with HCC model is written as follows;

$$\left(1 + \frac{C\rho_m^2}{\rho_g \rho_l}\right)\left(\frac{\partial u_g}{\partial t} - \frac{\partial u_l}{\partial t}\right) = \cdots \frac{\rho_m}{\rho_g \rho_l}(\rho_l - \rho_g) g H \frac{\partial\alpha_l}{\partial x} \quad (6)$$

Since the term $(\partial\alpha_l / \partial x)$ roughly can be a function of the displacement of the wave, it can be understood as the oscillator equation with the inertia of $(1 + C\rho_m^2/\rho_g\rho_l)$. If the virtual mass coefficient C is not zero, then it effectively increase the inertia of the oscillator and the period turns out to be longer. The virtual mass coefficients of RELAP5 and SPACE are a function of gas volume fraction and shown in Fig.2.

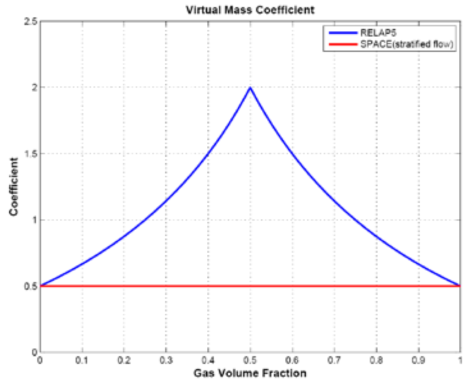

Fig.2. Virtual mass coefficients of RELAP5

Table II Code Predictions of periods for experiment (seconds)

	experiments	Linear theory	SPACE (C=0.5)	SPACE (C=0.0)
Air/water*	1.5	1.46	1.67	1.53
Salt/water [9]	80.1**	87.7	112.2	80.1**
*Mineral water bottle test **density is adjusted to match the values				

During this study it was found that if virtual mass coefficients are set 0.0, agreements between the calculations and the experimental results are much improved as shown in Table II. And linear theory developed in section 4 below perfectly matched with the code runs without virtual mass coefficients.

3. Theory of Kelvin-Helmholtz Oscillator

According to Milne-Thomson [4], the speed of long wave with wave number m is;

$$c = \sqrt{\frac{\alpha_g \alpha_l(\rho_l - \rho_g)}{\alpha_l \rho_g + \alpha_g \rho_l} gH} \quad (7)$$

And the speed of the wave in the channel with circular cross section is written as;

$$c = \sqrt{\frac{\alpha_g \alpha_l(\rho_l - \rho_g)}{\alpha_l \rho_g + \alpha_g \rho_l}\left(\frac{\pi}{4\sin\theta}\right) gH} \quad (8)$$

For the standing wave in the finite length of the cavity the period can be estimated as follow;

$$T = \frac{2L}{c} \quad (9)$$

Sometimes torus is better for the validation of the model because it can get away with the boundary condition effects. For the standing wave in the torus the period can be estimated with following equation;

$$\boldsymbol{T} = \frac{\boldsymbol{L}}{\boldsymbol{c}} \tag{10}$$

5. Validation of models against the KH oscillator

Two geometries are used, rectangular torus and circular torus as shown in Fig.3.

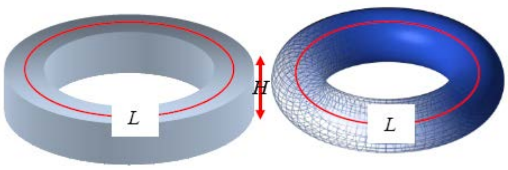

Fig.3. Rectangular and Circular Torus (L=10m, H=0.5m, 20 nodes, $\alpha_g = 0.5, \delta\alpha = 0.001\sin(2\pi x)/L$)

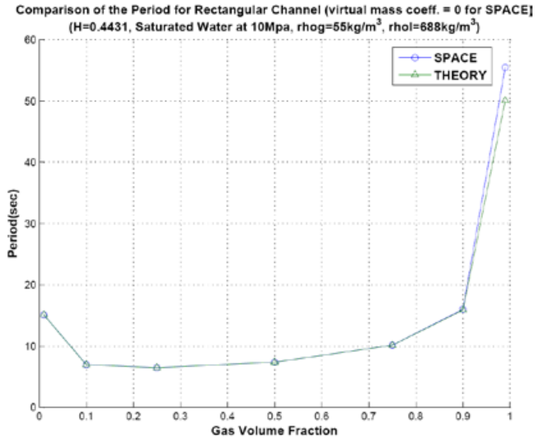

Fig.4. Periods for rectangular torus

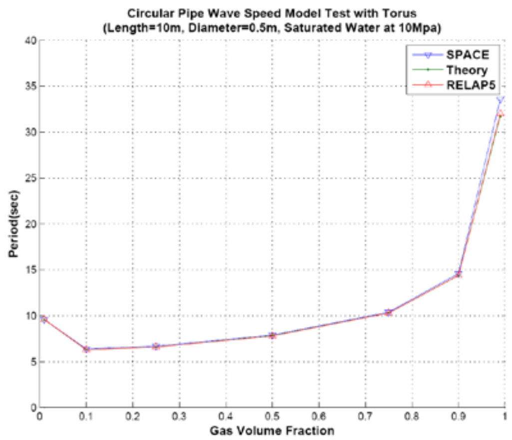

Fig.5. Period for circular torus

As one can see in Fig.4, predicted periods of rectangular torus by linear Kelvin-Helmholtz theory are perfectly matched with those of calculations. Similar comparisons are made for the circular torus as in Fig.5. Therefore it is concluded that the RELAP5 derivation of the HCC model is correct.

The model used in TRAC or TRACE code has the problem of the dependency of the period on the gas volume fraction. As depicted in Fig.6, periods predicted by TRAC code become smaller as the gas volume fraction gets larger. This behavior is quite opposite trend predicted by theory and predicted by SPACE code. Therefore, it can be said that TRAC has some deficiency in its HCC model.

7. Conclusions

The HCC models have been scrutinized to lead to the fact that the RELAP5 model has strong foundations. It is the most important fact that the RELAP5 model is identified to be equivalent to the Kelvin-Helmholtz oscillator theory in its linear application range.

It is also very important results of this study that the usage of virtual mass coefficients in the code strongly affects the behavior of the HCC model. Without the virtual mass coefficients RELAP5 and SPACE behaves exactly the same as the linear Kelvin-Helmholtz oscillator.

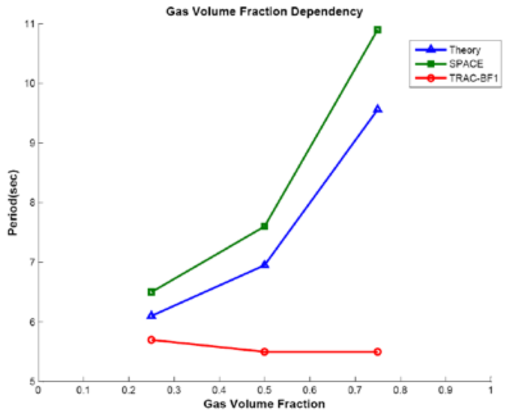

Fig.6. α_g dependency of periods for the rectangular torus

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