

Thermal Conductivity Measurement of Flowing Liquid Samples Based on a Thermal Wave Technique

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1. Introduction

The thermal conductivity of a liquid is generally measured under conditions that suppress bulk flow in the sample. However, *in situ* measurement of the thermal conductivity of a flowing liquid would be useful in various scientific and engineering applications. This work demonstrates that a thermal wave technique, called the three-omega method, can effectively measure the thermal conductivity of flowing water if the frequency range can be adjusted such that the thermal boundary layer is sufficiently thinner than the momentum boundary layer. A new dimensionless number was defined to assess the convection effect, and a single criterion for thermal conductivity measurements was obtained for water flowing in a circular tube.

2. Methods and Results

In the present work, a line heater was placed on the bottom of the tube and heated periodically to generate thermal waves that propagate into a liquid flowing in a circular tube (Fig. 1). A complex combination method can be used when the periodic boundary condition is sinusoidal, proportional to a sine or cosine function [1]. Consequently, a governing equation and a boundary condition can be obtained for the temperature response to the periodical heating:

$$\alpha \nabla^2 \tilde{T} = i2\omega \tilde{T} + \vec{v} \cdot \nabla \tilde{T}, \quad (1)$$

$$-\vec{n} \cdot (-k \nabla \tilde{T}) \Big|_{\text{heater surface}} = q_0. \quad (2)$$

Three-dimensional numerical calculations based on the finite element method were conducted to reveal the effects of convection on the temperature of the heater. Computational domains were divided into polycarbonate, liquid, and glass domains. Liquid flow in the liquid domain was assumed to be decoupled with the governing equation and to have a parabolic velocity

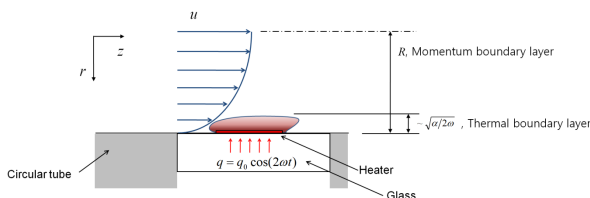


Fig. 1 Schematic diagram of the thermal wave problem in a circular tube.

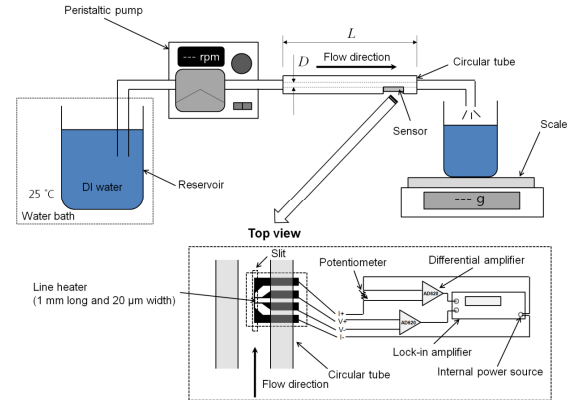


Fig. 2 Experimental setup.

profile for the fully-developed laminar regime because most of our experimental results were obtained in the laminar regime and the turbulent regime is not our major concern. U_0 was chosen for Re_D not to exceed 2100, and heating frequencies of 1, 10, and 100 Hz were used. The thermal mass of the thin-film heater in the 3ω method was neglected. Instead, a heat flux boundary condition with a size of $1 \text{ mm} \times 20 \mu\text{m}$ was set at the position of the heater. Eq. (2) was applied as the heat flux boundary condition ($q_0 = 100 \text{ kW/m}^2$), and the temperatures at all boundaries except the heat flux boundary were set to zero. To figure out the influence of variation of thermal properties, water and ethanol were chosen as flowing liquids in the liquid domain. Solving Eqs. (1) and (2) by numerical calculation gave a complex temperature field containing the real and imaginary parts (the amplitude and phase information) of the temperature oscillation by periodical heating. To obtain the temperature of the heater, the average temperature oscillation of the heat flux boundary was calculated.

The 3ω method [2] needs a thin-film heater patterned on a substrate (Fig. 2). A 200 nm thick Au film and a 20 nm thick Cr film were deposited on a glass substrate and patterned by a lift-off process. The width and length of the heater were $20 \mu\text{m}$ and 1 mm, respectively. The single metallic strip acts as both a heater and a sensor by the following procedure. A sinusoidal current of angular frequency ω is supplied to the line heater, causing a temperature fluctuation at 2ω , which is related to the thermal properties of the substrate and the fluid contacting the heater, and to the heater geometry. The oscillating temperature perturbs the heater resistance at 2ω , and consequently induces an oscillating voltage

signal at 3ω . The average temperature fluctuation \hat{T} of the heater for various configurations of the heater and its surroundings in the 3ω method has been analytically obtained from a pure conduction problem using the Fourier transform method. If the thermal penetration depth $\sim\sqrt{\alpha/2\omega}$ is much larger than the half-width of the heater ($10\ \mu\text{m}$), the solution can be expressed in a simple asymptotic form. Using the simple form, the thermal conductivity of the liquid sample is obtained from the slope of the curve relating $\ln\omega$ to \hat{T}_{real} ,

$$k_s + k_1 = -\frac{P}{2l\pi} \frac{d \ln \omega}{d \hat{T}_{\text{real}}} \quad (3)$$

An experimental setup for an internal flow in three circular tubes was developed to analyze the effects of flow on the thermal conductivity measurements using the 3ω method (Fig. 2). The circular tubes were made of polycarbonate and had three combinations of length L and inner diameter D . The lengths of the tubes were selected such that the flow was fully developed at the position of the sensor. L was chosen to be larger than the entrance lengths L_e for laminar and turbulent flows. The circular tubes were partially carved to make a slit along which the line heater was aligned and exposed to the internal flow. A peristaltic pump was used to force water to flow from a reservoir into the tube. It delivered flow rates from 0.9 to 565 ml/min during the measurements. The mass flow rate was measured using an electronic scale. All the measurements were conducted after waiting more than an hour for the water temperature to stay stable at 25°C in a water bath.

Based on the experimental results (Fig. 3), regardless of the flow regime, the flow effect started to appear at

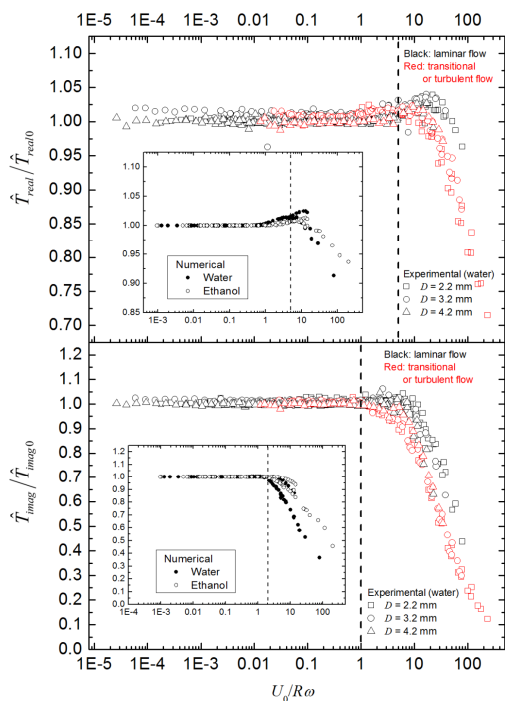


Fig. 3 Correlation between the dimensionless number describing the convection effects and the temperature response.

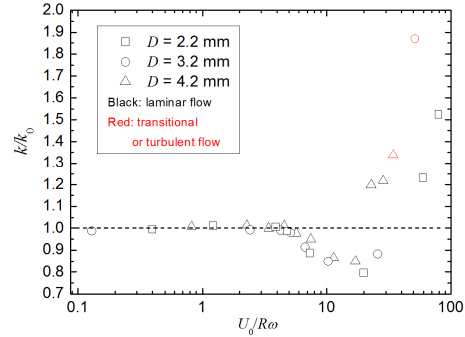


Fig. 4 Thermal conductivity measurement of water flowing in circular tubes

the similar value of $U_0/R\omega$. Also, the sensor signals in the transitional or turbulent flow regime decreased more than those in the laminar flow regime after $U_0/R\omega$ exceeded the critical value. The numerical results of ethanol showed a quite similar trend with those of water, but the temperature change by the flow effect was less severe. Although the proposed dimensionless number could not explain the magnitude difference of the temperature change by variation of the flow regime and thermal property of the liquid, $U_0/R\omega$ was found to be a good criterion to indicate the emergence of the flow effect. The experimental results showed that at $U_0/R\omega \approx 5$, the real part of the signal began to deviate from the static value and then decrease dramatically due to convection. Using the real part of the temperature signal, the thermal conductivity of water flowing in the circular tubes was measured by increasing the mean velocity. The measured thermal conductivity was displayed with respect to the value of $U_0/R\omega$ at 1.5 Hz (Fig. 4). The apparent thermal conductivities were normalized by the static value ($k_0 = 0.607\ \text{W/m}\cdot\text{K}$ at 25°C). When $U_0/R\omega < 5$, the measured thermal conductivities agreed well with the reference value measured without flow effects. The deviations were within $\pm 2.3\%$.

3. Conclusions

The thermal conductivity of water flowing in a circular tube was successfully measured using the 3ω method. When $U_0/R\omega < 5$, it was demonstrated experimentally that k of the flowing water could be measured using the real part of the thermal signal. The uncertainty of measuring k under the water flow condition was 1.5%. Deviations in k of the flowing water with respect to the value of still water did not exceed $\pm 2.3\%$. As future work, the thermal conductivity of flowing sodium is planned to be measured using the same technique.

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