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On the Time-Space Averaged Momentum Equation for Dispersed Flow

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1. Introduction

In the field of nuclear safety analysis, the thermal-hydraulic behavior is simulated by solving the time-space averaged balance equations for two-phase flow. Most thermal-hydraulic codes have the similar structure of the balance equations. However, since the equations are formulated based on continua, a question arises that they can be valid even for the spatially-dispersed phase. In this paper, we discuss the inherent problem of the existing momentum equation, and propose a modified momentum equation.

2. Existing momentum equation

The averaged equations can be obtained by the time-, area-, and/or volume averaging treatment. In spite of the various averaging procedures, the basic forms are similar one another. For a reference, the time-volume averaged momentum equation used in RELAP5 code [1] is given below.

$$\alpha_g\rho_g\frac{\partial v_g}{\partial t}+\alpha_g\rho_g v_g\frac{\partial v_g}{\partial x}=-\alpha_g\frac{\partial p}{\partial x}+\alpha_g\rho_g\vec{g}_x-\alpha_g\rho_g\mathrm{FWG}\cdot v_g+\Gamma_g(v_{gI}-v_g)-\alpha_g\rho_g\mathrm{FIG}\cdot(v_g-v_f)+\mathrm{VM}_g \quad (1)$$

$$\alpha_f\rho_f\frac{\partial v_f}{\partial t}+\alpha_f\rho_f v_f\frac{\partial v_f}{\partial x}=-\alpha_f\frac{\partial p}{\partial x}+\alpha_f\rho_f\vec{g}_x-\alpha_f\rho_f\mathrm{FWF}\cdot v_f-\Gamma_g(v_{fI}-v_f)-\alpha_f\rho_f\mathrm{FIF}\cdot(v_f-v_g)+\mathrm{VM}_f \quad (2)$$

The terms on the right-hand sides of Eqs. 1 and 2 are, respectively, the pressure gradient, gravity, wall drag, momentum transfer owing to phase change, interface drag, and virtual mass force.

Let us consider a bubbly flow under a simplified circumstance (horizontal, steady, fully-developed, no phase change), then we have

$$0=-\alpha_g\frac{\partial p}{\partial x}-\alpha_g\rho_g\mathrm{FWG}\cdot v_g-\alpha_g\alpha_f\mathrm{FIG}\cdot(v_g-v_f) \quad (3)$$

$$0=-\alpha_f\frac{\partial p}{\partial x}-\alpha_f\rho_f\mathrm{FWF}\cdot v_f-\alpha_g\alpha_f\mathrm{FIF}\cdot(v_f-v_g) \quad (4)$$

If we neglect the wall drag on bubbles, Eq. (3) becomes

$$-\alpha_g\frac{\partial p}{\partial x}=\alpha_g\alpha_f\mathrm{FIG}\cdot(v_g-v_f) \quad (5)$$

TRACE, COBRA-TF, and CHATHARE codes do not impose wall drag on dispersed phases (drop and bubble). The RELAP5 code [1] does not calculate a portion of the wall drag on droplets, but consider the wall drag on bubble. Most thermal-hydraulic codes contain the pressure gradient term. Equation 5 implies that the bubble velocity must be higher than the liquid velocity if a favorable pressure is applied. However, the dispersed phase is forced to move by the surrounding phase, the dispersed phase velocity must not exceed the carrier phase velocity in the simplified circumstance. According to the experimental works [2,3], the mean bubble velocity was measured to be slightly lower than the mean water velocity in horizontal bubbly flows, which might be attributed to the interaction between optical/electrical probes and bubbles. It is certain that the bubble velocity must not exceed the water velocity.

Recall that averaged momentum equation contains the pressure gradient term owing to the assumption of continua. If we could remove the pressure gradient term in Eq. (5), the velocities of two phases would be equal. In the next section, we will discuss the pressure gradient term and propose new momentum equations appropriate for disperse flows.

3. Proposed momentum equation

We have applied the volume-averaging to the local instantaneous momentum equation. As a result, we find out that the pressure gradient is devoted not to move the dispersed gas but to move the continuous liquid exclusively. In other words, the pressure gradient term appear only in the continuous phase momentum equation. Detailed derivation will be presented at the conference. As a matter of fact, a bubble is forced to move by both the asymmetric pressure distribution (form drag) and the shear stress force (shear drag). However, it is more proper to express the form drag in terms of the relative velocity rather than the global pressure gradient. Therefore, it is reasonable to remove the pressure gradient term in determining the bubble velocity, since the pressure difference between posterior and anterior is caused by the relative velocity rather than the global pressure gradient. However, in a vertical flow, the pressure gradient force should be expressed by the buoyancy force. Otherwise, the rising bubble velocity becomes slower than the surrounding liquid velocity.

Similarly, for a dispersed liquid flow, the pressure gradient term can be neglected in the momentum equation. Droplet flow could be categorized into a kind of particulate flow (dusty gas flow). There are several approaches to predict the particle movement in dusty gas flow. Of them, the point-particle approach assumes that the particles are sufficiently small that they perfectly follow the local carrier phase [4]. In the point-particle approach, the motion of particles is not described by the local pressure gradient. In other words, the pressure affects only the motion of the carrier phase. Of course, bubbles cannot be treated like point-particles, but droplets could be. As stated previously, neglecting the pressure gradient term in the dispersed phase is practicable when the control volume size is relatively larger than the bubble/droplet size. In this regard, neglecting the pressure gradient term in the dispersed phase is in harmony with the point-particle approach for a particulate flow.

Consequently, when the buoyancy force is negligible, we propose

$$\alpha_g \rho_g \frac{\partial v_g}{\partial t} + \alpha_g \rho_g v_g \frac{\partial v_g}{\partial x} = \Gamma_g (v_{gI} - v_g) - \alpha_g \rho_g \mathrm{FIG} \cdot (v_g - v_f) + \mathrm{VM}_g \quad (6)$$

$$\alpha_f \rho_f \frac{\partial v_f}{\partial t} + \alpha_f \rho_f v_f \frac{\partial v_f}{\partial x} = -\frac{\partial p}{\partial x} - \mathrm{FW} - \Gamma_g (v_{fI} - v_f) - \alpha_f \rho_f \mathrm{FIF} \cdot (v_f - v_g) + \mathrm{VM}_f \quad (7)$$

for a horizontal bubbly flow, and

$$\alpha_g \rho_g \frac{\partial v_g}{\partial t} + \alpha_g \rho_g v_g \frac{\partial v_g}{\partial x} = -\frac{\partial p}{\partial x} - \mathrm{FW} + \Gamma_g (v_{gI} - v_g) - \alpha_g \rho_g \mathrm{FIG} \cdot (v_g - v_f) + \mathrm{VM}_g \quad (8)$$

$$\alpha_f \rho_f \frac{\partial v_f}{\partial t} + \alpha_f \rho_f v_f \frac{\partial v_f}{\partial x} = -\Gamma_g (v_{fI} - v_f) - \alpha_f \rho_f \mathrm{FIF} \cdot (v_f - v_g) + \mathrm{VM}_f \quad (9)$$

for a dispersed droplet flow. For a vertical bubbly flow,

$$\alpha_g \rho_g \frac{\partial v_g}{\partial t} + \alpha_g \rho_g v_g \frac{\partial v_g}{\partial x} = \Gamma_g (v_{gI} - v_g) - S_{gf} \cdot (v_g - v_f) + \alpha_g (\rho_f - \rho_g) g + \mathrm{VM}_g \quad (10)$$

$$\alpha_f \rho_f \frac{\partial v_f}{\partial t} + \alpha_f \rho_f v_f \frac{\partial v_f}{\partial x} = -\frac{\partial p}{\partial x} - \mathrm{FW} - \Gamma_g (v_{fI} - v_f) - S_{gf} (v_f - v_g) - \rho_f g + \mathrm{VM}_f \quad (11)$$

Equations 8 and 9 can be used for a vertical dispersed liquid flow, since the buoyancy effect is negligible.

4. Test and result

SPACE code was modified according to the proposed equation. We simulated an annular flow in a pipe with D=0.1 m and L=20 m. The pressure difference 0.2 bar was applied between the inlet and the outlet. At the inlet, while the void fraction was set to 0.9, the drop fraction was adjusted between 0.01 to 0.05. To exclude any other effect, entrainment, deposition and phase change were disabled. The wall drag was not imposed on droplet. Figure 1 and 2 show the simulation results for the existing equation and for the proposed equation, respectively. It is observed in Fig. 1 that the droplet velocity is higher than the gas velocity for all cases. However, we can see that the droplet velocity equals the gas velocity in Fig. 2.

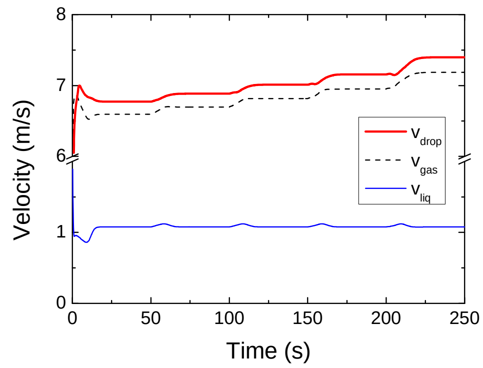

Fig.1 Test result of the existing momentum equation

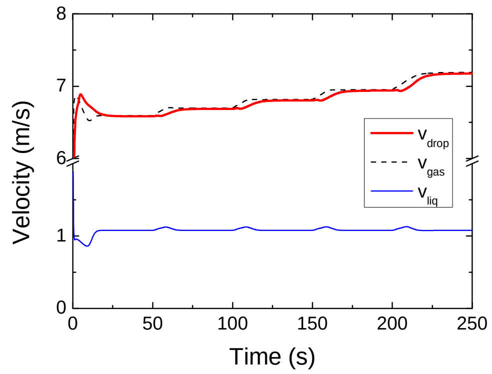

Fig.2 Test result of the proposed momentum equation

5. Conclusion

A new momentum equation has been proposed appropriated for dispersed flows.

Acknowledgments

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