Application of the Gaussian Quadrature to the Azimuthal Angle Discretization in MOC

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1. Introduction

Korea Atomic Energy Research Institute(KAERI) has been developing a 3-D whole-core neutron transport code, DeCART, based on the method of characteristic (MOC)[1]. A sensitivity study on the ray tracing parameters (ray spacing, number of azimuthal angles, number of polar angles) showed that the error introduced by the azimuthal angle discretization is quite large and the default input value of the DeCART code for the number of azimuthal angles, 4 currently, is not sufficient. In this paper, a new azimuthal angle discretization scheme using Gaussian quadrature in MOC was proposed and tested.

2. Methods and Results

2.1 Sensitivity Study on the Ray Tracing Parameters

A sensitivity study on the ray tracing parameters was performed for the VHTR(PMR-200) fuel element models shown in Figure 1(a) and Figure 1(b) : a double heterogeneity(DH) model and a reactivity equivalent physical transformation(RPT) model.

(a) VHTR DH (b) VHTR RPT (c) PWR

Fig. 1. Fuel element models.

Table 1 shows the sensitivity of the DeCART results on the number of polar angles (N_p). The 190-group DeCART library for the VHTR system was used in this calculation. The error introduced by the polar angle discretization is very small and the default input value of DeCART code for the number of polar angle, 2 currently, is sufficient.

Table 1 : Sensitivity of DeCART results on N_p

N_p	DH Model		RPT Model	
	k_∞	Δk (pcm)	k_∞	Δk (pcm)
Ext.	1.42105	-	1.42338	-
4	1.42104	-1	1.42334	-4
3	1.42103	-2	1.42331	-7
2	1.42097	-8	1.42295	-43

Table 2 and Table 3 show the sensitivity of the DeCART results on the ray spacing(δ) and the number of azimuthal angles(N_a) for the two unit cell models. The extrapolated result from the cases with (δ ,N_a, N_p)= (0.005,50,4), (0.005,40,4), (0.01,50,4), and (0.01,40,4) was taken as a reference solution. The errors in the RPT model are very large while those in the DH model are relatively small. We also observe that the magnitudes of errors are mainly dependent on N_a rather than δ .

Table 2 : DeCART error in the DH model (pcm).

N_a \ Δ	0.005	0.01	0.02	0.03	0.04	0.05
12	-13	-13	-1	+5	-63	-19
10	-8	-17	-14	-16	-68	+11
8	-22	-11	-32	-73	-31	+17
6	-42	-59	-48	-30	+5	-101
4	-102	-134	-96	-144	-200	-101

Table 3 : DeCART error in the RPT model (pcm).

N_a \ Δ	0.005	0.01	0.02	0.03	0.04	0.05
12	-133	-180	-27	+19	-358	-180
10	-73	-58	-146	-102	-293	-58
8	-113	-107	-101	+63	-291	-107
6	-160	+154	-277	-183	-139	+154
4	-512	-748	-420	-527	-620	-748

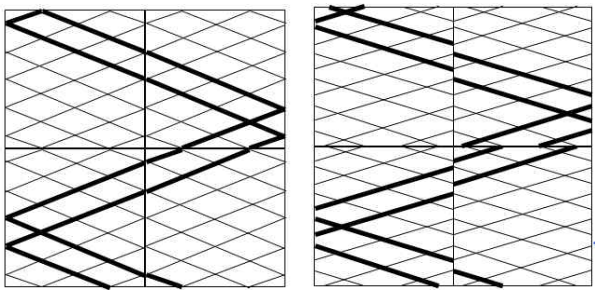
The same sensitivity study was performed for the PWR fuel element shown in Figure 1(c) wth the 47-group library. Table 4 shows the sensitivity of DeCART results on δ and N_a for the PWR fuel element. According to the extrapolated results obtained from those with δ=0.001 and δ=0.005, the error introduced by azimuthal angle discretization can be maintained less than 100[pcm] and 50[pcm] when N_a is larger than 10 and 20, respectively.

Table 4 : DeCART error in the PWR fuel model (pcm).

N_a \ δ	Extra-polated	0.001	0.005	0.01	0.02	0.03	0.04
24	-40	-40	-31	-54	-56	+31	+23
20	-51	-51	-55	-89	-36	+62	+33
18	-58	-58	-66	-81	-18	+64	+54
16	-69	-68	-48	-80	-121	+33	+55
14	-92	-91	-78	-134	-147	+84	+78
12	-88	-87	-73	-99	-97	+97	+117
10	-154	-153	-141	-163	-211	-109	+106
8	-198	-199	-216	-186	-202	-69	-163
6	-320	-320	-327	-302	-308	-279	-126
4	-511	-511	-506	-559	-475	-369	-292

2.2 Gaussian Azimuthal Angle Discretization

The error introduced by the azimuthal angle discretization can be reduced by adopting Gaussian quadrature because the MOC calculation procedure can be interpreted as the integration procedure over the phase variables (azimuthal angle, polar angle, space, and neutron energy). In the DeCART code, 90° in the azimuthal angle is divided into equally spaced N_a angles and then the angles are slightly modified so that the rays at the interface between the two unit cells are connected exactly as shown in Figure 2(a). The weights for the representative angles are determined according to the azimuthal angle ranges they cover. In the Gaussian scheme, 90° in the azimuthal angle is divided into N_a angles by using the Gaussian quadrature points. The weights for the representative angles are determined according to the Gaussian quadrature weights. In this case, the rays are not connected exactly at the interface between the two unit cells as shown in Figure 2(b). The ray tracing across the unit cells is performed by selecting the nearest ray with the same azimuthal angle at the interfaces, which introduces a new approximation that the incoming angular flux on a surface of a unit cell is the same as the outgoing angular flux at the nearest point on the surface of the adjacent unit cell. However, this additional approximation is not expected to introduce a large error because the ray spacing always remains very small in order to maintain the error of integration over the space variables small.



(a) DeCART scheme (b) Gaussian quadrature scheme

Fig. 2. Modular ray tracing schemes

A toy program that solves the multi-group neutron transport equation by MOC with the Gaussian azimuthal angle discretization was written and the Gaussian quadrature scheme was tested. Table 5 shows the sensitivity of the Gaussian quadrature scheme results on δ and N_a for the PWR fuel element. The error introduced by the azimuthal angles discretization can be maintained less than 100[pcm] and 50[pcm] when N_a is larger than 6 and 8, respectively, which means that the new scheme can reduce the computation time by minimizing the number of azimuthal angles.

To ensure that the improvements in the accuracy came from the adoption of Gaussian quadrature in azimuthal angle discretization, another scheme was tested. In this scheme, 90° in the azimuthal angle are divided into equally spaced N_a angles and the weights for the representative angles are assigned equally. The rays are not connected exactly at the interface between the two unit cells as they are in the Gaussian quadrature scheme. Table 6 shows the errors of the equally-spaced azimuthal angle scheme. We can find that the errors of this scheme are as large as those of the DeCART scheme, which means that the adoption of the Gaussian quadrature for azimuthal angle discretization was the cause of the error reduction in Table 5.

Table 5 : Error of Gaussian scheme in the PWR fuel model (pcm).

N_a \ δ	Extra-polated	0.001	0.005	0.01	0.02	0.03	0.04
10	-28	-27	-6	-72	-54	-85	-69
8	-89	-89	-85	-124	-141	-31	-108
6	-101	-101	-99	-132	-109	-70	-242
4	-149	-148	-132	-165	-207	-220	-299

Table 6 : Error of equally-spaced angel scheme in the PWR fuel model (pcm).

N_a \ δ	0.005	0.01	0.02	0.03	0.04
10	-137	-133	-148	-200	-118
8	-194	-215	-165	-164	-118
6	-328	-323	-358	-341	-242
4	-502	-501	-540	-549	-459

3. Conclusions

In this paper, the sensitivity of the DeCART results on the ray spacing and the number of azimuthal angles were analyzed not only for the VHTR fuel elements but also for the PWR fuel elements. The study showed that the error introduced by the azimuthal angle discretization in DeCART code is quite large and the default input value of the DeCART code for the number of azimuthal angles, 4 currently, is not sufficient. A new azimuthal angle discretization scheme which adopts Gaussian quadrature set for azimuthal angle discretization was proposed. Although the new scheme introduces an additional approximation, a sensitivity study showed that the new scheme improves the accuracy of the MOC calculation much. The next step for this study is to implement the new scheme into the DeCART code.

REFERENCES

[1] Jin Young Cho, et. al., "3-D Whole Core Transport Calculation Methodology for DeCART Code," KAERI Technical Report, KAERI/TR-2365/2003 (2003).