Calculation of Total Number of Disintegrations after Intake of Radioactive Nuclides Using the Pseudoinverse Matrix

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1. Introduction

Calculation of total number of disintegrations after intake of radioactive nuclides is indispensable to calculate a dose coefficient which means committed effective dose per unit activity (Sv/Bq). In order to calculate the total number of disintegrations analytically, Birchall's algorithm [1] has been commonly used. As described below, an inverse matrix should be calculated in the algorithm. As biokinetic models have been complicated, however, the inverse matrix does not exist sometime and the total number of disintegrations cannot be calculated. Thus, a numerical method has been applied to DCAL code [2] used to calculate dose coefficients in ICRP publication and IMBA code [3].

In this study, however, we applied the pseudoinverse matrix to solve the problem that the inverse matrix does not exist for. In order to validate our method, the method was applied to two examples and the results were compared to the tabulated data in ICRP publication. MATLAB 2012a was used to calculate the total number of disintegrations and *expm* and *pinv* MATLAB built-in functions were employed.

2. Materials and Methods

2.1 Calculation of Total Number of Disintegrations

Total number of disintegrations after intake of radionuclides can be analytically calculated using eqn. (1) [1].

$$u_i(t) = \lambda[A]^{-1}\left[e^{[A]t} - [I]\right]x_i(0), \quad (1)$$

where $u_i(t)$ is the total number of disintegrations up to time t in compartment i, $[A]^{-1}$ is the inverse matrix of [A], [I] is the identity matrix, and $x_i(0)$ is initial amount in compartment i. λ is 8.64×10^4 for unit conversion if $x_i(0)$ is given in Bq. The matrix [A] is formed by modifying a rate matrix [R] as follows

$$a_{ij} = r_{ji}, \text{ for i}\neq\text{j, and}$$

$$a_i = -\sum_{\substack{j=1 \\ j\neq i}}^{N} r_{ji}$$

where r_{ji} is a transfer coefficient from compartment i to j. $e^{[A]t}$ is the matrix exponential and defined by its series expansion [4]:

$$e^{[A]t} = \sum_{i=0}^{\infty} \frac{([A]t)^i}{i!}, A^0 = \text{I}. \quad (2)$$

In this study, we calculated the matrix exponential using *expm* MATLAB built-in function. The *expm* function uses the Padé approximation with scaling and squaring, which is one of the methods to improve both accuracy and efficiency [5].

2.2 Pseudoinverse Matrix

In order to calculate the total number of disintegrations, the inverse matrix of [A] should exist. If the [A] matrix is a singular matrix (i.e. a determinant of [A] is zero), however, the inverse matrix of [A] does not exist. Thus, we applied the pseudoinverse matrix to solve this problem.

The pseudoinverse matrix was first defined by the American mathematician Eliakim Moore in the 1920's and rediscovered by the influential British mathematician and physicist Roger Penrose in the 1950's, and often has their names attached [6]. The pseudoinverse matrix gives the least squares solution to the linear system and agrees with the inverse of A if A is a non-singular square matrix. The pseudoinverse matrix of A is represented by the symbol A^+ and can be obtained as eqn. (3) if A is not a zero matrix.

$$A^+ = Q\Sigma^{-1}P^T \quad (3)$$

In the eqn. (3), the each column of Q consists of first n normalized (unit) eigenvectors where n is the number of singular values. $\sum$ is a diagonal singular value matrix and each diagonal component is comprised of the square roots of positive eigenvalues and $\sum^{-1}$ is the inverse matrix of $\sum$. Each column of P is formed by multiplying A and the column of Q and dividing by the diagonal component of $\sum$. P^T is the transposition of P. In this study, we calculated the pseudoinverse matrix using *pinv* MATLAB built-in function.

Table 1. Comparison of U(50) in the regions of the alimentary tract for ingestion of ^{90}Sr

	Oral Cavity	Oesophagus (Fast)	Oesophagus (Slow)	Stomach	Small Intestine	Right Colon	Left Colon	Recto sigmoid
This Study	12	6.3	4	4.20×10^{3}	1.01×10^{4}	3.31×10^{4}	3.31×10^{3}	3.31×10^{3}
ICRP 100	12	6.3	4	4.2×10^{3}	1×10^{4}	3.3×10^{3}	3.3×10^{3}	3.3×10^{3}

2.3 Validation of Pseudoinverse Matrix Method

For validation of the proposed method using pseudoinverse matrix, two examples were used. The first example is calculation of the total number of disintegrations over 50 years, U(50), in the regions of the alimentary tract predicted by the human alimentary tract model (HATM) proposed in ICRP publication 100 [7] and the ICRP publication 67 model [8] for unit acute ingestion of ^{90}Sr. The U(50) for adult male and total diet were calculated and compared to the U(50) tabulated in ICRP Publication 100. Fig. 1 shows the compartment model of the first example. The second example is calculation of dose coefficient for ingestion and inhalation of ^{60}Co. The dose coefficients for inhalation with type M and S and for ingestion with f_1 of 0.1 and 0.05 were calculated. The compartment model to calculate the dose coefficient is identical to the compartment model used in ICRP publication 68 [9]. SEECAL 2.0 code [10] was used to calculate specific effective energy (SEE) for ^{60}Co. The calculated dose coefficient in this study was compared to the dose coefficient in ICRP publication 68.

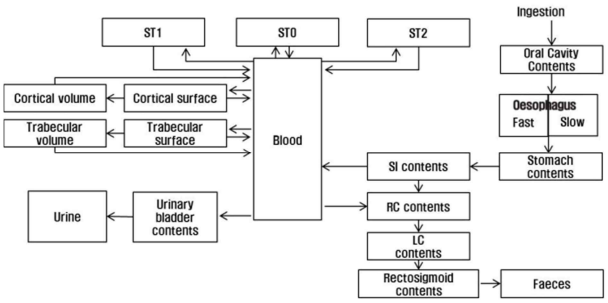

Fig. 1. The compartment model for ingestion of ^{90}Sr

3. Results and Discussion

Table 1 shows the U(50) in the regions of the alimentary tract in the first example calculated in this study and tabulated in ICRP publication 100. As shown in table 1, the U(50) in this study and ICRP publication 100 are equal except rounding error. Calculation time was only 0.001584 seconds by MATLAB 2012a at the Intel Core i5 2.80GHz.

Table 2 shows the dose coefficients for inhalation with type M and S and for ingestion with f_1 of 0.1 and 0.05 calculated in this study and in ICRP publication 68. All of the dose coefficients calculated using the pseudoinverse method for four cases are equal to those tabulated in ICRP publication 68 except rounding error. We confirmed applicability of the pseudoinverse matrix in calculation of U(50) and dose coefficients through the two example including sophisticated biokinetic model.

Table 2. Dose coefficients for intake of ^{60}Co

	Inhalation (Type M)	Inhalation (Type S)	Ingestion (f_1=0.1)	Ingestion (f_1=0.05)
This Study	7.14×10^{-9}	1.66×10^{-8}	3.42×10^{-9}	2.52×10^{-9}
ICRP 68	$7.1\times\times10^{-9}$	1.7×10^{-8}	3.4×10^{-9}	2.5×10^{-9}

4. Conclusion

In this study, the pseudoinverse matrix was applied in calculation of the total number of disintegrations to solve the problem that the inverse matrix of [A] does not exist for some compartment models. We confirmed that the U(50) and the dose coefficient using the pseudoinverse matrix are equal to those in ICRP publication using numerical method through two examples. This study has a great meaning in that the total number of disintegrations and the dose coefficient can be calculated simply even though the compartment model is sophisticated.

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