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Two-Node Nonlinear Iterative AFEN Method in Three-Dimensional Geometry

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Abstract

The nonlinear iterative scheme using two-node AFEN calculation is extended to three-dimensional rectangular geometry and applied to two well-known benchmark problems. This scheme is based on solving two-node problems with AFEN method flux expansion and use of two nonlinear correction factors at every interface instead of one factor in the conventional scheme. The use of two correction factors provides higher-order accurate interface fluxes as well as currents which are used as the boundary conditions of the two-node problem. The numerical results show that this method gives the same solution as that of the original AFEN method and the computing time is significantly reduced in comparison with the original AFEN method. However, the three-dimensional two-node acceleration scheme needs to update edge fluxes more frequently than that of the two-dimensional calculation to achieve stable convergence.

I. Introduction

The two-node nonlinear iteration scheme[1,2] based on the nodal expansion method (NEM) was successfully applied to two- and three-dimensional geometries and turned out to be very effective in reducing computing time and implementing effort associated with higher order nodal methods. This scheme solves the modified finite difference method (FDM) current equation given for interface of nodes n and $n+1$ with side-length h :

$$J_g^{n,n+1} = -\frac{2D_g^n D_g^{n+1}}{h(D_g^n + D_g^{n+1})}(\bar{\phi}_g^{n+1} - \bar{\phi}_g^n) - \frac{\hat{D}_g^{n,n+1}}{h}(\bar{\phi}_g^{n+1} + \bar{\phi}_g^n), \qquad (1)$$

where D_g^n, D_g^{n+1} and $\hat{D}_g^{n,n+1}$ represent the diffusion coefficients and nonlinear correction factor, respectively. Determining the nonlinear correction factor $\hat{D}_g^{n,n+1}$ so that the interface current $J_g^{n,n+1}$ preserves the value of a higher-order nodal method makes the solution of this modified FDM scheme equivalent to that of the higher-order nodal method itself. In conventional nodal methods using transverse-leakages, the two-node problem is solved for the interface current of the two nodes with currently available node-average fluxes and transverse-leakages shapes of both nodes as boundary conditions.

A two-node nonlinear scheme[3,4,5] based on the AFEN method[6,7] was developed and applied to two-dimensional problems. In this paper, the two-node nonlinear AFEN method is extended to three-dimensional geometry and computationally efficient procedures are investigated for the convergence of the solution. Using the three-dimensional AFEN method, the two-node problem is solved for the interface current of the two nodes with two node-average fluxes, 20 edge fluxes and 8 other direction interface currents as boundary conditions. The higher-order correction factors at the interface are updated by equating the resultant higher-order interface current with the modified FDM current given by Eq. (2) in Section II. The results show that the two-node AFEN method in three-dimensional geometry reproduces the same solution as that of the original AFEN method and that the computing time is significantly reduced in comparison with the original AFEN method.

II. Theory and Methodology

The two-node acceleration scheme for three-dimensional AFEN adopts two correction factors per interface. The correction factors at the interface of node n and node $n+1$ are defined respectively as follows :

$$J_{g,R}^n = -\frac{2D_g^n}{h}(\tilde{\phi}_g^{n,n+1} - \bar{\phi}_g^n) - \frac{2D_g^{n\to n+1}}{h}(\tilde{\phi}_g^{n,n+1} + \bar{\phi}_g^n) \tag{2a}$$

$$J_{g,L}^{n+1} = \frac{2D_g^{n+1}}{h}(\tilde{\phi}_g^{n,n+1} - \bar{\phi}_g^{n+1}) + \frac{2D_g^{n+1\to n}}{h}(\tilde{\phi}_g^{n,n+1} + \bar{\phi}_g^{n+1}) \tag{2b}$$

where

$\tilde{\phi}_g^{n,n+1}$: interface flux between nodes n and $n+1$,

$D_g^{n\to n+1}$: correction factor for node n,

$D_g^{n+1\to n}$: correction factor for node $n+1$,

$\tilde{\phi}_g^{n,n+1}$: interface flux of node n and $n+1$,

$\bar{\phi}_g^n, \bar{\phi}_g^{n+1}$: node average fluxes of nodes n and $n+1$, respectively,

D_g^n, D_g^{n+1} : diffusion coefficients of nodes n and $n+1$, respectively.

Employing the two factors makes it possible for this scheme to carry the higher-order accurate interface current and flux simultaneously during the FDM routine. By equating Eq. (2a) and Eq. (2b), the interface current and flux are obtained :

$$\begin{aligned} J_g^{n,n+1} &= J_{g,R}^{n} = J_{g,L}^{n+1} \\ &= \frac{2(D_g^n - D_g^{n\to n+1})(D_g^{n+1} + D_g^{n+1\to n})\bar{\phi}_g^{\,n}}{(D_g^n + D_g^{n+1} + D_g^{n\to n+1} + D_g^{n+1\to n})h} \\ &\quad - \frac{2(D_g^{n+1} - D_g^{n+1\to n})(D_g^n + D_g^{n\to n+1})\bar{\phi}_g^{\,n+1}}{(D_g^n + D_g^{n+1} + D_g^{n\to n+1} + D_g^{n+1\to n})h} \end{aligned} \tag{3a}$$

$$\tilde{\phi}_g^{\,n,n+1} = \frac{(D_g^n - D_g^{n\to n+1})\bar{\phi}_g^{\,n} + (D_g^{n+1} - D_g^{n+1\to n})\bar{\phi}_g^{\,n+1}}{D_g^n + D_g^{n+1} + D_g^{n\to n+1} + D_g^{n+1\to n}} \tag{3b}$$

Note that the equation system for the node-average fluxes derived from the current equation has the same structure to that of the FDM scheme, that is 7-point difference equation in the three-dimensional rectangular geometry. We easily note that if we set the two nonlinear correction factors to zero, Eq.(3a) and Eq.(3b) degenerate into the usual FDM equations for the interface current and flux, respectively. If the above FDM equation is solved with initially guessed nonlinear correction factors, the node-average fluxes, interface currents and interface fluxes are obtained and these are then used in calculating edge fluxes and solving two-node problems. At present, the edge fluxes are calculated by the edge balance method. The simplified form of the edge balance equation is given by

$$\begin{aligned} &\boldsymbol{t}_{ijk}^{L}\boldsymbol{\phi}_{i-1,j,k} + \boldsymbol{t}_{ijk}^{R}\boldsymbol{\phi}_{i+1,j,k} + \boldsymbol{t}_{ijk}^{T}\boldsymbol{\phi}_{i,j+1,k} + \boldsymbol{t}_{ijk}^{B}\boldsymbol{\phi}_{i,j-1,k} \\ &\quad + \boldsymbol{t}_{ijk}^{U}\boldsymbol{\phi}_{i,j,k+1} + \boldsymbol{t}_{ijk}^{D}\boldsymbol{\phi}_{i,j,k-1} + \boldsymbol{t}_{ijk}^{C}\boldsymbol{\phi}_{i,j,k} = \boldsymbol{p}_{i,j,k}. \end{aligned} \tag{4}$$

Numerical test shows that Eq.(4) must be solved once in every outer iteration for stable convergence in three-dimensional problems. After edge flux calculation, the equations for updating nonlinear correction factors are derived. Solving the two-node problem by the AFEN method starts from expanding the intranodal flux distribution of a node into the same non-separable analytic functions as in the original AFEN method except the constant term. The constant term is deleted since Eq.(3) already implies the neutron balance in the node. The flux expansion becomes

$$\begin{aligned} g_\mu^u(u) &= A_{\mu,1}^u SN[k_\mu u] + A_{\mu,2}^u CS[k_\mu u], \\ h_\mu^{u,v}(u,v) &= B_{\mu,11}^{u,v} SN[\frac{k_\mu u}{\sqrt{2}}]\cdot SN[\frac{k_\mu v}{\sqrt{2}}] + B_{\mu,12}^{u,v} SN[\frac{k_\mu u}{\sqrt{2}}]\cdot CS[\frac{k_\mu v}{\sqrt{2}}] \\ &\quad + B_{\mu,21}^{u,v} CS[\frac{k_\mu u}{\sqrt{2}}]\cdot SN[\frac{k_\mu v}{\sqrt{2}}] + B_{\mu,22}^{u,v} CS[\frac{k_\mu u}{\sqrt{2}}]\cdot CS[\frac{k_\mu v}{\sqrt{2}}], \\ \xi_\mu(x,y,z) &= g_\mu^x(x) + g_\mu^y(y) + g_\mu^z(z) + h_\mu^{x,y}(x,y) + h_\mu^{y,z}(y,z) + h_\mu^{z,x}(z,x). \end{aligned} \tag{5}$$

All the notations used in this equation are given in Ref. 8. In Eq.(5), all the coefficients are represented in terms of node-average flux (1), interface currents (5) and edge fluxes (12). The two-node equation can be derived by using the continuity condition of the neutron flux across the interface as follows :

$$\begin{aligned}
\tilde{\phi}_x^{n,n+1} &= a_x^n J_x^{n,n+1} + b_x^n \bar{\phi}^n + c_x^n, \\
\tilde{\phi}_x^{n,n+1} &= a_x^{n+1} J_x^{n,n+1} + b_x^{n+1} \bar{\phi}^{n+1} + c_x^{n+1}, \\
J_x^{n,n+1} &= (a_x^n - a_x^{n+1})^{-1} \{ b_x^{n+1} \bar{\phi}_x^{n+1} - b_x^n \bar{\phi}_x^n + c_x^{n+1} - c_x^n \},
\end{aligned} \tag{6}$$

where

$$c_x^n = c_x^n(J_{0,y}^n, J_{1,y}^n, J_{0,z}^n, J_{1,z}^n, \phi_{uv}^{00,}, \phi_{uv}^{01,}, \phi_{uv}^{10,}, \phi_{uv}^{11,}; uv = xy, yz, zx).$$

During implementation of this equation, nodal values used in c_x^n are all known values, and this fact simplifies the derivation and calculation of Eq. (6). After solving the neutron current and flux at the interface of the two-node problem by the AFEN method, the nonlinear correction factors are updated. The equation for updating the nonlinear correction factors are obtained by solving Eq.(2) in terms of the nonlinear correction factors :

$$\begin{aligned}
D_g^{n \to n+1} &= \frac{-J_g^{n,n+1} h - 2D_g^n (\tilde{\phi}_g^{n,n+1} - \bar{\phi}_g^n)}{2(\tilde{\phi}_g^{n,n+1} + \bar{\phi}_g^n)}, \\
D_g^{n+1 \to n} &= \frac{J_g^{n,n+1} h + 2D_g^{n+1} (\tilde{\phi}_g^{n,n+1} - \bar{\phi}_g^{n+1})}{2(\tilde{\phi}_g^{n,n+1} + \bar{\phi}_g^{n+1})}
\end{aligned} \tag{7}$$

Once we update all the correction factors over the whole core, the calculation flow return to the next FDM outer iteration. The whole calculational sequence is shown in Fig. 1. According to the results, two sweepings of LSOR FDM inner iterations are most effective in reducing the computing time than the case of one sweeping. In the three-dimensional two-node scheme, the edge fluxes must be updated in every outer iteration. This is due to the large number of the edge fluxes relative to the interface fluxes than the two-dimensional case and during iteration, consistent convergence (convergence of all nodal unknowns together) of the edge fluxes must be made. As the results in the next section show, we also found that the correction factors need not be fully converged. We can get the accurate solution using roughly converged correction factors and this reduces further computing time spent in the edge flux update and the two-node AFEN solution.

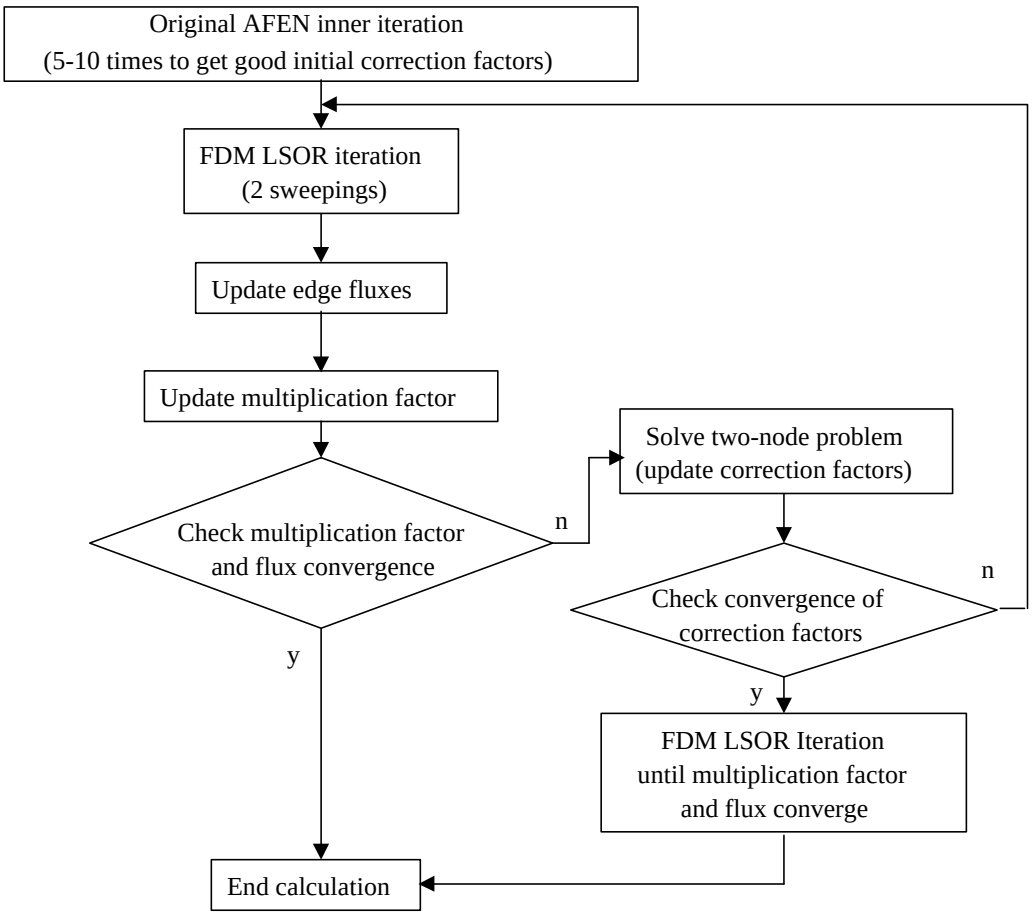
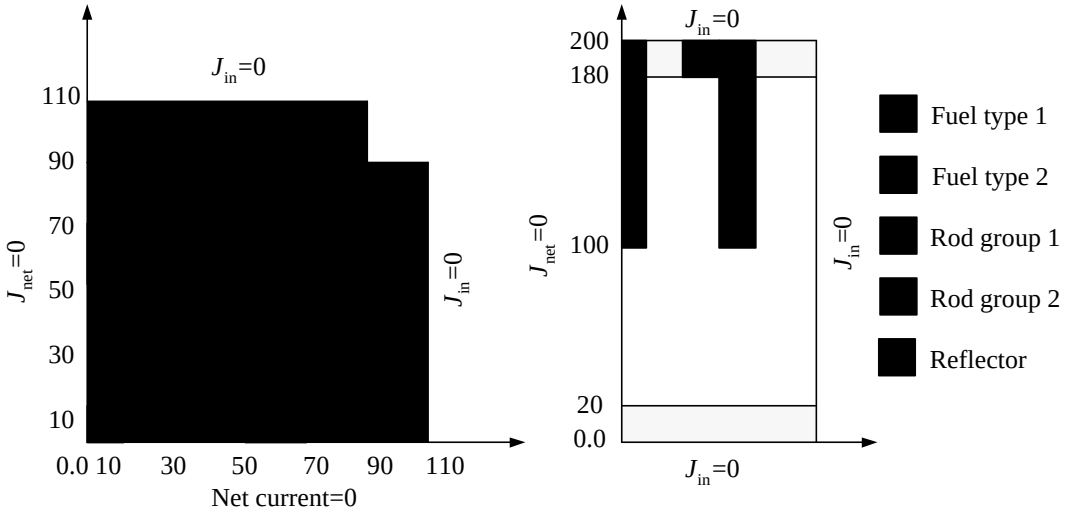


Fig. 1. Calculational flowchart of AFEN-3D two-node calculation.

III. Results and Discussion

For the verification of our three-dimensional two-node acceleration scheme, two benchmark problems were tested. Benchmark problem I is the well-known LMW core which has two-zone core containing 35 fuel assemblies (width 20cm) in a quadrant. The core is reflected both radially and axially by 20 cm of water with the active core height 160cm. The configuration of the LMW reactor is shown in Fig. 2. To investigate a more realistic and much larger problem, IAEA-3D benchmark problem was tested as the second problem. It has 76 assemblies per quadrant and the core height is 380cm. To compare the solutions, the power distributions averaged over z-direction of the nonlinear iterative AFEN method and of the original AFEN method are compared with each other in Fig. 3. The two solutions are exactly the same if the correction factors are fully converged and the minor truncation errors are neglected. The case of using the correction factor criteria of 0.05 provides no meaningful diffenrece with the converged solution.



a) quadrant of reactor b) vertical cross section

Fig. 2. Configuration of LMW benchmark problem

keff= 1.028710 (original AFEN)
1.028710 (two-node AFEN)

0.002	0.002	0.002	0.001	0.000	-0.001	-0.002	-0.002
0.002	0.002	0.002	0.001	0.000	-0.001	-0.002	-0.002
0.002	0.002	0.004	0.001	0.000	-0.001	-0.002	-0.002
0.001	0.001	0.001	0.001	0.000	-0.001	-0.002	
0.000	0.000	0.000	0.000	-0.001	-0.001	-0.002	
-0.001	-0.001	-0.001	-0.001	-0.001	-0.002		
-0.002	-0.001	-0.001	-0.002	-0.002			
-0.002	-0.002	-0.002					

Fig. 3. % difference of the power distributions between original AFEN and two-node AFEN for IAEA-3D benchmark problem
(correction factor convergence criteria : 0.05).

Tables 1 and 2 show the computing times of the original three-dimensional AFEN and two-node AFEN with various correction factor criteria. They show that the computing time decreases about 4 times with very tight correction factor criteria and we can obtain computing time reduction of about 8 times using correction factor criteria of 0.5 with the practically same results.

Table 1. Computing Time of LMW Benchmark Problem (6×6×10 Node)

correction factor convergence criteria	computing time(sec) on HP-780	keff difference(%)	max power difference(%) (z direction averaged)
original AFEN	42.4	0.9997150	reference
5.0E-1	4.82	-1.0E-4	0.087
5.0E-2	6.34	1.0E-5	-0.007
5.0E-3	7.78	0	0.001
5.0E-4	9.90	0	0.001
5.0E-5	10.98	flux converged before correction factor	

(relative convergence criteria : flux=1E-6, keff=1E-7)

Table 2. Computing Time of IAEA-3D Benchmark Problem (9×9×20 Node)

correction factor convergence criteria	computing time(sec) on HP-780	keff difference(%)	max power difference(%) (z direction averaged)
original AFEN	401	1.02870985	reference
5.0E-1	48	-3E-6	0.011
5.0E-2	60	0	0.004
5.0E-3	95	0	0.003
5.0E-4	112	flux converged before correction factor	

(relative convergence criteria : flux=1E-6, keff=1E-7)

III. Conclusions

In this paper, nonlinear two-node acceleration scheme for three-dimensional AFEN method is presented and tested on two benchmark problems. The method is based on solving two-node problems and use of two nonlinear correction factors to provide higher-order accurate interface fluxes as well as currents which are used as the boundary conditions of the two-node problem. The numerical results show that the converged solutions of the two-node nonlinear AFEN method are the same with those

of the original AFEN method. Also, we found that the use of weaker convergence criteria on the correction factor than on the flux or multiplication factor can be used to get the accurate solutions. The results indicate that the computing time can be reduced by a factor of 4~8 for usual three-dimensional problems. This value is slightly smaller than that of two-dimensional problems due to updating of the edge fluxes at every outer iteration. However, the computing times of the method are significantly reduced in comparison with those of the original AFEN method. Therefore, the nonlinear two-node acceleration scheme for the AFEN method can be effectively used in practical nuclear design problems.

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