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Estimation of CANDU Reactor Zone Controller Level by Generalized Perturbation Method

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ABSTRACT

The zone controller level change due to refueling operation has been studied using a generalized perturbation method. The generalized perturbation method provides sensitivity of zone power to individual refueling operation and incremental change of zone controller level. By constructing a system equation for each zone power, the zone controller level change was obtained. The details and a proposed model for future work are described.

INTRODUCTION

The zone controller unit (ZCU) of a Canada deuterium uranium (CANDU) reactor is a unique mechanism that maintains reference power distribution during normal operation which involves daily refueling. There are 14 ZCU's which are composed of light water and feed tubes. The water level of ZCU changes depending on zone power. Therefore the ZCU level represents excess power (or reactivity) of individual zone and can be used as an index for selecting refueling channel of a CANDU reactor.

In the previous work,[1] the zone power fractions, defined as the ratio of zone power over total power, were considered as system characteristics of interest when the ZCU level was estimated. The expression for variation of zone power fraction caused by a refueling perturbation was obtained by a conventional perturbation technique combined with a flux-difference method.

However, the zone power fraction and its variation are very small, which causes a considerable truncation error during the calculation. In this study, we have revised system equations such that the zone power is used as system response and the variational approach is used to derive generalized perturbation theory (GPT) expressions for the response.

DERIVATION OF PERTURBATION EQUATIONS

System Characteristics of Interest

The zone power and the total reactor power can be represented as linear functionals:

$$P_l = \langle H_o, \phi_o \rangle_l \tag{1}$$

$$P_T = \langle H_o, \phi_o \rangle_R \tag{2}$$

where P_T is the total reactor power to which the flux is normalized and subscripts R and l indicate the whole reactor core and zone l, respectively.

Refueling Perturbation

For the reference unperturbed system, the steady-state diffusion and adjoint equations are written in matrix form as follows:

$$(\boldsymbol{M_o} - \lambda_o \boldsymbol{F_o})\phi_o = 0 \tag{3}$$

$$(\boldsymbol{M_o^*} - \lambda_o \boldsymbol{F_o^*})\phi_o^* = 0 \tag{4}$$

The perturbation expression for reactivity can be easily obtained by conventional perturbation theory (CPT) such as:

$$\Delta\lambda = \frac{\langle \phi_o^*, (\Delta\boldsymbol{M} - \lambda_o \Delta\boldsymbol{F})\phi_o \rangle}{\langle \phi_o^*, \boldsymbol{F_o}\phi_o \rangle} + O(\Delta\phi)^2 . \tag{5}$$

Now, the goal is to obtain the variation in zone power caused by any refueling perturbation. In the variational approach for a response functional P_l, an augmented functional K is constructed as:

$$K[\alpha, \phi_o, \Gamma^*, P^*, \lambda_o] = P_l - P^*(\langle H_o, \phi_o\rangle_R - P_T) - \langle \Gamma^*, (\boldsymbol{M_o} - \lambda_o \boldsymbol{F_o})\phi_o\rangle \tag{6}$$

where P^* and Γ^* are Lagrange multipliers for the flux normalization and the neutron balance equation, respectively. The functional K depends on each data parameter α (appearing in $\boldsymbol{M_o}$ and $\boldsymbol{F_o}$), neutron flux, Lagrange multipliers (Γ^* and P^*), and eigenvalue λ_o. The first variation of functional K is written as

$$\delta K = \left\langle \frac{\partial K}{\partial \alpha}, \Delta\alpha \right\rangle + \left\langle \frac{\partial K}{\partial \phi_o}, \Delta\phi_o \right\rangle + \left\langle \frac{\partial K}{\partial \Gamma^*}, \Delta\Gamma^* \right\rangle + \left\langle \frac{\partial K}{\partial P^*}, \Delta P^* \right\rangle + \left\langle \frac{\partial K}{\partial \lambda_o}, \Delta\lambda_o \right\rangle \tag{7}$$

and stationary conditions for variations of ϕ_o, Γ^*, P^* and λ_o are given by following equations:

$$(\boldsymbol{M_o^*} - \lambda_o \boldsymbol{F_o^*})\Gamma_l^* = S_l^* \equiv \frac{\partial P_l}{\partial \phi_o} - P^* H_o(\vec{r}) \tag{8}$$

$$(\boldsymbol{M_o} - \lambda_o \boldsymbol{F_o})\phi_o = 0 \tag{9}$$

$$\langle H_o, \phi_o\rangle_R = P_T \tag{10}$$

$$\langle \Gamma^*, \boldsymbol{F_o}\phi_o\rangle = 0 \tag{11}$$

The adjoint power P^* is obtained by requiring the adjoint source be orthogonal to the flux, i.e.,

$$\langle S_l^*, \phi_o\rangle = 0 \tag{12}$$

which results in

$$P^* = \frac{\langle H_o, \phi_o\rangle_l}{\langle H_o, \phi_o\rangle_R} \equiv p_l. \tag{13}$$

Therefore, the adjoint source in Eq.(8) can be written as

$$S_l^* = \{\delta_l(\vec{r}) - p_l\} H_o(\vec{r}), \tag{14}$$

where $\delta_l(\vec{r}) = 1$ only for regions in zone l.

If the stationary conditions given by Eqs.(8) to (14) are satisfied, the perturbation of response P_l is given by the variation of functional K with respect to α:

$$\Delta P_l = \langle \Delta H, \phi_o \rangle_l - p_l \{ \langle \Delta H, \phi_o \rangle_R - \Delta P_T \} - \langle \Gamma^*, (\Delta \boldsymbol{M} - \lambda_o \Delta \boldsymbol{F}) \phi_o \rangle + O(\Delta \phi)^2. \quad (15)$$

Since P_T is a constant number provided by the user, the variation of zone power due to the refueling perturbation of channel j can be rewritten by

$$\Delta P_l^{(j)} \approx \langle \Delta H^{(j)}, \phi_o \rangle_l - p_l \langle \Delta H^{(j)}, \phi_o \rangle_R - \langle \Gamma^*, (\Delta \boldsymbol{M}^{(j)} - \lambda_o \Delta \boldsymbol{F}^{(j)}) \phi_o \rangle \ , \quad (16)$$

where Γ^* is the so-called "generalized adjoint flux" obtained by solving Eq.(8), which is the adjoint source equation for each control zone l with the adjoint source term given by Eq.(14). The first two terms on the right hand side of Eq.(16) are called the "direct effect" of the perturbation, and the last one is called the "indirect effect" which is caused by the flux perturbation.

Eq.(16) contains only operator perturbations and unperturbed flux. Once generalized adjoint functions are calculated, then the variation of zone power can be estimated to the second order accuracy by simple inner products without solving perturbed diffusion equations for individual perturbation.

Unit ZCU Level Displacement Perturbation

The variation of power in zone l caused by a unit ZCU level displacement perturbation in zone m can be obtained by the same approach. Introducing this perturbation into Eq.(16), the change of zone power due to unit zone level change can be estimated as;

$$\Delta Q_{l(m)} \approx \langle \Delta H_{(m)}, \phi_o \rangle_l - p_l \langle \Delta H_{(m)}, \phi_o \rangle_R - \langle \Gamma^*, (\Delta \boldsymbol{M}_{(m)} - \lambda_o \Delta \boldsymbol{F}_{(m)}) \phi_o \rangle . \quad (17)$$

Estimation of ZCU Level Change

When there is a refueling perturbation, the ZCU level changes such that the reference zone power is maintained. For a refueling perturbation in channel j, the displacement of ZCU level in zone m, $\Delta z_m^{(j)}$, can be obtained by solving the following linear equation

$$\sum_{m=1}^{14} \Delta Q_{l(m)} \Delta z_m^{(j)} + \Delta P_l^{(j)} = 0 \quad (l = 1, 2, \cdots, 14). \tag{18}$$

NUMERICAL TEST

A numerical test of the GPT program for a CANDU reactor loaded with natural uranium fuel has been performed. Table I shows estimates of effective multiplication factor for refueling perturbation in channels M-9 and G-4, and results are compared to those of RFSP direct calculation. The eigenvalue was predicted by conventional perturbation theory (CPT) which uses fundamental adjoint flux. Considering the convergence criteria of the forward/adjoint flux calculation, eigenvalue predicted by the CPT is in a good agreement with that by direct calculation.

Table II gives the zone power variations caused by a refueling perturbation and a unit ZCU level displacement perturbation (10% of total ZCU level) and estimates of ZCU level changes due to the refueling perturbation. The direct and GPT calculations have shown good agreements when the zone power variation is relatively large. When the magnitude of the perturbation is relatively small, it was found that truncation errors imbedded in the sensitivities of zone power variations are not negligible. However such a small perturbation in ZCU level has only a minor effect on reactor physics calculations.

CONCLUSION

A generalized perturbation method has been developed for the purpose of application to the CANDU core fuel management study. A sample calculation has shown that the ZCU level which is an index to represent the zone power distribution is reasonably estimated. However it is inevitable that the direct inversion method used to estimate the ZCU level includes certain amount of error in the solution vector because the coefficient matrix already has truncation error. Therefore it is recommended to find the optimum ZCU level that minimize the difference of current and reference zone powers, which is more close to the procedure used to determine the ZCU level in the forward calculation.

ACKNOWLEDGEMENT

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REFERENCE

1. D.H. Kim et al., "A Generalized Perturbation Program for CANDU Reactor", Proc. of the Korean Nuclear Society Spring Meeting, Seoul, 1998.

Table I. Comparison of Effective Multiplication Factor for Simultaneous Refueling of Channels M-9 and G-4

Effective Multiplication Factor, k_{eff}		
Direct	CPT	Error (%)
1.00056	1.00050	-0.006

Table II. Comparison of Zone Power Variations by Refueling Channels M-9 and G-4, Comparison of Zone Power Variations by Unit ZCU Level Displacement in Zone 1, and Comparison of Zone Level Changes Due to Refueling Perturbation

Zone Number	ΔP_l (kW)		$\Delta Q_{l(1)}$ (kW)		Δz (fraction)	
	Direct	GPT	Direct	GPT	Direct	GPT
1	9341.8	8424.6	-2581.2	-2851.6	0.208	0.215
2	3092.5	3050.4	-357.5	-313.7	0.028	0.038
3	-171.7	-241.1	-983.1	-984.1	-0.163	-0.208
4	1919.9	1898.2	-122.0	-258.9	0.108	0.144
5	-2924.8	-2584.6	551.1	649.8	-0.032	-0.007
6	-5542.8	-5158.2	476.8	500.1	-0.086	-0.044
7	-5831.9	-5443.3	737.2	781.5	-0.081	-0.034
8	9442.2	8464.8	-456.6	-501.8	0.222	0.225
9	3126.1	3052.3	80.2	153.4	0.027	0.043
10	-127.3	-219.4	33.1	-32.4	-0.148	-0.196
11	1966.4	1914.4	399.6	464.0	0.119	0.153
12	-2894.2	-2561.7	750.5	848.5	-0.033	-0.010
13	-5557.9	-5162.7	643.6	670.6	-0.081	-0.042
14	-5837.8	-5433.6	829.5	874.5	-0.082	-0.033