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Parametric Robustness of the Reactor Control System

Yoon Joon Lee, Sin Kim
Cheju National University

Abstract

The parametric analysis method is applied to the reactor control system. The mathematical reactor model is discussed in terms of the parametric uncertainties. The Tsyplin-Polyak locus, based on the boundary crossing theorem, shows the reactor plant has an intrinsic stability. A simple controller is incorporated to the plant to configure the overall closed loop system. Then parametric stability margins are obtained for the controller constants. The results show that a new design constraint of the controller robustness with respect to overall system should be considered for the controller design of an uncertain system.

1. Introduction

The fundamental problem of the control system design is to define the plant, which is to be controlled, in an exact manner. But it is almost impossible to describe the plant exactly due to various reasons such as linearization of the governing equations, change of operating conditions and the drift with the aging. Even if the plant, the RLC system for an example, could be described exactly, the actual physical properties of the components might deviate from the design values up to 50%[1]. Because of these uncertainties, the robustness problem is drawing more attention from control fields.

The robustness means the operation capability of the system in an actual situation with the stability and performance as intended in design stage. The robustness of the control system is not a new issue but has been well known from the classic era. The typical classic approach for the robustness is the loop shaping by considering the frequency characteristics of plant perturbations and external noises.

One of the modern approaches to the robust control was established by Zames in 1970's[2]. It is the H infinity control with several offshoots such as mu-theory with the precise mathematical background. The H infinity control can be regarded as the optimization of the system norms in frequency domains, and its paradigm provides a synthetic approach to robust problem, particularly for the system with unstructured perturbations. But for the system with parametric uncertainty, it is however quite deficient in addressing the same issues[3]. The performance of the controller under real parameter uncertainty, as well as mixed parametric-uncertainty, is an important issue to all of the control systems. However H infinity or H_2 optimal theory is incapable of providing a direct and non-conservative answer this problem.

The parametric approach was first proposed in 1950's and has been developed to one of the control theory entity with the advent of Kharitonov theorem. This theory provides an exact solution to the calculation of the real parametric stability margin. Also it can be used to determine the stability of the system under mixed parametric and unstructured perturbations and to evaluate the robust performance measured in H infinity norm over a prescribed parametric uncertainty set.

The nuclear reactor can be thought to have all the typical characteristics of an uncertain plant. First of all, the mathematically derived model has uncertainties due to the simplification with various assumptions and linearization. And the model itself varies depending on the operating conditions. The performance and stability of the control system might be different from the intended ones, if the design is made with such an inexact model. Therefore, it is quite natural to consider the robustness of the system during the design stage. In this paper, the robustness of the controller has been studied by the parametric approach.

The sensitivity of the controller constant with respect to plant parameter perturbation has been analyzed also, and this sensitivity should be considered in addition to the existing stability criteria for the case of uncertain system.

2. Parametric Uncertainty of Reactor Plant

The reactor dynamics is described by use of the point kinetics equations with one group delayed neutrons. A singly lumped energy balance equation is incorporated to consider the moderator and fuel temperature feedback effects on the reactivity. Even this simple description yields the fifth order MIMO (multi input, multi output) system. In addition to the simplification and linearization of the governing equations, almost all of the physical properties which constitutes the reactor model are subject to change depending on the operating conditions, that is the reactor power, P. These errors in modeling and inexact properties are major causes of the system uncertainty.

With assumptions of that the coolant inlet temperature and coolant flow rate be constant, the MIMO reactor plant reduces to SISO(single input, single output) and is described in the following linear state variable equations[4].

$$\dot{\mathbf{x}}(\mathrm{P}) = \mathbf{A}(\mathrm{P})\mathbf{x} + \mathbf{B}(\mathrm{P})\mathrm{u}, \quad \mathrm{y}(\mathrm{P}) = \mathrm{C}\mathbf{x}(\mathrm{P}) + \mathrm{Du} \qquad (1)$$

In addition to the physical properties which depend on the reactor power, it is found that the moderator temperature coefficient a_c, fuel temperature coefficient a_f, and the fuel gap heat transfer coefficient h_g have great effects on the plant parameters. For example, the heat transfer coefficient has a wide range of 2,500 to 11,000 w/m^2 ^{o}K. And the moderator feedback temperature coefficient as well as fuel temperature coefficient depend on the boron concentration, reactor life time, fuel temperature and rod position, so on. The FSAR of Kori Unit 2 reads that they have the values ranging over

$$a_c \; \hat{I} \; (-57\,\mathrm{pcm}/{}^o\mathrm{K},\; 13.5\,\mathrm{pcm}/{}^o\mathrm{K}) \;\; a_f \; \hat{I} \; (-4.7\,\mathrm{pcm}/{}^o\mathrm{K},\; -2.8\,\mathrm{pcm}/{}^o\mathrm{K}) \qquad (2)$$

For convenience, the ‘nominal’ plant is defined in this paper as the plant of h_g = 4,850 w/m^2 ^{o}K, a_c = 0pcm/^{o}K, and a_f = -3.7 pcm/^{o}K. On the other hand, the ‘optimal’ plant is such a plant of h_g = 10,000 w/m^2 ^{o}K, a_c = -57pcm/^{o}K, a_f =-4.7pcm/^{o}K, and finally the ‘worst’ one has properties of h_g = 2,000 w/m^2 ^{o}K, a_c = 13.5pcm/^{o}K, and a_f = -2.8 pcm/^{o}K.

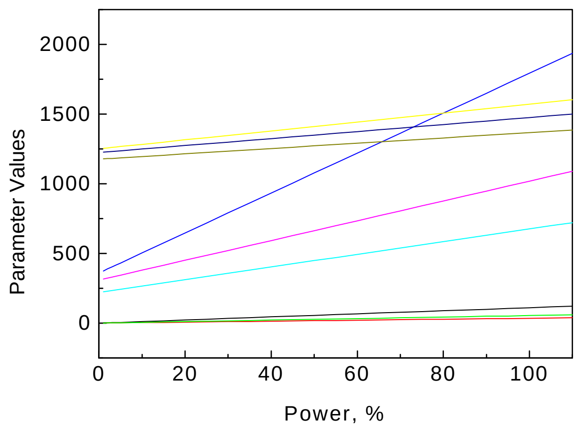

Figure 1. Range of Plant Parameters

For all the cases above, the plant equations has the form of

$$G(s) = \frac{z_3 s^3 + z_2 s^2 + z_1 s + z_0}{s^5 + 406.3 s^4 + p_3 s^3 + p_2 s^2 + p_1 s} \tag{3}$$

Of interest is the characteristic polynomials of the plant, and Fig. 1 shows the parameter sets of nominal, optimal and worst plants, respectively. As shown in the figure, the parameter set $\boldsymbol{p} = (p_1 \quad p_2 \quad p_3)$ varies over a wide range, and each parameter has a bounded value of

$$p_3 \in (1177.8 \quad 1602.6),\ p_2 \in (225 \quad 1935.6),\ p_1 \in (0.3 \quad 121.8) \tag{4}$$

And the nominal value at 90% power is $\boldsymbol{p}^0 \in (p_3^0 \quad p_2^0 \quad p_1^0) = (1449.6 \quad 947.1 \quad 48.9)$.

3. Stability of the Control System

A simple controller is incorporated based on the nominal plant at 90% power. The controller is designed in such a manner that the overall system output characteristics be comply with the FSAR requirement, that is, the maximum overshoot should not exceed 103% when the steady power of 90% is subject to 10% step increase. The controller used in this study is $C(s) = 0.73 + 7s$. Then it is quite natural to ask whether this controller guarantees the Hurwitz stability when the parameters have a great degree of the uncertainty as in Eq. (4). The characteristic polynomials of the closed loop system is

$$\begin{aligned} d(s,\boldsymbol{p}) &= (s^5 + 2006s^4 + 5140s^3 + 2122s^2 + 263s + 10)(1) \\ &\quad + (s)(p_3 s^2 + p_2 s + p_1) = F_1(s)P_1(s) + F_2(s)P_2(s) \end{aligned} \tag{5}$$

For the linear interval family of $d(s,\boldsymbol{p}) = F_1(s)P_1(s) + \ ....\ F_m(s)P_m(s)$, the Hurwitz stability is guaranteed if and only if the Tsyplin-Polyak locus in the complex plane should not cross the maximal radius of the stability ball for all $w = [0, \ \infty)$. The Tsyplin-Polyak, locus is obtained from the following equation

$$z(w) = \frac{A(jw)}{|A(jw)|} m(w), \text{ where } A(jw) = d(jw, \boldsymbol{p}^0) \tag{6}$$

In the above equation, $m(w)$ is determined from the conditions of the zero exclusion principle. For the polynomials of Eq.(5), the common margin of perturbations of all parameter is found to be 1. Figure 2 shows the Tsyplin-Polyak locus of the characteristic polynomials of Eq.(5), and it does not cross the maximal radius of the stability ball.

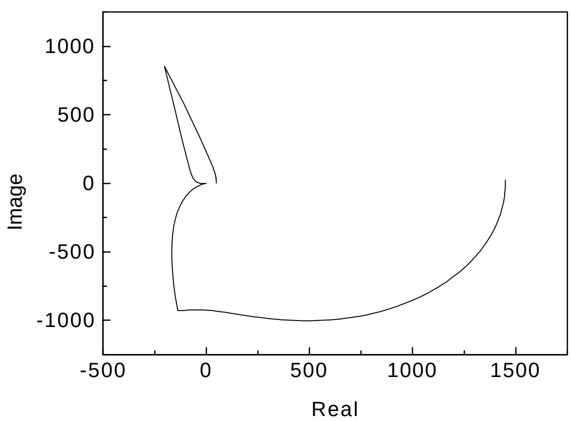

Figure 2. Tsyplin-Polyak Locus of the Characteristic Polynomial

As shown in Fig.2, the reactor control system is Hurwitz although the parameters have a great degree of uncertainties as of Eq.(4). The robust stability margin of the system can be obtained by use of a weighted l_p norm in the coefficient space. For a family of polynomials centered at a nominal coefficient point, the robust stability margin is a weighted l_p ball of radius r.
That is, for a polynomial of $A(s) = a_n s_n + a_{n-1} s_{n-1} + \ \dots \ + a_1 s + a_0$, the margin is

$$r_p := \left\{ \sum_k \left| \frac{a_k - a_k^0}{\alpha_k} \right|^p \right\}^{1/p} \le r \text{ for p = integer, or } r_\infty := \left\{ \max_k \left| \frac{a_k - a_k^0}{\alpha_k} \right| \le r \right\} \text{ for infinity norm} \tag{7}$$

where α_k is a weight of perturbation of the corresponding coefficients.
To determine this margin, the polynomial is to be divided into even and odd parts in the frequency domain, and the least weighted distance in the complex plane should not intersect with l_p norm radius.

The characteristic equation of Eq.(5) is rewritten as

$$d(s, \boldsymbol{p}) = s^5 + 2006s^4 + (5140 + p_3)s^3 + (2122 + p_2)s^2 + (263 + p_1)s + 10 \tag{8}$$

Since $\boldsymbol{p}^0 \in (p_3^0 \quad p_2^0 \quad p_1^0) = (1449.6 \quad 947.1 \quad 48.9)$, the nominal coefficients are

$$\boldsymbol{a} = [1,\ 2006,\ 6589,\ 3069,\ 312,\ 10] \tag{9}$$

It is assumed that $\max(\boldsymbol{p}) - \min(\boldsymbol{p})$ is applied to the perturbed terms and 20% of perturbation to other terms. Then with Eq.(4), the perturbation weight vector is

$$\boldsymbol{\alpha} = [0.2,\ 401,\ 525,\ 1711,\ 122,\ 2] \tag{10}$$

With these coefficient and perturbation vectors, the robust stability margins of l_2 norm and l_∞ norm is calculated as in Figs. 3 and 4.

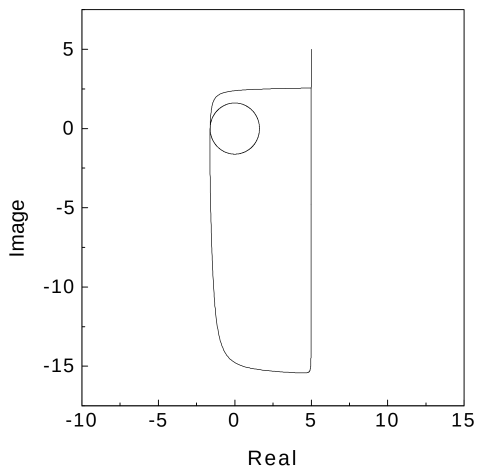

Figure 3. l_2 Stability Margin

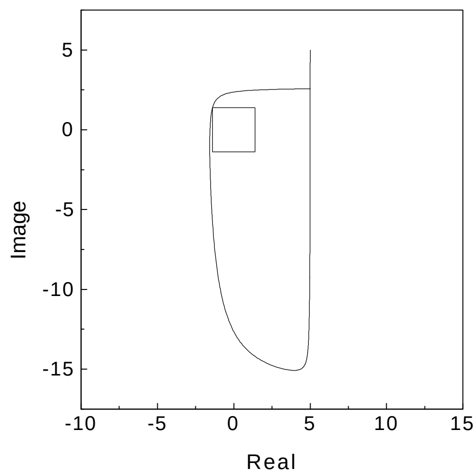

Figure 4. l_∞ Stability Margin

The robust stability margin has a different value depending on the applied norm. In the above, the robust margin is 1.6143 for the case of l_2 norm, and is 1.3947 for l_∞ norm. Further, as the degree of the perturbation, $\boldsymbol{\alpha}$, becomes larger, the stability becomes smaller as is expected. Also it is to be noted that the system is marginal stable when the locus meets the stability ball or square of above figures.

4. Robustness of Controller Parameters

The controller applied in this study is assumed to be of PD controller. Usually, the controller constants or parameters are determined taking the system performance and stability into consideration. But for the case of which the plant has uncertainties, there is another factor to be considered. It is the parametric stability margin, which implies the sensitivity of controller constant with respect to overall system stability.

The controller has the form of $C(s) = K_p + K_i s$. With the assumption that K_i has constant value of 7, the characteristic polynomial has the arguments of parameter vector $\boldsymbol{p}$ and K_p, that is $d(s, \boldsymbol{p}^0 + \Delta\boldsymbol{p}, K_p)$, where $\boldsymbol{p}^0$ is the nominal parameters, $\Delta\boldsymbol{p}$ is the parameter perturbations and K_p is the controller constant. Then, the characteristic polynomial is described as

$$\begin{aligned} d(s, \Delta\boldsymbol{p}, K_p) = {} & s^5 + 2006s^4 + (6423 + \Delta p_3)s^3 + (2551 + \Delta p_2)s^2 + (145 + \Delta p_1)s \\ & + K_p(229s^3 + 710s^2 + 229s + 14) \end{aligned} \tag{11}$$

The characteristic polynomial of Eq.(11) is rewritten, by dividing into real and imaginary parts, as

$$\begin{bmatrix} 0 & -w^2 & 0 \\ -w^2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta p_3 \\ \Delta p_2 \\ \Delta p_1 \end{bmatrix} = \begin{bmatrix} -2006w^4 + 2551w^2 + 710w^2K_p - 13.7K_p \\ -w^4 + 6422w^2 - 145 + 229w^2K_p - 229K_p \end{bmatrix} \tag{12}$$

By letting this equation be $\boldsymbol{AP} = \boldsymbol{K}$, the parametric stability margin of the proportional coefficient is found to be

$$m(K_p) = \inf_{w} \left\| \boldsymbol{A}^T(\boldsymbol{AA}^T)^{-1}\boldsymbol{K} \right\|_2 \tag{13}$$

As in the same manner, the parametric stability of the differential coefficient is obtained by fixing K_p to 0.73. The characteristic polynomial is

$$\begin{bmatrix} 0 & -w^2 & 0 \\ -w^2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta p_3 \\ \Delta p_2 \\ \Delta p_1 \end{bmatrix} = \begin{bmatrix} 406w^4 + 1466w^2 - 229w^4K_d + 229w^2K_d - 10 \\ -w^4 + 1616w^2 - 216 + 710w^2K_d - 14K_d \end{bmatrix} \tag{14}$$

The parametric stability margins calculated for the proportional and differential coefficients are shown in the following figures. Figure 5 shows that the parametric stability margin of proportional coefficients is least around 3.7. This means it is more desirable for the proportional constant be farther from the value of least stability margin, although the stability is maintained for all the positive values. This condition, which is not shown in the existing control theory, is a new concept for the robustness design of uncertain systems. From the same viewpoint, it can be said that a larger value of differential constant is preferable within the frame of prescribed performance specification.

5. Conclusion

The stability and performance of the control system is strongly dependent on the exactness and reliability of the plant to be controlled. But all the real plants have uncertainties, and it is questionable that the designed controller would work as intended in the real circumstances. The robust control takes all the possible uncertainties during the design process. One of the robust controls is the parametric analysis which can handle the system parameters directly. Also it can be used to design the robust controller for the system under mixed parametric and unstructured perturbations. This parametric approach is applied to the nuclear reactor control system. The reactor model has a great degree of uncertainties, hence the controller should provide a sufficient robustness to the system. The parametric analysis shows the reactor plant has an intrinsic stability. However, it also shows that the controller should be designed by taking the parametric stability margins, or sensitivity of the controller coefficients on the system stability into account. This is a new design constraint in addition to the existing stability and performance analyses for the uncertain systems.

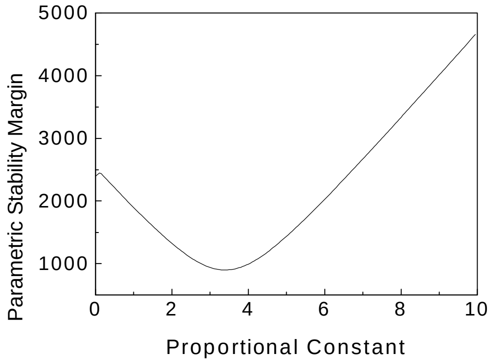

Figure 5. Parametric Stability Margin of K_p

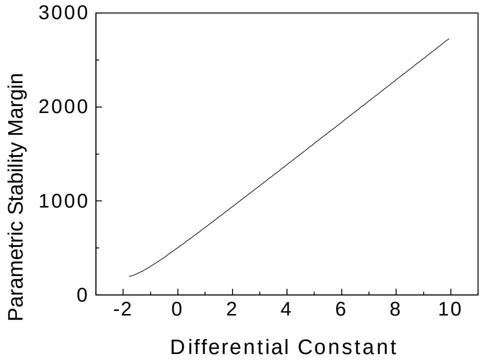

Figure 6. Parametric Stability Margin of K_d

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