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Development of Discrete-Time H_∞ Filtering Method for Time-Delay Compensation of Rhodium Incore Detectors

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Abstract

A method is described to develop an H_∞ filtering method for the dynamic compensation of se neutron detectors normally used for fixed incore instruments. An H_∞ norm of the filter transfer matri as the optimization criteria in the worst-case estimation error sense. Filter modeling is perform discrete-time model. The filter gains are optimized in the sense of noise attenuation level of H_∞ s introducing Bounded Real Lemma, the conventional algebraic Riccati inequalities are converted into Matrix Inequalities (LMIs). Finally, the filter design problem is solved via the convex optimization fra using LMIs. The simulation results show that remarkable improvements are achieved in view of t response time and the filter design efficiency.

I. Introduction

Digital compensation of the self-powered neutron detectors (SPNDs) has been particularly emph to its importance in reactor surveillance and operation monitoring. Previously, several filtering meth proposed for the compensation of delayed signals from Rhodium fixed incore detectors extensively Asea Brown Boveri-Combustion Engineering (ABB-CE) PWRs.[1,2,3,4] Korean Standard Nuclear Power also adopted the same type of detectors for core monitoring purpose. Today, there is a growing need the slow response of the detectors for the enhancement of the power maneuvering capability and the of uncertainties in thermal margin estimation. Recently, an open-loop observer type estimation metho proposed in [3] and standard Kalman filter was applied in this field.[4] Although Kalman filter me considerably improved the slow response of the Rhodium detectors, there still remain some difficulties design such as the requirement of the knowledge of noise covariance and the limited performance c relax these limitations, we introduced an LMI-based H_∞ filtering method.[5,6,7] Section II presents the framework of the LMI-based linear filtering theory on the H_∞ setting. Section III describes the application results of the method for dynamic compensation of the delayed signal from Rhodium incore detectors.

II. LMI-Based H_∞ Filtering Theory

The state-space model based filtering methods has been widely applied in the fields of sig estimation and fault diagnostics, etc. The most well known estimator is the Kalman filter which h applications in wide variety of industries including the compensation of delayed signal from Rhodium incore detectors.[4] Kalman filter is an estimation method which minimizes the average estimation erro precisely, Kalman filter minimizes the variance of the estimation error. But the Kalman filter assumes noise properties are known. That means the optimality of the Kalman filter relies on the knowledg covariance matrices and another tuning process after installation of the filter would be required. limitations gave rise to H_∞ filtering, also known as minimax filtering. The H_∞ filter gives hard uppe on the estimation errors, no matter what the disturbances are as long as they are of finite energy. filtering minimizes the worst-case estimation error. Recently, the conventional H_∞ filtering method applied to the problem of estimating time-varying reactivity[8,9], which is based on solving the equation and requires an iteration scheme to find the optimal noise attenuation level. This kind of approach can fail if the Hamiltonian matrix of the filter has pure imaginary eigenvalues during the The LMI-based approach can overcome this kind of limitations by solving the convex optimization meth instead of the closed-form Riccati equation. Due to the dramatic growth in computing power and the very powerful numerical optimization algorithms, the LMI problem can be solved within a com computing time required to find a closed-form solution.[11]

II.1 Discrete Time LMI-Based *a Priori* H_∞ Filtering Problem

Consider the following linear time-invariant discrete-time system given by

$$\begin{aligned} x_{k+1} &= Ax_k + Bw_k \\ y_k &= Cx_k + Dw_k \\ z_k &= Lx_k. \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $y \in R^r$ is the measurement output vector, $w \in R^q$ is a disturbanc containing both process and measurement noise and $z \in R^p$ is the signal to be estimated. The A, B, C, D and L are real and of appropriate dimensions. We are interested in designing a filter of the

$$\begin{aligned} \hat{x}_{k+1} &= A\hat{x}_k + K(y_k - C\hat{x}_k) \\ \hat{z}_k &= L\hat{x}_k \end{aligned} \tag{2}$$

where $K \in R^{n \times r}$ is the filter constant gain to be determined. Defining the state error as $e_k = x_k - \hat{x}_k$ estimation error dynamics is given by

$$\begin{aligned} e_{k+1} &= (A - KC)e_k + (B - KD)w_k \\ \tilde{z}_k &\equiv z_k - \hat{z}_k = Le_k. \end{aligned} \tag{3}$$

The key important feature of the H_∞ filtering problem is to find the estimate $\hat{z}_k$ of the signal z_k by minimizing the worst-case estimation error energy $\|e\|_2$ for all bounded energy disturbance w, that is,

$$\min \|H_{we}\|_\infty = \min \sup_{w \in l_2[0,\infty)} \frac{\|e\|_2}{\|w\|_2}\ . \tag{4}$$

where H_{we} is the transfer function from the disturbance w to the estimation error e. Since the in norm of the signal does not require any knowledge except to be bounded, the H_∞ filtering problem tu be a powerful strategy. The γ-suboptimal H_∞ filtering problem is defined to find (if it exists) a filter $\|H_{we}\|_\infty < \gamma$, where the positive scalar γ is a prescribed noise attenuation level. The construction of an filter is to find a symmetric positive definite matrix P which can be derived from the following dis Bounded Real Lemma.[11,12]

Discrete-Time Bounded Real Lemma :
$(A-KC)$is asymptotically stable and $\|H_{we}\|_\infty < \gamma$ if and only if there exists a positive definite s matrix $P \in R^{n\times n}$ satisfying the following linear matrix inequality

$$\begin{bmatrix} A-KC & B-KD \\ L & 0 \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A-KC & B-KD \\ L & 0 \end{bmatrix} - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0. \tag{5}$$

□

To transform this inequality to a solvable form, define the filter gain as $K = P^{-1}W$ where $W \in R^{n\times r}$ Eq. (5) becomes

$$\begin{bmatrix} A-P^{-1}WC & B-P^{-1}WD \\ L & 0 \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A-P^{-1}WC & B-P^{-1}WD \\ L & 0 \end{bmatrix} - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0. \tag{6}$$

It is straightforward to rewrite Eq.(6) as

$$\begin{bmatrix} PA-WC & PB-WD \\ L & 0 \end{bmatrix}^T \begin{bmatrix} P^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} PA-WC & PB-WD \\ L & 0 \end{bmatrix} - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0. \tag{7}$$

By using Schur complement[12], this can be rewritten as

$$\begin{bmatrix} P & 0 & A^TP-C^TW^T & L^T \\ 0 & \gamma^2 I & B^TP-D^TW^T & 0 \\ PA-WC & PB-WD & P & 0 \\ L & 0 & 0 & I \end{bmatrix} < 0. \tag{8}$$

This is an LMI feasibility problem for discrete-time optimal H_∞ filter. The γ-optimal H_∞ filter is obt solving the LMI γ-optimization problem subject to the LMI constraint Eq. (8).

II.2 Discrete Time LMI-Based *a Posteriori* H_∞ Filtering Problem

The discrete-time H_∞ filter, Eq. (2), uses measurements in one step delay, i.e., *a priori* filter. Cu sampling time step size of fixed incore detector system is 2 sec which is a rather large time step size. So we are interested in using the current measurement, the *a posteriori* filter. [14] Using the filter form that follows Eq. (2), it can be written as

$$\begin{aligned} \hat{x}_{k+1} &= A\,\hat{x}_k + K(y_{k+1} - CA\,\hat{x}_k) \\ \hat{z}_k &= L\,\hat{x}_k . \end{aligned} \tag{9}$$

Then the filter error dynamics becomes

$$e_{k+1} = (A-KCA)e_k + (B-KCB)w_k - KDw_{k+1}. \tag{10}$$

For this error dynamics equation, it is not easy to construct the proper LMI system due to the $(k+1)th$ exogenious term KDw_{k+1}. To simplify the filter design problem, we assume $D=0$. Then Eqs.(5) and transformed into

$$\begin{bmatrix} A-KCA & B-KCB \\ L & 0 \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A-KCA & B-KCB \\ L & 0 \end{bmatrix} - \begin{bmatrix} P & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} P & 0 & A^TP - Y^TX^T & L^T \\ 0 & \gamma^2 I & B^TP - Z^TX^T & 0 \\ PA - YX & PB - XZ & P & 0 \\ L & 0 & 0 & I \end{bmatrix} < 0 \tag{12}$$

where $Y=CA$, $Z=CB$ and the filter gain is given by $K=P^{-1}X$. These system of LMIs are convex and can be easily solved by the following algorithm with LMI Control Toolbox[11].

II.3 Tradeoffs between Response Time and Noise Gain

In the previous sections, we considered the minimization of γ only. However, the filter resp becomes faster and the noise gain increases as γ decreases and vice versa. That means the reduction pros and cons. In the design process of SPND's dynamic response, the noise gain should be considered to prevent any excessive overshoot in filter response induced by random noise. The noise gain is defin square root of the sum of the squares of filter impulse response as time approaches infinity. In this applied a simple tradeoffs between response time and noise gain by introducing a weighted sum of two filter gains attained from separate H_∞ filter design with different tuning parameter. That is,

$$K = dK_s(1-d)K_f, \tag{13}$$

where K_s = H_∞ filter gain with slow response and small noise gain

K_f = H_∞ filter gain with fast response and large noise gain

d = weighting parameter less than 1.

The weight value d is a key design parameter in the tradeoff of the filter performance. But it can determined by checking the closed loop stability, response time and noise gain.

III. Application of H_∞ Filtering Scheme to SPNDs

Dynamic response of Rhodium SPNDs is mainly governed by the (n,β) reaction of β-emitting This gives rise to delay, depending on the β-decay constant, of the signal of measured neutron level in the reactor core. The dynamic model of Rhodium SPNDs are well known and one can find published results[1,2,4].

The discrete-time detector dynamic model is given by;

$$x_{k+1}=\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{p}(q-\frac{rg}{104\lambda})(1-e^{-104\lambda T_s}) & e^{-104\lambda T_s} & 0 \\ \frac{1}{p}\frac{rg}{104\lambda}(1-e^{-104m\lambda T_s}) & 0 & e^{-104m\lambda T_s} \end{bmatrix} x_k+\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w_k, \tag{14}$$

$$y_k=[p\ p\ p]x_k,$$

where $x=[\phi, x_g, x_f]^T$, is ϕ the neutron flux to be estimated and x_g, x_f are fictitious state variab definitions of the filter constant and values used in this paper can be found in [4]. In this study, the are performed to demonstrate the applicability of the discrete-time H_∞ filter with various desire attenuation levels. The tuning parameters of the filter are the prompt fraction p and weighting para which determine the response time and noise gain. The advantage of the H_∞ filter is that there is tuning parameter and the noise covariance need not be known.

In this paper, the simulation is performed for T_s= 1 and 2 secs. Table 1. shows the filter gain resulting closed-loop poles of the H_∞ filter. Table 2. summarizes the filter response time and noise g H_∞ filter compared with Kalman filter. The response time is defined as the time taken to reach 9 response and interpolated between sampling interval. The H_∞ filter response tine can be reduced to 4. sec with T_s= 1 and 2 sec, respectively, compared with 6.9 and 6.5 sec of the Kalman filter. The H_∞ gives improved noise gain. Figures 1 and 2 show the step and ramp responses of the filter, respectiv the signal from SPND is transferred to core monitoring system which performs extensive calculations core operational margin, the update time of the SPND signal is currently limited to 2 sec. As the sam size becomes larger, the effect of the noise gain becomes significant. If the sampling step size could b the uncertainty of estimating thermal margin in core monitoring can also be decreased. As shown in Fi 2, the H_∞ filter shows reduction in response time for step response.

Table 1. Filter Gains and Closed-Loop Poles of H_∞ Filters

	H_∞ Filter* (T_s=1 sec)	H_∞ Filter* (T_s=2 sec)
Filter Gain (K)	$\begin{bmatrix} 5.2662 \\ 0.1445 \\ -0.0035 \end{bmatrix}$	$\begin{bmatrix} 5.3682 \\ 0.7740 \\ -0.1010 \end{bmatrix}$
Closed -Loop Poles	0.5496 0.8222 0.9972	0.6107 + 0.1009i 0.6107 - 0.1009i 0.9944

* *a posteriori* filter with $K_s=K(p=0.22)$, $K_f=K(p=0.1)$

Table 2. Performance Comparison of H_∞ Filters with Kalman Filter

Method	Response Time (sec	Noise Gain
Discrete-Time H_∞ Filter (T_s=1 sec, d=0.87)	4.8	5.78
Discrete-Time H_∞ Filter (T_s=2 sec, d=0.75)	5.7	5.71
Kalman Filter (T_s=1 sec	6.9	5.88
Kalman Filter (T_s=2 sec	6.5	5.89

* *a posteriori* filter with $K_s = K(p=0.22)$, $K_f = K(p=0.1)$

IV. Conclusions and Recommendations

A new method for dynamic compensation of Rhodium self-powered neutron detectors is developed H_∞ filtering scheme. The method is based on the minimization of the worst-case estimation error via optimization algorithm. The optimization problem is constructed as a linear matrix inequality problem wh overcome the limitations of the conventional method based on the solution of Riccati equation. The app of the developed method is demonstrated by simulations. The developed filtering method shows im performance in spite of the totally unknown noise covariance and gives simple and efficient filter design It is recommended that the applicability of the filtering method be considered for monitoring and p system of commercial power reactors. It is recommended that the applicability of the filtering me considered for monitoring and protection system of commercial power reactors.

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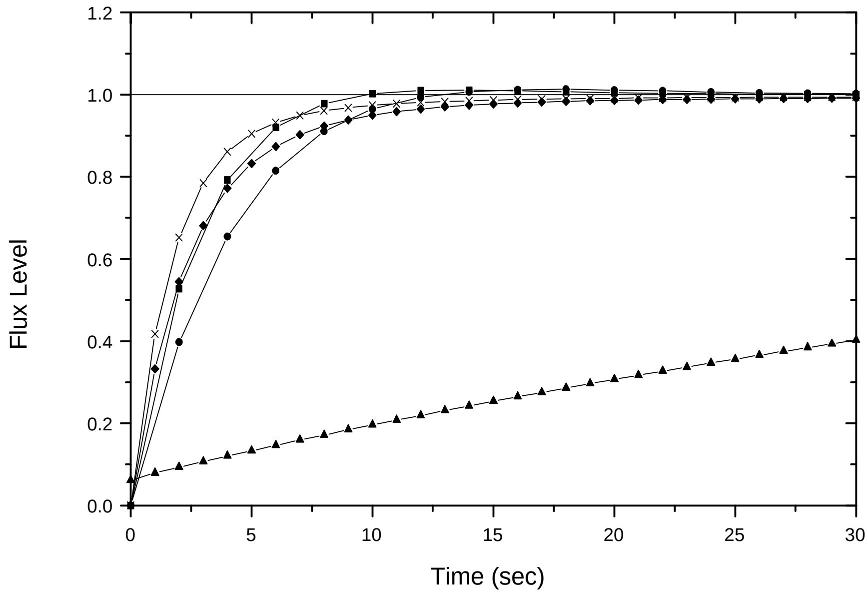

Figure 1. Step Response of H_∞ filter (Continuous-Time)

(— : Reference, ▲ : Uncompensated,

x : Discrete-Time *a posteriori* H_∞ Filter with T_s= 1 sec, d = 0.87,

■ : Discrete-Time *a posteriori* H_∞ Filter with T_s= 2 sec, d = 0.75,

◆ : Kalman Filter with T_s= 1 sec, ● : Kalman Filter with T_s= 2 sec)

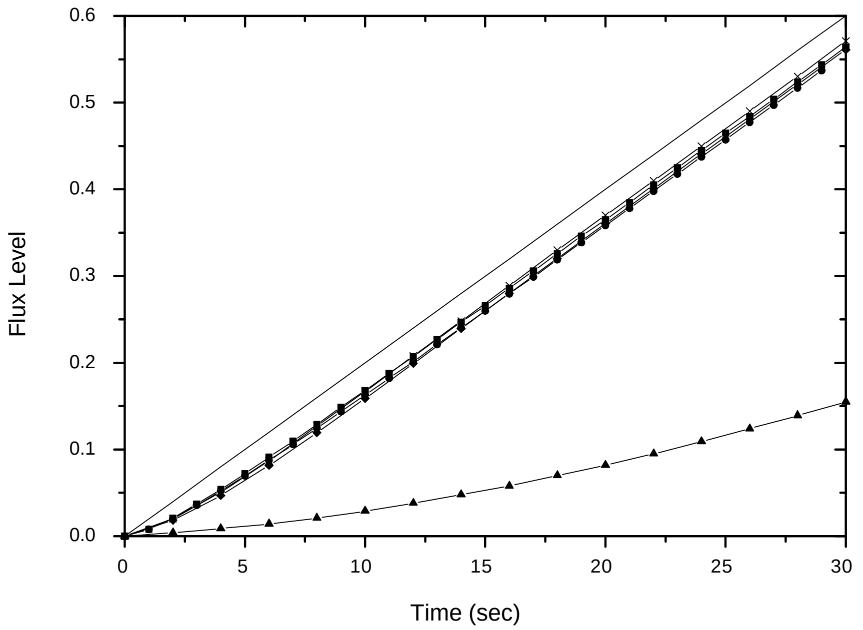

Figure 2. Step Response of H_∞ filter (Discrete-Time)

(— : Reference, ▲ : Uncompensated,

x : Discrete-Time *a posteriori* H_∞ Filter with T_s= 1 sec, d = 0.87,

■ : Discrete-Time *a posteriori* H_∞ Filter with T_s= 2 sec, d = 0.75,

◆ : Kalman Filter with T_s= 1 sec, ● : Kalman Filter with T_s= 2 sec)