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Use of Inverse Heat Conduction Solution for Determination of Temperature Distribution in the Pressurizer Surge Line

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Abstract

A prerequisite for reliable assessment of the thermal stratification impact on mechanical damage of pressurizer surge lines is to determine as accurately as the temperature distributions during transients where the highest temperature difference between pressurizer and hot leg occurs. In order to monitor the occurrence of thermal stratification in the pressurizer surge lines, various surge line outer surface temperature measurement programs have been widely implemented for pressurized water reactors. This paper presents a method to determine the temperature distributions of the inner pipe wall of pressurizer surge line from the circumferential temperature distributions measured on the outer surface of the piping by applying the inverse steady-state heat conduction solution. The steady-state inverse heat conduction solution is obtained using the boundary element method in conjunction with regularization procedure. The present method is validated by comparing the predicted results of a sample problem to the finite volume analysis results of fluid flow and heat transfer.

1. Introduction

Thermal stratification can occur in a piping system where both hot and cold fluids flow in and form two fluid layers due to the difference in fluid density (or temperature), with little mixing. In this situation, the cold (denser) fluid occupies the lower position of the pipe while the hot fluid the upper space. The difference in temperature of the two fluids may lead to considerable circumferential temperature gradients in the pipe wall, which can result in excessive differential expansion at the top and bottom of the pipe threatening the integrity of the piping system.

In operating nuclear power plants, some safety-related pipings connected to reactor coolant systems are known to be potentially susceptible to this phenomenon. Those include pressurizer surge lines, emergency core cooling lines, residual heat removal lines, pressurizer spray lines, charging lines etc. [1, 2]. Especially the thermal stratification in the pressurizer surge lines has been addressed as one of the significant safety and technical issues in most countries holding nuclear plants since the USNRC issued Bulletin 88-08 and 88-11in 1988, requesting licensees to take proper actions for resolution of the issue. To assess the potential for damage of piping due to the thermal stratification, it is necessary, first of all, to determine the transient temperature distributions in the wall of the pipe in which thermally stratified flow occurs.

Several investigators [3, 7] have made efforts to determine the temperature distributions in the pipe wall by means of laboratory testing of the particular geometry, measurement of the temperatures on the outer surface of the pipe in the field, or theoretical predictions. There are much difficulties and limitations in applying the first two approaches for operating plants. Therefore various surge line outer surface temperature measurement programs have been widely implemented for the pressurized water reactors in order to monitor the occurrence of thermal stratification in the pressurizer surge lines.

If the thermal stratification in the pressurizer surge line persists at a steady-state condition for a long time, the temperature distributions in the piping wall can be predicted by using the inverse heat conduction solution. The inverse heat conduction solution is obtained with a couple of specified boundary conditions at the outer surface of the piping. The two outer boundary conditions can be given as both direct measurements of temperature and heat flux (or, in stead, ambient temperature with known heat transfer coefficient) around the circumference, with accuracy.

This paper presents a method to determine the temperature distributions in the pipe wall of pressurizer surge line from the circumferential temperature distributions measured on the outer surface of the piping by applying the inverse steady-state heat conduction solution. The steady-state heat conduction solution is obtained using the boundary element method in conjunction with regularization procedure.

Inverse problems have been studied in many fields of science and engineering [8, 9]. Typical examples include determining the location and size of coolant passage in an internally cooled turbine blade to produce a desired heat flux distribution [10], and identifying heat conductivity from boundary temperature distribution, etc. Also, the inverse heat conduction solution have been extensively applied to the area of nuclear engineering [11~13].

Boundary element method (BEM)[14] has been popular as an effective numerical method for the solution of this kind of inverse problem. Several investigators [15~18] have developed various techniques of BEM applications to inverse heat conduction problems. In the present study, BEM together with Tikhonov's regularization method [19] is used. Formulated boundary element solution matrix $AX=B$ is ill posed in the Hadamard sense [8, 19]. Therefore trying to solve for the unknown X directly leads to highly error-sensitive solutions at the boundary, which in turn gives ill-behaved solution field at certain conditions. Numerical regularization procedure is introduced to achieve stable solution with a partial loss of accuracy. The method is applied to determine the unknown temperature distributions in the pipe wall of surge line with a couple of specified boundary conditions at the outer surface of pipe. The present method has been validated by comparing the predicted results of a sample problem to the numerical analysis results of stratified flow and heat transfer.

2. Problem and Mathematical Formulation

The cross-sectional view of the circular pipe is shown in Fig. 1(a). The outer boundary Γ_a is subjected to a Robin condition, while it is assumed that a Dirichlet condition is imposed on the inner boundary Γ_c. However the actual boundary conditions at the inner boundary are unknown. The problem to be addressed in this study is to determine the temperature distribution in the pipe wall of which the outer boundary conditions are only known. In this situation, the problem considered here is formulated in a two-dimensional space as follows:

Governing equation

$$\nabla^2 T(x) = 0, \qquad x \in R^2 \tag{1}$$

Boundary conditions on Γ_a

$$k\frac{\partial T(x)}{\partial n} + h_a(x)T(x) = h_a(x)\overline{T}_e(x), \quad x \in \Gamma_a \tag{2}$$

$$T(x) = \overline{T}_a(x), \quad x \in \Gamma_a \tag{3}$$

where T, k and h denote temperature, thermal conductivity, and convective heat transfer coefficient, respectively. The bar (-) over T stands for the specified value of T, and $\overline{T}_e$ represents the specified ambient temperature with which the surface heat exchange by convection. The partial differential of T is carried out along an outward normal to a boundary surface.

This kind of inverse heat conduction problem can be solved by various numerical methods such as the finite element method or finite difference method or finite volume method etc. Among the methods, the boundary element method is the most straightforward technique for determining the temperature distributions in the wall when the temperature and heat flux at the outer boundary are known.

3. Derivation of Boundary Element Nodal Equations and Regularization Procedure

The auxiliary problem is solved with the governing Eq. (1). The following boundary conditions are considered.

$$k\frac{\P T(x)}{\P n}+h_a(x)T(x) = h_a(x)\overline{T_e}(x),\ \ x\in\Gamma_a \qquad (4)$$

$$T(x)=\overline{T_b}(x),\ \ x\in\Gamma_b \qquad (5)$$

Following the common practice of the development of the BEM, governing Eq. (1) and boundary conditions (4) and (5) are weighted with T^* and their results are combined as

$$\int_\Omega T^* k\nabla^2 T\, d\Omega=\int_{\Gamma_a} T^*[k\frac{\P T}{\P n}+h_a(T-\overline{T_e})]d\Gamma-\int_{\Gamma_b}\frac{\P T^*}{\P n}(T-\overline{T_b})d\Gamma \qquad (6)$$

Then by performing integration by parts of terms involving partial differentials, it is derived as

$$-\int_{\Gamma_a} T k\frac{\P T^*}{\P n}d\Gamma+\int_{\Gamma_b} T^* k\frac{\P T}{\P n}d\Gamma+\int_\Omega kT\nabla^2 T^*\, d\Omega$$
$$=\int_{\Gamma_a} h_a(T-\overline{T_e})T^* d\Gamma+\int_{\Gamma_b}\overline{T_b}\, k\frac{\P T^*}{\P n}d\Gamma \qquad (7)$$

The area integral $\int_\Omega kT\nabla^2 T^*\, d\Omega$ in Eq. (7) can be changed to a local temperature T_i by taking T^* to be the solution of

$$\nabla^2 T^* + \Delta_i = 0 \qquad (8)$$

where Δ_i represents the Dirac delta function. Then, Eq. (7) becomes

$$kT_i=\int_\Gamma T q^* d\Gamma-\int_\Gamma aT^* d\Gamma \qquad (9)$$

here $q=-k\frac{\P T}{\P n}$, and $q^*=-k\frac{\P T^*}{\P n}$ (10)

Equation (9) is known as the Green's third identity, which is derived here by using the boundary conditions given in Eqs. (4) and (5). It is noted that Eq. (9) is strictly valid for evaluating the temperatures at interior points. In order to develop the method as a boundary element method, the boundary Γ must be discretized into elements as shown in Fig. 1(b), and the point i is moved to the nodes on the boundary. As a result, boundary integral equation is derived as

$$C_i\ \ T_i=\int_\Gamma T\quad d\Gamma-\int_\Gamma q\quad ^* d\Gamma \qquad (11)$$

where $C_i=\begin{cases}1\ \ for\ x_i\in\Omega\\ \frac{q}{2p}\ \ for\ x_i\in\Gamma,\end{cases}$ (12)

where q is the angle between the tangent to Γ on either side of x_i .

For the two-dimensional domain of interest in this study, T^*, q^* can be derived by using Eq. (8) as [7]

$$T^*=\frac{1}{2p}1n(\frac{1}{r}),\ \ q^*=\frac{k}{2p\,r}\nabla r\cdot\hat{n} \qquad (13)$$

where r stands for distance between points x_i and x .

In the numerical solution of the integral Eq. (12), the T and q in the integrals are modeled using linear interpolation functions. A general integral is introduced as

$$\int_{\Gamma_j} UV^* d\Gamma = U_j \hat{V}_{ij}^{j} + U_{j+1} \hat{V}_{ij}^{j+1} \tag{14}$$

where

$$\hat{V}_{ij}^{i} = \int_{\Gamma_j} f_1 V^* d\Gamma \; and \; \hat{V}_{ij}^{j+1} = \int_{\Gamma_j} f_2 V^* d\Gamma \tag{15}$$

Here the first subscript of $\hat{V}$ refers to the specific position of the point where the temperature is evaluated. The second subscript of $\hat{V}$ refers to the boundary element over which the contour integral is carried out. The superscript j and j+1 designate the linear interpolation function f_1 and f_2 respectively, with which the V^* function is weighted in the integrals in Eq. (15). With this notation, the boundary nodal equations can be formulated in a compact form and the terms in these equations can be readily checked for error. For the boundary $\Gamma = \Gamma_a \cup \Gamma_b$ discretized into N elements,

$$\int_{\Gamma} UV^* d\Gamma^j = \sum_{j=1}^{N} \int_{\Gamma_j} UV^* d\Gamma = \sum_{j=1}^{N} [\hat{V}_{i(j-1)}^{j} + \hat{V}_{ij}^{j}] U_j \tag{16}$$

where $\hat{V}_{i0}^{1} = \hat{V}_{in}^{1}$.

Introducing Eq. (16) into Eq. (11) and manipulating results yields a boundary element equation as [14]

$$\sum_{j=1}^{N} \{[\hat{q}_{i(j-1)}^{j} + \hat{q}_{ij}^{j}] - [\hat{T}_{i(j-1)}^{j} + \hat{T}_{ij}^{j}] h_j - d_{ij} C_i k\} T_j = \sum_{j=1}^{N} [\hat{T}_{i(j-1)}^{j} + \hat{T}_{ij}^{j}] q_j \tag{17}$$

where d_{ij} is Kronecker delta.

In Eq. (17), if the temperature is given at a nodal point j , then T_j at this point is $\overline{T}_j$; q_j is unknown, and h_j will be taken to be zero. However when a Robin condition is imposed at point j , q_j will be taken to be -$h_a \overline{T}_e$, a known quantity; T_j is unknown, and h_j becomes h_a . In practice, Eq. (17) can be used to construct a matrix equation to solve for unknowns. For the inverse problem given by Eq. (1) to (3), T_j and q_j are fully specified on Γ_a , Whereas all T_j and q_j on Γ_b are unknowns. Then by discretizing Γ_a and Γ_b with each number of nodal points M¡ ¢ and M respectively, Eq. (17) is used to construct a system of (M¡ ¢ + M) linear equations with 2M unknowns on Γ_b . Rearranging the unknowns in a vector X yields

$$AX = B \tag{18}$$

where matrix A is dimensioned ((M¡ ¢ +M),2M). Assuming M¡ ¢ ≥ M, it is then possible to obtain an approximate solution to Eq. (18).

The classical way to solve Eq. (18) is to finds X which minimizes the Euclidean norm of the residual

$$r = |AX - B|^2 \tag{19}$$

By using the normal equation of least square problems, solution is

$$X = (A^T A)^{-1} A^T B \tag{20}$$

However, as the problem is ill-posed, trying to solve X by Eq. (20) will give highly error-sensitive solutions including the case M¡ ¢ = M. Therefore, to obtain satisfying results, regularization procedure [12] is introduced as

$$\min [|AX - B|^2 + m|RX|^2] \tag{21}$$

where the matrix R sets the order of regularization and coefficient m sets its magnitude.

Then the explicit solution of Eq. (22) can be written as

$$X = (A^T A + m R^T R)^{-1} A^T B \tag{22}$$

As a note, zeroth order of regularization is used in this study so that R becomes identity matrix.

Once the temperatures and heat fluxes at the boundaries are fully determined, it is straightforward to use them to find the temperatures at interior points $x \in \Omega'$. Equation (17) is again used, but this time C is taken to be unity as in Eq. (12).

4. Numerical Experiments and Discussions of the Results

The sample problem chosen here for numerical experiments of the present inverse method is the same as the case addressed in [20]. That is about the prediction of the temperature distributions in the piping wall of PWR pressurizer surge line, of which the outer surface is insulated, subjected to internally stratified flow of two distinct fluids at different temperatures. As, in [20], it is assumed that the outer surface of the pipe is perfectly insulated and axial conduction heat transfer through the pipe wall is negligible. A specified amount of hot water flows into the horizontal pipe of surge line initially filled with cold water at the same temperature of the pipe wall, and then occupies the upper position of the pipe suddenly. Jo et al. [20] calculated the unsteady stratified flow and temperature distributions in the pipe using the finite volume approach. In the numerical experiments to validate the inverse heat conduction solution technique, the transient temperature distributions in the circumferential direction at the outer surface of the piping provided in reference [20] are used as input data of the specified Dirichlet condition imposed on the outer surface, and the approximations of the temperature distributions at the inner surface by the present method are compared with those in [20].

In this study, only the thermal stratification model in which the fluid interface level is at a height of 0.25 d_i (d_i is the diameter of the pipe) is investigated for the numerical experiments. Because the outer surface of the pipe wall is adiabatic, the specified value of heat transfer coefficient h involved in Eq. (4) is zero. The outer and inner boundaries of the solution domain are comprised of both circumferential surfaces of the pipe. For solving the inverse problem, each boundary is discretized as 25 elements, and the temperature values provided in [20] are specified to the corresponding nodes on the outer boundary. The nodal points on the outer boundary are numbered in a counterclockwise direction, while those on the inner boundary are numbered in a clockwise direction. This follows the convention of the direction of contour integration.

The values of computational parameters used here are as follows:

- Initial temperature of the cold water and pipe wall: 66 °C
- Initial temperature of the hot water: 232 °C
- Thermal conductivity of the pipe wall material: 15.4 W/m°C
- Outer diameter of the pipe: 0.305m
- Thickness of the pipe: 0.036m

The circumferential temperature distributions at the inner surface of pipe wall were determined with two different values of the regularization coefficient, μ=0.0 and 0.1. The test results showed that the effects of regularization coefficient on the predictions are negligible. The reasons are that the inverse solution coefficient matrix is closely at a well-condition because of small number of elements and the error-free input data was used in the calculations. If the number of elements used for the analysis increases, the solution coefficient matrix becomes ill conditioned and the regularization coefficient will affect the calculation results.

Figs. 2(a)-(e) show the comparisons of the present predictions of circumferential temperature distributions at the outer surface of pipe to the results obtained by Jo et al. [20], for the cases where the non-dimensional elapsed times are 500, 750, 1000, 1500 and 2000. In each figure, the temperature distributions at the specified time are plotted along the circumference of pipe. The values of angle 0° and 180° indicate bottom and top positions of the inner wall surface, respectively. As can be seen in the figures, for all the elapsed times the two temperature distribution profiles obtained by the different methods have similar shapes. The position of inner wall surface area at the highest temperature is located at the level of fluid interface the wall, while the bottom is at the lowest temperature. During the whole transient period the approximations by the present method are under-estimated. The deviation between the two profiles is considerably large in the early stage of the transient, but it decreases as time elapses and vanishes at steady state condition.

It is seen in Figs. 2(a)-(e) that the maximum temperature differences in the circumferential temperatures by both calculations comes out at the dimensionless elapsed time zone ranging from 500 to 1000 as was discussed in [20]. Although the deviations between the two temperature profiles are not negligible, the maximum temperature differences in both profiles make little difference. Therefore it is considered that the present method may be applicable to determine the transient temperature distributions in the piping wall, using measured circumferential temperature distributions and available data of the pipe at the input to the inverse heat conduction solution. To obtain reliable and correct results by the present method, it is necessary to take the input data as correctly as possible.

5. Conclusions

This paper presented a method to determine the temperature distributions in the pipe wall of pressurizer surge line from the circumferential temperature distributions measured on the outer surface of the piping by applying the inverse steady-state heat conduction solution. The steady-state inverse heat conduction solution was obtained using the boundary element method in conjunction with regularization procedure. As a result of the numerical experiments, it is considered that the present method may be applicable to determine the transient temperature distributions in the piping wall, using measured circumferential temperature distributions and available data of the pipe at the input to the inverse heat conduction solution. To obtain reliable and correct results by the present method, it is necessary to use noise-free input data as practicable.

Nomenclature

A	Coefficient matrix
B	Vector containing linear elements
C	Coefficient
$\boldsymbol{h}$	Convective heat transfer coefficient
k	Thermal conductivity
M	Number of nodal points
N	Number of boundary elements
$\mathbf{n}$	Unit normal vector
$\boldsymbol{q}$	Heat flux
R	Regularization matrix
r	Distance from the source point to points on the boundary element
T	Temperature
U, V	Functions in the general expression
X	Vector of unknowns
x	Coordinates of two dimensional domain

<u>Greek symbols</u>

G	Boundary
D	Dirac delta function
d_{ij}	Kronecker delta
q	Angle
m	Regularization coefficient
Ω	Domain of the original problem
Ω'	Domain of the auxiliary problem

<u>Subscripts</u>

a	Outer boundary
b	Inner boundary
e	Ambient condition

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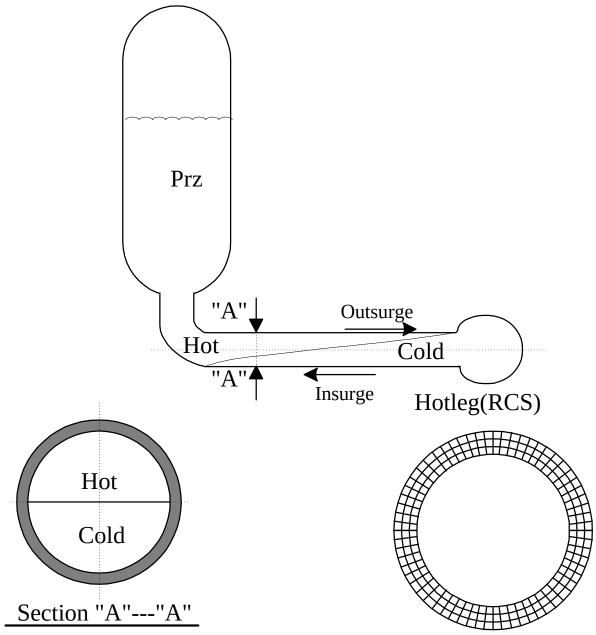
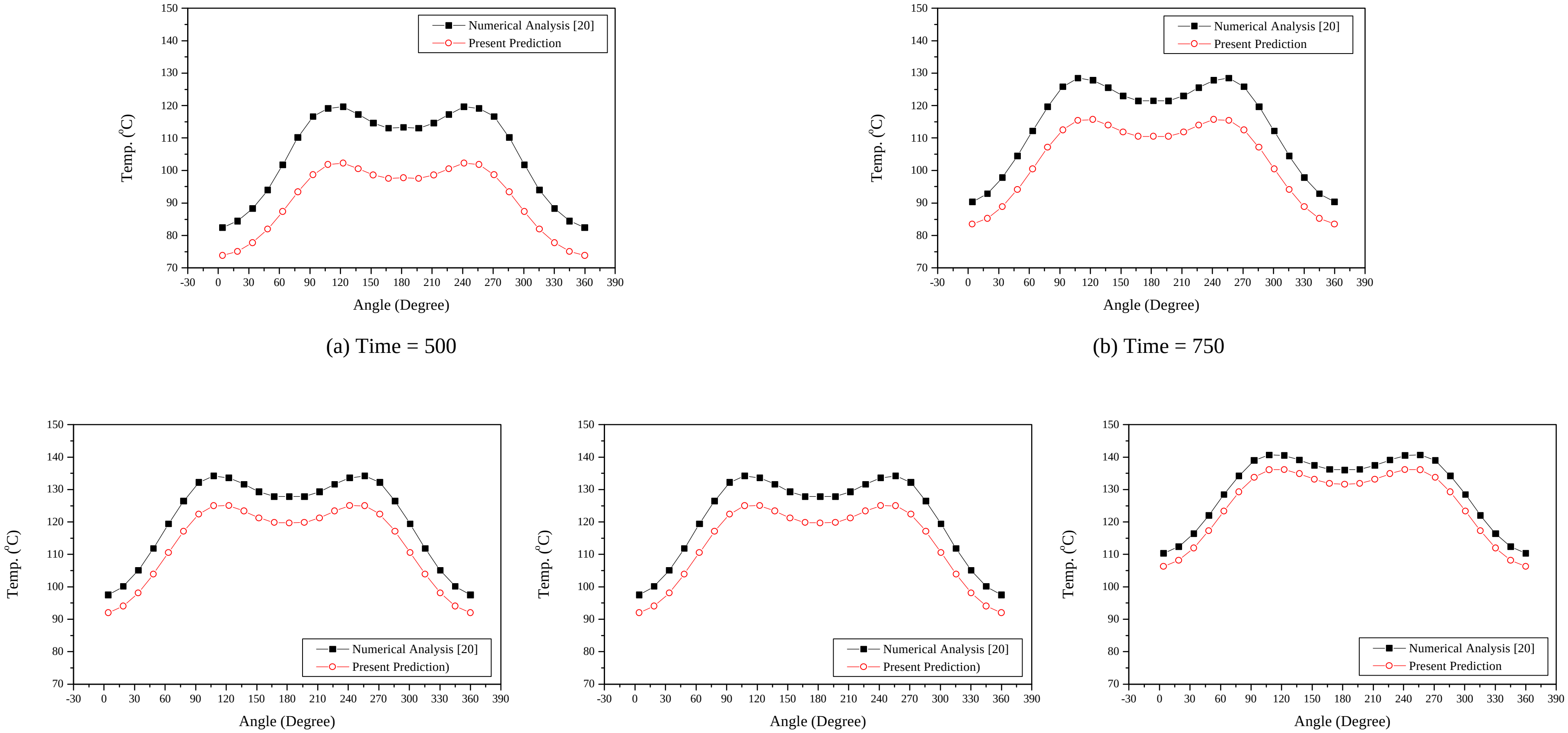


(a) Cross-sectional view of the circular pipe (b) Nodes on the boundary

Fig. 1 Stratified flow in the pressurizer surge line



(a) Time = 500

(b) Time = 750

(c) Time = 1000

(d) Time = 1500

(e) Time = 2000

Fig. 2 Comparisons of the circumferential temperature distributions predicted by the present method to the results provided in [20].