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One Dimensional Analysis of the End Effect of an EM Pump

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Abstract

Longitudinal end effect due to finite length of the pump are analyzed one dimensionally on an annular linear induction electromagnetic (EM) pump for the transportation of the electrically conducting liquid metal. The mathematical regions of the modeled pump is divided into three of the inlet, outlet and developing zone in large parts. Solving governing equations with the applied boundary condition, the distributions of magnetic field and developing force are investigated according to the coordinate of axial direction and compared with those of the pump with infinite length. At both ends of the pump, it is shown that the radial magnetic field is distorted and even the opposite force, which may cause local separation of the flow as the velocity of the pumping fluid is increased, is generated at the inlet region. In the present study, frequency control is suggested as one of nthe methods for the reduction of the end effect of the pump.

1. Introduction

Studies on the annular linear induction EM pumps have been carried out for the circulation of liquid metal since old Soviet employed the electromagnetic pumping method by moving magneic field in the beginning of the 20th century[1]. Especially, due to its advantages such as contactless operation with liquid metal, no noise, no bearings, and no rotary part over mechanical pump, attention gas been given to it in the nuclear Liquid Metal Reactor(LMR)system that uses chemically reactive liquid sodium coolant. By the way, because of the finite length of a pump, there is distortion of magnetic field and thus developing force at both ends of it. That is, phenomenon of the end effect[2] brings about reduction of the efficiency of the pump in result. Besides, reverse force at the inlet region of the pump prevents sodium coolant from continuous flow and so eventually the pump system may get unstable. By setting governing equation and obtaining non-trivial solutions on the simplified model of the pump radial magnetic field and developing force distributions are found out on the pump with flowrate of 200 l/min. Parameter conditions for lessening the end effect are discussed from the result. Mainly, electrical methods are suggested rather than geometrical modification.

2. Mathematical Model of the Pump

Fig. 1 shows a real EM pump of the annular linear induction type. Practical arrangements of three-phase coils for the generation of the exciting current are converted to equivalent sheet current[3] as is seen in the analytical model of Fig. 2. Then sodium fluid is assumed to be steady and incompressible and in addition, it has constant axial flow velocity[4] with axisymmetry through entire annular channel where its density is ρ, electrical conductivity σ, viscosity μ, and magnetic permeability μ_0. In the Fig. 2 outer and inner core of the pump has much larger magnetic permeability(5,000 15,000 times) than that of vacuum of μ_0 and surface current flows azimuthally in the surface of the outer cylinder of the annular channel. On the other hand, equivalent sheet current of Fig. 2 is given as[3]

$$\begin{aligned} \mathbf{J_a}(r_b, z, t) &= J_m cos(\omega t - kz)\hat{\theta} \\ &= \mathrm{Re}[J_m e^{j(\omega t - kz)}]\hat{\theta} \end{aligned} \tag{1}$$

where J_m is amplitude of sheet current represented as $\frac{3\sqrt{2}k_w NI}{p\tau}$ where k_w is winding coefficient, N number of turns, I exciting current, p number of pole pairs and τ pole pitch, ω angular frequency and k wave number.

3. Calculation of Radial Magnetic Field and Force Density

MHD governing equations for the analysis of the incompressible flow are described using dimensionless parameters of Reynolds number,R_e, Hartmann number, H_a and magnetic Reynolds number, R_m as follows

$$\begin{gathered} \nabla \cdot \mathbf{V} = 0 \\ \frac{\partial \mathbf{V}}{\partial \mathbf{t}} + (\mathbf{V} \cdot \nabla) \cdot \mathbf{V} = \nabla P + \frac{1}{R_e}\nabla^2 \mathbf{V} + \frac{H_a^2}{R_e}\mathbf{J} \times \mathbf{B} \\ \nabla \times \mathbf{B} = R_m \mathbf{J} \\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathbf{t}} \\ \nabla \cdot \mathbf{B} = 0 \\ \mathbf{J} = \mathbf{E} + \mathbf{V} \times \mathbf{B} \end{gathered} \tag{2}$$

where $R_e = \frac{\rho R_0 V_s}{\mu}$, $H_a = \sqrt{\frac{\sigma B_0^2 R_0^2}{\mu}}$ and $R_m = \mu_0 \sigma R_0 V_s$. Then, reference values for the non-dimensionalization are given as

$$B_0 = \frac{\mu_0 J_m}{\sqrt{2} R_0 k}, \qquad R_0 = r_a - r_b, \qquad V_s = \frac{\omega}{k}$$

Extending Eq. (2) by components, reduced MHD equations are obtained for the radial magnetic field and azimuthal induced current density. First of all, equivalent sheet current exists in the active region of the pump with finite length, L for the purpose of developing the pump. Therefore the pump is divided into three regions of inlet, active zone and outlet of the pump and then analytic solutions are obtained from the boundary conditions between each zone. In the Fig. 2, let the left side of the pump be the inlet, the right be outlet and the region of the pump cores and equivalent sheet current between both sides be active zone. Then reduced equations in each zone are shown as follows.

(1) Active Zone ($0 \leq z \leq L$)

$$\frac{\partial^2 B_r}{\partial z^2} - R_m(1-s)\frac{\partial B_r}{\partial z} - kR_o R_m B_r j = -\sqrt{2} k^2 R_o^2 e^{-jkR_o z} j \tag{3}$$

$$J = \frac{1}{R_m}\frac{\partial B_r}{\partial z} - \frac{\sqrt{2} k R_o}{R_m} e^{-jkR_o z} \tag{4}$$

$$f_z = \frac{H_a^2}{2R_e} Re(J B_r^c) \tag{5}$$

Then, in the Eq. (3) solution of the magnetic field is obtained as

$$B_{ractive} = A e^{\gamma_1 z} + B e^{\gamma_2 z} + B_n e^{-jkR_o z} \tag{6}$$

where

$$\begin{aligned} \gamma_1 &= \frac{R_m(1-s) + \sqrt{R_m^2(1-s)^2 + 4kR_o R_m j}}{2} \\ \gamma_2 &= \frac{R_m(1-s) - \sqrt{R_m^2(1-s)^2 + 4kR_o R_m j}}{2} \\ B_n &= \frac{\sqrt{2}}{-j + \frac{sR_m}{kR_o}} \end{aligned} \tag{7}$$

(2) Inlet and Outlet Zone ($z < 0$ and $z > L$)

$$\frac{\partial^2 B_r}{\partial z^2} - R_m(1-s)\frac{\partial B_r}{\partial z} - kR_o R_m B_r j = 0 \tag{8}$$

$$J = \frac{1}{R_m}\frac{\partial B_r}{\partial z} \tag{9}$$

In the Eq. (8) solutions at inlet and outlet of the pump are given as

$$\begin{aligned} B_{r\ inlet} &= Ce^{\gamma_1 z} && (Z < 0) \\ B_{r\ outlet} &= De^{\gamma_2 z} && (Z > L) \end{aligned} \tag{10}$$

Coefficients A, B, C and D of Eq. (6) and Eq. (10) of the general solutions are determined from the boundary conditions followed by Eq. (11) and Eq. (12) that induced current density and magnetic field are continuous at the each boundary.

- Continuity on the magnetic field

$$\begin{aligned} B_{r\ inlet}|_{z=o} &= B_{r\ active}|_{z=o} \\ B_{r\ outlet}|_{z=L} &= B_{r\ active}|_{z=L} \end{aligned} \tag{11}$$

- Continuity on the induced current density

$$\begin{aligned} J_{inlet}|_{z=o} &= J_{active}|_{z=o} \\ J_{outlet}|_{z=L} &= J_{active}|_{z=L} \end{aligned} \tag{12}$$

Unique solution can be obtained for the radial magnetic field and induced current of the active region from the coefficients of Eq. (13) found out by applied boundary conditions of Eq. (11) and Eq. (12) to general solution of Eq. (6)

$$\begin{aligned} A &= \frac{-\{\sqrt{2}kR_o + (\gamma_2 + jkR_o)B_n\}}{\gamma_2 - \gamma_1} e^{-\gamma_1} \\ B &= \frac{\sqrt{2}kR_o + (\gamma_2 + jkR_o)B_n}{\gamma_2 - \gamma_1} \\ C &= A(1 - e^{\gamma_1}) \\ D &= B(1 - e^{-\gamma_2}) \end{aligned} \tag{13}$$

In the Fig. 3 (a) and (b), the distribution of radial magnetic field and force density obtained by using the Eq. (6) and Eq. (5) are plotted in the active region according to axial coordinates. As is seen in the Fig. 3 (a), radial magnetic field appears distorted at both ends of the pump, that is, due to end effect of the pump, the magnetic field deviates from sinusoidal shape which appears in the pump of the infinite length without end effect. Especially, it is shown that developing force acts reversely at the inlet region of the pump in the Fig. 3 (b). Reversed force by end effect reduces the overall efficiency of the pump resultantly. Then, there may be installation of compensation coils or operation with low input frequency or increased numbers of magnetic pole pairs as methods for the reduction of end effect. Generally, it is known that end effet is decreased by the reduced synchronized velocity of the pump for the increased number of pole pairs or decreased frequency in the constant length of the pump[5]. But, consideration should be taken into decreased fluid velocity by reduced speed of moving magnetic field due to too low synchronized velocity. Fig. 4 shows the ratio of the developing force of the case with end effect to that without end effect according to frequency change. It is understood in the Fig. 4 that the ratio is almost close to unity for the low frequency around teen Hz and then pump has little end effect. That is because fluid velocity gets slow due to the synchronized speed decreased by the lower frequency.

4. Conclusion

Analysis on magnetic field and force density has been carried out by solving MHD equations one dimensionally. It was shown that magnetic field was distorted at both ends of the pump and especially caused reverse developing force at the inlet region. Then, for the purpose of reduction of this end effect, the method of frequency control has been suggested. An EM pump is shown to have little end effect in the low input frequency of teen Hz from the analysis. Generally considering that the higher velocity of the EM pump is, the larger end effect is, Practically, low frequency operation is predicted to be feasible as far as developing force and efficiency are too much decreased.

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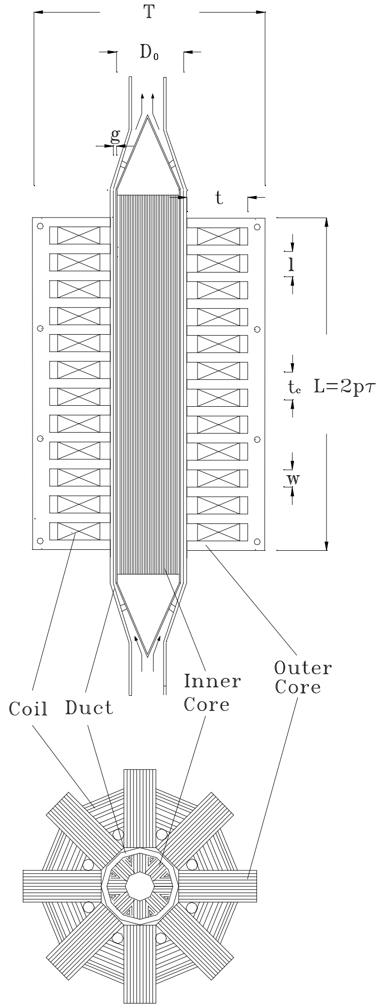


Fig. 1 Cross-Sectional View of the Annular Linear Induction EM pump

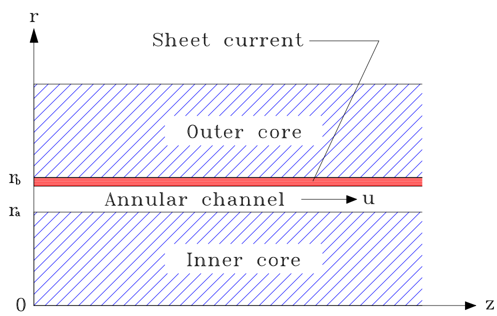
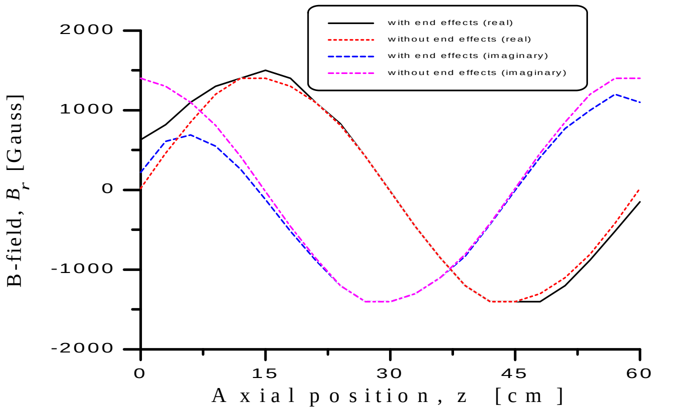
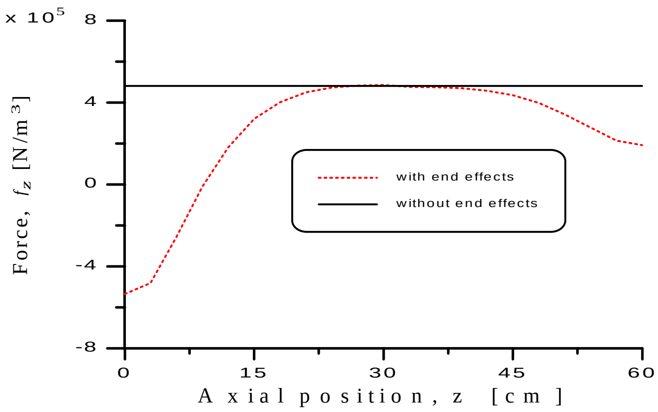


Fig. 2 Mathematical Model of the Annular Linear Induction EM Pump



(a)



(b)

Fig. 3 Distribution of (a) Magnetic Field and (b) Force Density according to Axial Direction in the Case with and without End Effect

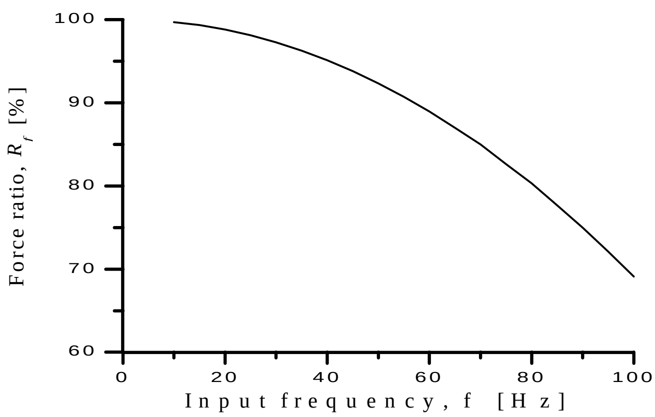


Fig. 4 Ratio of Both Forces according to Frequency Change in Case with and without End Effect