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Application of High-Order Uncertainty for Severe Accident Management

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ABSTRACT

The use of probability distribution to represent uncertainty about point-valued probabilities has been a controversial subject. Probability theorists have argued that it is inherently meaningless to be uncertain about a probability since this appears to violate the subjectivists' assumption that individual can develop unique and precise probability judgments. However, many others have found the concept of uncertainty about the probability to be both intuitively appealing and potentially useful. Especially, high-order uncertainty, i.e., the uncertainty about the probability, can be potentially relevant to decision-making when expert's judgment is needed under very uncertain data and imprecise knowledge and where the phenomena and events are frequently complicated and ill-defined. This paper presents two approaches for evaluating the uncertainties inherent in accident management strategies: "a fuzzy probability" and "an interval-valued subjective probability". At first, this analysis considers accident management as a decision problem (i.e., "applying a strategy" vs. "do nothing") and uses an influence diagram. Then, the analysis applies two approaches above to evaluate imprecise node probabilities in the influence diagram. For the propagation of subjective probabilities, the analysis uses the Monte-Carlo simulation. In case of fuzzy probabilities, the fuzzy logic is applied to propagate them. We believe that these approaches can allow us to understand uncertainties associated with severe accident management strategy since they offer not only information similar to the classical approach using point-estimate values but also additional information regarding the impact from imprecise input data.

Keywords: High-Order Uncertainty, Severe Accident Management, Fuzzy Probability, Bayesian Probability

1. Introduction

Severe accident management involves the assessment of various phenomena under uncertain and imprecise conditions. For each phenomenon that is poorly understood there may be several different models, each incomplete with respect to various aspects of the strategy to be assessed. For example, liner melt-through is one of the concerns in NUREG-1150 for BWRs [1]. NUREG-1150 considers liner melt-through as a "possible early containment failure mechanism", even with the presence of water in the pedestal area. However, Theofanous [2] has argued that liner melt-through under this circumstance may not be credible. The physical processes regarding this issue are extremely complex and difficult to model. Major uncertainty is associated with the behavior of molten debris when it leaves the vessel and interacts with the concrete and the water in the pedestal area. Current PRA methodology uses expert opinion in the assessment of this kind of rare event probabilities. The problem is that these probabilities may be difficult to estimate even though reasonable engineering judgment is applied. This occurs because expert opinion under incomplete knowledge and limited data is inherently imprecise and uncertain. Hence, methods using point-valued probabilities provided by experts may not adequately reflect this imprecision. This kind of characteristic in human judgment under very uncertain situations should be reflected in the analysis of severe accident management.

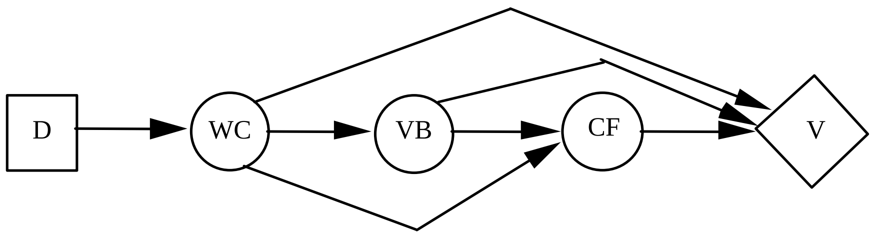
The use of distribution to represent uncertainty about probabilities has been a controversial subject. Probability theorists have argued that it is inherently meaningless to be uncertain about a probability since it appears to violate the subjectivists' assumption that individual can develop unique and precise probability judgments. However, many others have found the concept of uncertainty about the probability to be both intuitively appealing and potentially useful. Especially, high-order uncertainty, i.e., the uncertainty about the probability, could be potentially relevant to decision-making when one needs a expert's judgment under very uncertain data and imprecise knowledge and where the phenomena and events are frequently complicated and ill-defined.

The main purpose of this paper is to apply two famous approaches for evaluating this kind of uncertainties inherent in accident management. These approaches are "an approach using fuzzy theory" and "an approach using interval-valued subjective probability". The analysis includes the representation of uncertain or imprecise probabilities in an influence diagram, and propagation of these probabilities through the diagram and deduction of values for decision making. We believe that these approaches can allow us to increase the understanding of uncertainty in the PSA for severe accident management because they offer not only information similar to the classical approach using point-estimate values but also additional information regarding the impact from imprecise input data.

2. Decision-Oriented Framework for Accident Management

Decision analysis is a logical framework for decision making based on the principles of probability theory and utility theory. In this paper, the influence diagram representation of a decision problem is shown as a method to evaluate the drywell flooding strategy for BWRs[3]. Influence diagrams are a graphical and mathematical representation of probabilistic decision problems. An influence diagram is an acyclic directed graph, in which the variables of the decision model are represented as chance nodes and the probabilistic dependencies among variables are represented as directed arcs among nodes. Unlike decision trees, influence diagrams are compact and show an unambiguous representation of dependencies in a decision model. This section describes a simple example of the application using an influence diagram and fuzzy set theory in a decision problem.

Consider the simplified example of a PWR cavity flooding strategy as shown in Figure 1. Each circle indicates a chance node with two outcomes. The value node depends upon the three chance nodes (WC, VB and CF) which are defined in Figure 1. When a reactor vessel fails with a flooded cavity, containment failure due to an ex-vessel steam explosion is possible. When the reactor vessel fails with no water in the cavity, containment failure due to a core-concrete interaction may occur. Suppose that we have the following values for the probabilities and consequences:



D: Decision VB: Vessel Failure
WC: Cavity Flooding CF: Containment Failure
V: Value (Expected Risk)
Figure 1. Example of a cavity flooding strategy

P_1= P(WC|D) : the probability of successful flooding on time, given the decision to flood,
P_2= $P(-VB \mid WC)$: the probability of no vessel failure given successful flooding,
P_3= $P(-VB \mid -WC)$: the probability of no vessel failure given unsuccessful flooding,
P_4= $P(-CF \mid VB, WC)$: the probability of no containment failure given vessel failure and successful flooding,
P_5= $P(-CF \mid VB, -WC)$: the probability of no containment failure given vessel failure and unsuccessful flooding,
C_1= consequences of no vessel failure (= 0.0 early fatality per year),
C_2= consequence for no containment failure (= 0.0 early fatality per year),
C_3= consequence for containment failure with a flooded condition, i.e., with an ex-vessel steam explosion after vessel failure (= 0.01 early fatality per year), and
C_4= consequence for containment failure without a flooded condition, i.e., due to a CCI after vessel failure (= 0.01 early fatality per year).

The risk measure, conditional on the decision, can be obtained as follows:

$$EV\ (\text{Do Nothing}) = C_4\ (1 - P_3)\ (1 - P_5)$$

$$EV\ (Flooding) = C_3\ P_1\ (1 - P_2)\ (1 - P_4) + C_4\ (1 - P_1)\ (1 - P_3)\ (1 - P_5) \quad (1)$$

The probabilities associated with each node can be determined using PRA methodology. In the assessment of a conventional influence diagram, the probabilities are usually treated as point values. Hence, the calculation is straightforward and the results can be compared directly. As already mentioned however, due to very limited data and knowledge, it is often difficult to quantify exact values for the probabilities regarding these events. In this case, the analysis should handle uncertain and imprecise values. Conventional sensitivity analysis may now be applied. However, sensitivity analysis usually varies only one variable, while the other variables remain constant. Therefore it may not show the combined effect of uncertain input data. Also, some variables which are correlated with each other may not be treated adequately in the analysis.

One of the natural ways of quantifying these probabilities is to use interval values. It is possible to choose a best estimate value from the interval values by applying reasonable engineering judgment. The following is an example:

" Due to limited knowledge, the value of probability may lie between 0.7 and 0.9. However, based on reasonable engineering judgment, the value of 0.8 could be highly preferred as a best estimate."

Again, conventional sensitivity analysis may not capture this kind of a confidence level expressed by an expert. For example, the value of 0.8 represents a high confidence compared to other two values (0.7 and 0.9). Analysis should reflect this kind of confidence regarding input data.

3. Classification of Uncertainties

A higher order uncertainty is an uncertainty about one's uncertain values. First of all, we need to distinguish between two major types of uncertainty:

a) Uncertainty due to stochastic variability, and
b) Uncertainty due to a lack of knowledge.

The first type of uncertainty is due to the actual, random behavior in some physically measurable quantity. Examples of the stochastic variability are variations in weather, variations in component failure times from one observation to another, and variations in consequences from one accident to another. The second type of uncertainty is quite different from the stochastic variability. It is vagueness or imprecision in an analysis, or stated value. The uncertainty exists because of a lack of knowledge; if we gained more information and more knowledge, the uncertainty would decrease or would not exist. Examples of this uncertainty are uncertainties associated with an estimated value of a value, or uncertainties in the appropriateness of an consequence model.

The first case assesses the uncertainty when the end point is an unknown distribution of values. The assessment end point is a true but unknown distribution of values representing random variability in the parameters or measured data used in the model. On the other hand, the second involves a case when the end point is a fixed but unknown value due to the imprecision in the analysists knowledge about models, their parameters, and/or their predictions. The subjective confidence interval can be used for the unknown value in this case. The distribution used represents a range of "degrees of belief" that the true but unknown value is equal to or less than any value selected from the distribution.

This study focuses on the second uncertainty, a called "**epstemic uncertainty**"[4]. This uncertainty arises that one cannot give a single value of probability about an event. Consider an event tree branch point that is representing steam explosion occurrence after vessel failure. The traditional Bayesian view assumes precise point-valued probabilities. In this case, one states that the yes branch is 1 with probability p, and the no branch with the probability 1-p. Although problems such as cognitive imprecision or vagueness about determining this probability have been recognized, their uncertainties are not considered. The Bayesian probability theorists have argued that it is inherently meaningless to be uncertain about a probability since this appears to violate the subjectivists' assumption that individual can develop unique and precise probability judgments. However, many others have found the concept of uncertainty about the probability to be both intuitively appealing and potentially useful. Especially, high-order uncertainty, i.e., the uncertainty about the probability, can be potentially relevant to decision-making when expert's judgment is needed under very uncertain data and imprecise knowledge and where the phenomena and events are frequently complicated and ill-defined.

In recent, there are researchers who believe that the requirement for point-valued probabilities is too strong and suggested an interval-based approach. On the other hand, a theory termed fuzzy set theory has undergone rapid development in the past several years. Fuzzy set theory attempts to address the uncertainty due to lack of knowledge which is not addressed by conventional approaches[5].

4. Application of a High-Order Uncertainty

To evaluate the uncertainty of probabilities inherent in accident management strategies described in the previous section, two approaches are used : "an approach using fuzzy theory" and "an approach using interval-valued subjective probability".

4.1 Representation of uncertainty

First of all, the representation of these probabilities is considered for each approach. This study uses a simple triangular representation of these probabilities. In this representation, the modal value (i.e., a_2) is interpreted as the best -estimated value and the two values (i.e., $a_1$and a_3) are extremes in the distribution. It gives a rational approximation with an appropriate bound for an uncertain probability. In addition, it has a simple form and therefore is easy to handle. Also, it has a relatively small number of parameters for estimation. These properties of the representation provide a good basis for combining information when available information is limited.

For the fuzzy approaches, one can use the concept of "a fuzzy probability". It is called a possibility distribution of probability, which represents an imprecise probability by means of subjective possibility measures associated with judgment uncertainty. This can simultaneously model the probability and its degree of possibility expressed by an expert. In this case, the value of a_2 is defined as a modal value and interpreted as the most possible value (possibility is one). The two values (i.e., a_1 and a_3) are considered as two extremes and the least possible values (possibility is zero).

In the framework of interval-valued subjective probability, the value of a_2 is called a mode and interpreted as the best-estimated value in the range. The two values (i.e., a_1 and a_3) are represented as extremes in the range. Compared with fuzzy probability, the value of a_2 is not necessary to be one but the CDF of its distribution should be one. Figure 2 depicts the triangular representation used in the two approaches.

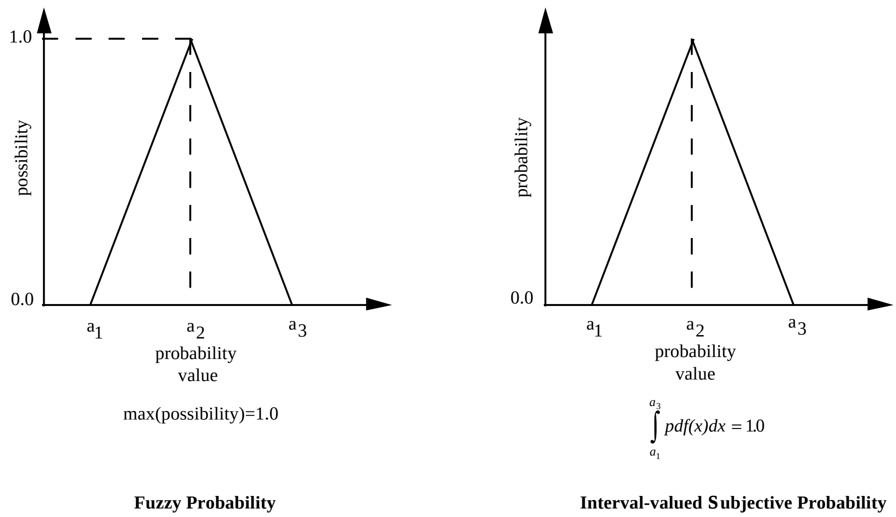


Figure 2. Triangular representation used in two approaches

4.2 Propagation of Uncertainty

A basic principle for the calculus of the fuzzy framework is known as the "Extension Principle" [6]. However, the implementation of the solution procedure is not trivial using this approach. The reason is that the solution procedure corresponds to a nonlinear programming problem which is very complex, except for the simplest mapping functions. A simple approach is to use the discretization technique on the variable domain. However, this technique would fail and lead to irregular and fuzzier results because the min-max operation on fuzzy sets can lead to irregular membership functions. Hence, in the present study, the calculational procedure is implemented by the method proposed by Dong and Wong [7]. This algorithm is based on the discretization technique on the possibility measure or membership value domain, instead of on the variable domain, and an interval analysis. A general computational algorithm is provided below:

1) Discretize the range of possibility [0,1] into a finite number of values. Call these a_1, a_2, ..,a_n. This is called an "α-cut" on the possibility measure domain.
2) For each value α_j, find the corresponding intervals on the value domain in x_i, i=1,..,N. Denote the end points of these interval by [a_1, b_1], [a_2, b_2],..., and so on.
3) Taking one end point from each of the intervals, the end points can be combined into an N-ary array. There are 2^Ndistinct permutations, giving 2^N combinations for the vector (x_1, x_2,..,x_N).
4) Evaluate the function f(x_1, x_2,..,x_N)for each of the 2^N combinations and obtain 2^N values for y. Denote these by y_1, y_2,...,y_{2N}. The desired interval for y is given by [min y_k, max y_k]. These points define the support of α_j-cut of the final solution.

For the propagation of the interval-valued subjective probability, this study uses a Monte-Carlo analysis [8]. The method is based on performing evaluations with probabilistically selected input parameters, and then using the results of these evaluations to determine both the uncertainty in model predictions and the input variables that give rise to this uncertainty. In general, the analysis involves four steps.

In the first step, a range and distribution are selected for each input variable. These selections will be used in the next step in the generation of a sample from input variables. In the second step, a sample is generated from the ranges and distributions specified in the first step. The result of the step is a sequence of sample elements of the form

$$\mathbf{x_i} = [x_{i1}, x_{i2},...,x_{in}], \qquad i=1,2,..m \qquad (2)$$

where *n* is the number of input variable and *m* is the sample size. The most widely used sampling techniques are random sampling, importance sampling and Latin hypercube sampling.

In the third step, the model is evaluated for each sample element shown in (2). This creates a sequence of results of the form

$$y_i = f\,(x_{i1}, x_{i2}, x_{i3} .., x_{in}) = f\,(\boldsymbol{x_i}), \qquad i=1,2,...,m \qquad (3)$$

In essence, these model evaluations create a mapping from the analysis input (i.e., the $\boldsymbol{x}_i$) to the analysis results (i.e., the y_i) that can be used studied in subsequent uncertainty analysis.

In the fourth step, the results are used as the basis for an uncertainty analysis. Describing uncertainty in the output variable, y, involves the quantification of the range of y, its arithmetic mean value, the arithmetic standard deviation of y, and upper and lower percentiles of y, such as 5 % lower and 95 % upper bound. Convenient tools for presenting such information are the probability density function (PDF), or the cumulative distribution function (CDF) for y.

4.3 Interpretation of results

For the purpose of the comparison, this study characterizes 3 important values for each approach.

The approach using fuzzy theory

1) Knowledge Interval

The interval of confidence of a fuzzy value is defined as the interval that contains all the elements of the fuzzy value corresponding to any specific possibility measure. Then the specific possibility measure indicates a degree of belief that a true value of the fuzzy quantity may exist within the interval. The endpoints of this interval are obtained by the specific "α-cut" on the possibility measure or membership function value domain. The knowledge interval of a fuzzy value can be obtained by zero-valued α-cut on the possibility measure or membership function value domain. In this sense, it represents a possible maximum interval where a true value may exist within. This value reflects the impact of the imprecise input data regarding the flooding case. In other words, we have an imprecise or incomplete knowledge to evaluate the flooding strategy. As we have more precise input data, the knowledge interval is reduced. Therefore, the interval is equal to zero when we can assign the exact point values of the input data.

2) Modal Value

The peak of the possibility distributions (with a possibility value one) is called a modal value. The value is equivalent to the value obtained by best estimate point-valued probabilities. In this respect, the modal value can be regarded as an element with the highest confidence in the fuzzy outcome. This is why the concept of a fuzzy probability in fuzzy logic is consistent with the classical approach using point-valued probabilities.

3) Average Index

The ranking function [9], a so-called average index of fuzzy outcome $\tilde{A}$, can be defined by

$$V_p = \int_0^1 f_a(a)dp(a) \quad (4)$$

By means of $V_P(\bullet)$, a comparison relation on real value is built:

$$\text{For } \tilde{A} \text{ and } \tilde{B},\ \tilde{A} \le \tilde{B} \Leftrightarrow V_p(\tilde{A}) \le V_p(\tilde{B}) \quad (5)$$

We will say that $\tilde{A}$ is indifferent to $\tilde{B}$ if and only if their average indices coincide. In general, the definition of function of f_A could be made arbitrarily by a decision-maker. However, the following function is usually used:

$$f_A^{\lambda}(\alpha) = \lambda b_\alpha + (1-\lambda) a_\alpha \quad (6)$$

where $\lambda \in [0,1]$, $A_\alpha = [a_\alpha, b_\alpha]$. b_α and a_α are "α-cut" on the possibility distribution of $\tilde{A}$.

The parameter λ is an optimism-pessimism degree, which should be selected by a decision-maker. When the most advantageous decision is to choose the smallest quantity (i.e., the smallest risk), an optimistic person would think of the lower extreme of the interval a_α ($\lambda = 0$) . On the contrary, a pessimistic person would prefer the upper extreme of the interval b_α ($\lambda = 1$) .

In this analysis, the index proposed by Yager [10] is used, which uses λ as $\frac{1}{2}$: In this case, it is assumed that we have an "unbiased degree" to make a decision.

$$V_p(\tilde{A}) = \int_0^1 (b_a + a_a)\frac{1}{2}\, da \quad (7)$$

The approach using interval-valued subjective probability

1) Mean

The mean is given by

$$\bar{x} = \sum_{i=1}^{n} x_i p_i \quad (8)$$

where n = the no. of samples

2) Median

The median is the value that the an result has a 50 % probability of exceeding, i.e.

$$F(x_{0.5}) = 0.5 \qquad (9)$$

3) Maximum and minimum value

These two value can be obtained after the random sampling or LHS. These are the values for the determination of ranges of a result obtained.

5. Results

For uncertain probabilities on the influence diagram, suppose that the four probabilities; P_1, P_2, P_4 and P_5 are considered. The following interval-valued probabilities are assumed:

P_1= [0.5, 0.7, 0.9]
P_2= [0.7, 0.8, 0.9]
P_3= [0.4, 0.5, 0.6]
P_5= [0.1, 0.2, 0.3] (10)

where the central values mean best-estimated values and the other two are extreme values of the probability. Given the four uncertain values of probabilities, the calculation is performed using Eq. (1).

Table 1 summarizes three values for the fuzzy approach in the example for both cases: "flooding" and "do nothing". As is seen, the value of the knowledge interval exactly reflects the impact of imprecise input data for both cases. That is, this value for "flooding" is broader than that of "do nothing" because of the four imprecise probabilities; P_1, P_2, P_4, and P_5. Since this value represents a possible maximum interval where a true value may exist within, the broader interval of "flooding" means that there exists considerable uncertainty in the result. However, even though there exists uncertainty in the results, the two values for "flooding" are smaller than those of "do nothing": the modal value ($3.1\text{x}10^{-3}$ vs. $8.0\text{x}10^{-3}$) and the average index ($3.14\text{x}10^{-3}$ vs. $8.0\text{x}10^{-3}$). It is usually difficult to make a decision based on results from imprecise input data. However, the above three values characterizing the fuzzy results for the simplified PWR cavity flooding example clearly show that "flooding" would be preferred to "do nothing".

Table 1. Results for the fuzzy approach for the example

	Knowledge Intervals	Modal Values	Average Indices
Flooding	[$1.1\text{x}10^{-3}$, $5.4\text{x}10^{-3}$]	$3.1\text{x}10^{-3}$	$3.1\text{x}10^{-3}$
Do Nothing	[$7.0\text{x}10^{-3}$, $9.0\text{x}10^{-3}$]	$8.0\text{x}10^{-3}$	$8.0\text{x}10^{-3}$

Table 2 shows three values for the subjective probability approach in the example for both cases: "flooding" and "do nothing". The min and max values in a range varies with the number of samples. However, the mean and median values which have similar property with the average indexes and modal values in the fuzzy approach do not show any big differences as the number of samples increases.

Table 2. Results for the interval-valued subjective probability approach for the example

Samples=1,000	Range (min. and max.)	Medians	Means
Flooding	[$1.55\text{x}10^{-3}$, $4.77\text{x}10^{-3}$]	$3.10\text{x}10^{-3}$	$3.11\text{x}10^{-3}$
Do Nothing	[$7.03\text{x}10^{-3}$, $8.95\text{x}10^{-3}$]	$8.0\text{x}10^{-3}$	$8.0\text{x}10^{-3}$

Samples=10,000	Range (min. and max.)	Medians	Means

Flooding	[1.36×10^{-3} , 4.92×10^{-3}]	3.10×10^{-3}	3.10×10^{-3}
Do Nothing	[7.03×10^{-3} , 8.95×10^{-3}]	8.0×10^{-3}	8.0×10^{-3}

As can be seen, it is surprised that the results show almost same between two approaches even though they have different ways of the representation of uncertain and calculational procedures. These approaches offer not only information similar to the classical approach using point-estimate values but also additional information regarding the impact from imprecise input data. However, the application for real domains is needed to validate the interpretation of this study.

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