

# **A Semi-linguistic Approach Based on Fuzzy Set Theory: Application to Expert Judgments Aggregation**

Seong Ho Ghyym

Korea Electric Power Research Institute  
Center for Advanced Reactor Development  
Munji-Dong 103-16, Yuseong-Gu  
Taejon, 305-380, South Korea

## **Abstract**

In the present work, a semi-linguistic fuzzy algorithm is proposed to obtain the fuzzy weighting values for multi-criterion, multi-alternative performance evaluation problem, with application to the aggregated estimate in the aggregation process of multi-expert judgments. The algorithm framework proposed is composed of the hierarchical structure, the semi-linguistic approach, the fuzzy R-L type integral value, and the total risk attitude index. In this work, extending the Chang/Chen method for triangular fuzzy numbers, the total risk attitude index is devised for a trapezoidal fuzzy number system. To illustrate the application of the algorithm proposed, a case problem available in literature is studied in connection to the weighting value evaluation of three-alternative (i.e., the aggregation of three-expert judgments) under seven-criterion. The evaluation results such as overall utility value, aggregation weighting value, and aggregated estimate obtained using the present fuzzy model are compared with those for other fuzzy models based on the Kim/Park method, the Liou/Wang method, and the Chang/Chen method.

## **1. Introduction**

In the areas of every level of probabilistic risk assessment and management in nuclear power plants, the application of expert judgments is broadly in demand. The behind reason is probably the fact that the assessment is usually performed, based on the limited data due to the scarcity of operating experience and/or on the incomplete knowledge due to the complexity of phenomena embedded in severe accidents [Cojazzi et al., 1996]. By expert judgment, we mean any judgment requiring special expertise [Otway and von Winterfeldt, 1992]. For example, the human reliability analysis is carried out using expert judgment due to sparse empirical human error data [Py and Jacobsson, 1998].

Recently, the problem of aggregation has received remarkable attention in the field of engineering based on the expert opinions or judgments. In the aggregation process, multi-expert judgments can be combined to achieve the aggregated value, according to various approaches. Two main directions can be generally distinguished: (1) non-fuzzy; and (2) fuzzy approaches.

In the non-fuzzy case, Zio [Zio, 1996] investigated the capability of the analytic hierarchy process (AHP) method as an analytical tool for treating an aggregation problem. As indicated by him, however, in assigning the importance intensity according to the Saaty's 9-scale, the decision-makers' (DMs') diverse and subjective opinion is incorporated into the resulting priority vectors.

Indeed, subjectivity of opinion or vagueness of assignment of ill-known numerical quantities can be more effectively manipulated in the framework of fuzzy set theory [Kim, 1987]. Regarding the fuzzy approach, Yu [Yu, 1997] suggested an approach to combination of fuzzy probabilities using Dempster-Shafer's theory. The method restricted to simultaneous treatment of only two fuzzy sets is exact but rather more difficult to use due to  $\alpha$ -cut arithmetic

operations used. Moon and Kang [Moon and Kang, 1998] studied a fuzzy approach associated with arithmetic operations on triangular fuzzy numbers. Using the five-member scale conversion table described in Ref. [Chang and Chen, 1994], they constructed linguistic matrices corresponding to AHP-based results [Zio, 1996]. For obtaining the fuzzy preference indices, they reduced the Chang/Chen method for multiple decision-makers to the formulation for single decision-maker. Then they coupled these with the total integral value, after the Liou/Wang method as one of ranking methods of triangular fuzzy numbers.

The major aim of the work is to propose a semi-linguistic fuzzy algorithm for treating the aggregation of judgments of multi-expert viewed as alternatives in the multi-criteria evaluation problem. In addition, extending the Chang/Chen method for triangular fuzzy numbers, the total risk attitude index for trapezoidal fuzzy numbers is suggested.

## 2. Semi-linguistic Approach Based on Fuzzy Set Theory

### Semi-linguistic Approach

Since these linguistic values have a property that the boundary of value is not crisp but vague, the approximate (or inexact) reasoning based on the concept of fuzzy sets [Zadeh, 1965] can be applied. The form of uncertainty associated with the property is vagueness (or fuzziness) rather than randomness (or ambiguity) [Klir and Folger, 1988, p.138]. Thus, linguistic fuzzy approaches, depending on the degree of combination of the linguistic variable and fuzzy set concepts, facilitate the DMs' subjective assignments of the importance weight and the preference rating.

In the case of a linguistic approach at the post-stage of the modeling, the evaluation results are presented by linguistic terms via a translation of fuzzy numbers using a similarity measure. In a multi-expert judgment aggregation problem, however, linguistic approach can be applied only to the pre-stage model of a fuzzy modeling of performance evaluation, not to the post-stage model. In the present work, therefore, the modeling is viewed as a semi-linguistic approach. A scale conversion table is shown in Table 2-1.

### Fuzzy Numbers

According to the fuzzy set theory, a fuzzy number in  $\mathbf{R}$  (i.e., set of real numbers) is a fuzzy set (or a fuzzy subset) that has both normal and convex properties [Kaufmann and Gupta, 1991]. Here, normality means that the highest value of membership function (m.f.) is equal to 1. By convexity we mean that every  $\alpha$ -cut is a closed interval of  $\mathbf{R}$ . For the readers with more interest, the mathematical definitions of fuzzy set and its  $\alpha$ -cut as well as normality and convexity may be referred to in Ref. [Klir et al., 1997].

A triangular fuzzy number  $N$ , represented by  $(a, b, c)$ , can be analytically formulated by the corresponding m.f.  $f_N(x)$  with  $a, b$ , and  $c \in \mathbf{R}$ :

$$f_N(x) = \begin{cases} 0 & \text{for } -\infty < x \leq a, \\ \frac{x-a}{b-a} & \text{for } a \leq x \leq b, \\ \frac{x-c}{b-c} & \text{for } b \leq x \leq c, \\ 0 & \text{for } c \leq x < +\infty. \end{cases} \quad (2-1)$$

Similarly, for a trapezoidal fuzzy number  $N = (a, b, c, d)$ ,

$$f_N(x) = \begin{cases} 0 & \text{for } -\infty < x \leq a, \\ \frac{x-a}{b-a} & \text{for } a \leq x \leq b, \\ 1 & \text{for } b \leq x \leq c, \\ \frac{x-d}{c-d} & \text{for } c \leq x \leq d, \\ 0 & \text{for } d \leq x < +\infty. \end{cases} \quad (2-2)$$

In case of  $b = c$  in the trapezoidal fuzzy number  $N = (a, b, c, d)$ , the trapezoidal fuzzy number is reduced to the triangular fuzzy number  $N = (a, b, d)$ .

#### Algorithm of Fuzzy Aggregation of Expert Judgments

In this work, an algorithm is proposed for aggregating multi-expert judgments. The algorithm consists of both the utility estimate model and the risk attitude index estimate model. The algorithm used for obtaining an aggregated judgment can be described as follows:

##### **Stage 1: Construction of hierarchical structure**

- (1) Identify objective of performance evaluation at the top level; define alternatives to be evaluated  $A = \{A_i \mid i = 1, 2, \dots, n\}$  at the bottom level; and choose evaluation criteria  $C = \{C_t \mid t = 1, 2, \dots, k\}$  at the middle level in a hierarchical structure.

##### **Stage 2: Construction of linguistic matrices**

- (2) Select linguistic importance scale as elements of a set  $W_t$  of linguistic importance weight of evaluation criteria; and determine linguistic preference scale as elements of a set  $S_{it}$  of linguistic preference rating of alternatives under each criterion  $C_t$ .
- (3) Assign a linguistic importance value to  $W_t$  for each criterion  $C_t$  of interest; and similarly, give a linguistic preference value to  $S_{it}$  for alternatives  $A_i$  under each criterion  $C_t$ ; then form a linguistic importance matrix and a linguistic preference matrix for each evaluation criterion.

##### **Stage 3: Fuzzification of linguistic matrices using a scale table**

- (4) Form a scale conversion table assigning fuzzy number associated with each linguistic scale to the importance weight set and the preference rating set.
- (5) Convert linguistic scale values of each set into fuzzy numbers using a scale conversion table.
- (6) Approximate fuzzy preference index  $F_i$  for each alternative  $A_i$  using the fuzzy weighted averaging operator.

##### **Stage 4: Defuzzification of fuzzy preference index**

- (7) Obtain the R-L type utility values  $U_R$  and  $U_L$  of fuzzy preference index  $F_i$ .
- (8) Evaluate the total risk attitude index  $\alpha_T$ .
- (9) Calculate the overall utility value  $U_T$  using the R-L type utility values  $U_R$  and  $U_L$  and the total risk attitude index  $\alpha_T$ .

##### **Stage 5: Aggregation of multi-expert judgments**

- (10) Find the weighting value  $b(F_i)$  of aggregation for each alternative  $A_i$ .
- (11) Determine the aggregated estimate  $P_T$  using multi-expert judgment's estimates  $P(A_i)$  and aggregation weights  $b(F_i)$ .

#### Fuzzy Preference Index

Using fuzzy multiplication operator  $\otimes$  and fuzzy addition operator  $\oplus$ , the average fuzzy preference ratings  $S_{it}$  of alternatives  $A_i$  under evaluation criteria  $C_{ij}$  and the average fuzzy importance weights  $W_t$  of evaluation criteria  $C_t$  can be expressed, respectively, by

$$S_{it} = \frac{1}{n} \otimes (S_{it1} \oplus S_{it2} \oplus \dots \oplus S_{itj} \oplus \dots \oplus S_{itm}), \quad (2-3a)$$

$$W_t = \frac{1}{n} \otimes (W_{t1} \oplus W_{t2} \oplus \dots \oplus W_{tj} \oplus \dots \oplus W_{tm}). \quad (2-3b)$$

Here  $i (= 1, 2, \dots, m)$  denotes the alternative index;  $t (= 1, 2, \dots, k)$  the evaluation criterion index;  $j (= 1, 2, \dots, n)$  the decision-maker index;  $m$  the number of alternatives;  $k$  the number of criteria; and  $n$  the number of decision-makers.

Based on fuzzy weighted averaging, the average preference ratings  $S_{it}$  of alternatives  $A_i$  under evaluation criteria  $C_t$  and the average importance weights  $W_t$  of evaluation criteria  $C_t$  are aggregated to yield the fuzzy preference index  $F_i$  of alternatives  $A_i$  as follows:

$$F_i = \frac{1}{k} \otimes [(S_{i1} \otimes W_1) \oplus (S_{i2} \otimes W_2) \oplus \dots \oplus (S_{it} \otimes W_t) \oplus \dots \oplus (S_{ik} \otimes W_k)], \quad (2-4)$$

where  $i (= 1, 2, \dots, m)$  stands for the alternative index;  $t (= 1, 2, \dots, k)$  the evaluation criterion index;  $m$  the number of alternatives; and  $k$  the number of criteria.

Based on the fuzzy number arithmetic, the approximated fuzzy preference ratings  $S_{it}$  is

$$S_{it} \cong (o_{it}, p_{it}, q_{it}, r_{it}), \quad (2-5a)$$

with

$$o_{it} = \sum_{j=1}^n \frac{o_{ij}}{n}, p_{it} = \sum_{j=1}^n \frac{p_{ij}}{n}, q_{it} = \sum_{j=1}^n \frac{q_{ij}}{n}, r_{it} = \sum_{j=1}^n \frac{r_{ij}}{n}. \quad (2-5b)$$

In the same manner,

$$W_t \cong (a_t, b_t, c_t, d_t), \quad (2-6a)$$

with

$$a_t = \sum_{j=1}^n \frac{a_{tj}}{n}, b_t = \sum_{j=1}^n \frac{b_{tj}}{n}, c_t = \sum_{j=1}^n \frac{c_{tj}}{n}, d_t = \sum_{j=1}^n \frac{d_{tj}}{n}. \quad (2-6b)$$

The fuzzy preference index  $F_i$  is approximated as

$$F_i \cong (V_i, X_i, Y_i, Z_i), \quad (2-7a)$$

with

$$V_i = \sum_{t=1}^k \frac{o_{it} a_t}{k}, X_i = \sum_{t=1}^k \frac{p_{it} b_t}{k}, Y_i = \sum_{t=1}^k \frac{q_{it} c_t}{k}, Z_i = \sum_{t=1}^k \frac{r_{it} d_t}{k}. \quad (2-7b)$$

Here  $i = 1, 2, \dots, m$  is the alternative index and  $t = 1, 2, \dots, k$  the criterion index.

### Overall Utility Value

Based on the R-L type integral values, as described in Ref. [Liou and Wang, 1992], the total integral value for a trapezoidal fuzzy number  $N = (a, b, c, d)$  is assessed, following the Zadeh's convex combination [Zadeh, 1965], by

$$U_T(N) = \alpha_T I_R(N) + (1-\alpha_T) I_L(N), \quad (2-8a)$$

with

$$I_R(N) = 0.5 [c + d], I_L(N) = 0.5 [a + b]. \quad (2-8b)$$

### Treatment of DMs' Attitude towards Vagueness and Risk

In the present work, for a trapezoidal fuzzy number  $N = (a, b, c, d)$ , the number  $\alpha = (b-a)/[(d-c)+(b-a)]$  can be referred to as the individual risk attitude index. It is based on the evaluation data assigned at the data input stage of the evaluation procedure, instead of given at the data output stage. The DM's attitude toward risk can be taken into account by means of the index  $\alpha \in [0, 1]$ . For a trapezoidal fuzzy number  $N = (a, b, b, d)$ , the index is degenerated to the index  $\alpha = (b-a)/(d-a)$  defined in Ref. [Chang and Chen, 1994] for the triangular fuzzy

number  $N = (a, b, d)$ .

For a multiple decision-makers problem, using the trapezoidal fuzzy numbers such as  $S_{ij} = (o_{ij}, p_{ij}, q_{ij}, r_{ij})$  and  $W_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ , the total risk attitude index  $\alpha_T$  can be obtained by

$$\mathbf{a}_T = \frac{\mathbf{a}_W + \mathbf{a}_S}{k \times n + m \times k \times n}. \quad (2-8d)$$

Here the individual risk attitude index  $\alpha_W$  for the fuzzy importance weight  $W_{ij}$  and the individual risk attitude index  $\alpha_S$  for the fuzzy preference rating  $S_{ij}$  are defined as, respectively,

$$\mathbf{a}_W = \sum_{i=1}^k \sum_{j=1}^n \frac{b_{ij} - a_{ij}}{(d_{ij} - c_{ij}) + (b_{ij} - a_{ij})}, \quad (2-8e)$$

$$\mathbf{a}_S = \sum_{i=1}^m \sum_{t=1}^k \sum_{j=1}^n \frac{p_{ij} - o_{ij}}{(r_{ij} - q_{ij}) + (p_{ij} - o_{ij})}. \quad (2-8f)$$

Similarly, for a single or an individual decision-maker problem, using the trapezoidal fuzzy numbers such as  $S_{it} = (o_{it}, p_{it}, q_{it}, r_{it})$  and  $W_t = (a_t, b_t, c_t, d_t)$ , the total risk attitude index  $\alpha_T$  is reduced to

$$\mathbf{a}_T = \left[ \sum_{t=1}^k \frac{(b_t - a_t)}{[(d_t - c_t) + (b_t - a_t)]} + \sum_{i=1}^m \sum_{t=1}^k \frac{(p_{it} - o_{it})}{[(r_{it} - q_{it}) + (p_{it} - o_{it})]} \right] / (k + m \times k). \quad (2-8g)$$

### Weighting Values

The normalized utility value is called the weighting value  $\mathbf{b}(F_i)$  and formed as follows:

$$\mathbf{b}(F_i) = \frac{U_T(F_i)}{\sum_{i=1}^m U_T(F_i)}. \quad (2-9a)$$

For the alternatives  $A_i, i = 1, 2, \dots, m$ , the aggregated estimate  $P_T$  is determined using  $\mathbf{b}(F_i)$  as

$$P_T = \frac{\sum_{i=1}^m \mathbf{b}(F_i) P(A_i)}{\sum_{i=1}^m \mathbf{b}(F_i)}. \quad (2-9b)$$

Here,  $P(A_i)$  = estimate based on expert  $A_i$ 's judgment regarding the model output parameter  $P$ .

### **3. Application to Aggregation of Multi-Expert Judgments**

The proposed algorithm is applied to a case problem available in the literature. The problem was studied using the AHP-based aggregation model [Zio, 1996]. The case study is related to aggregation of three-expert judgment on the containment pressure increment due to the breach of reactor pressure vessel that can occur in the context of a hypothetical severe accident in the Sequoyah nuclear power plant.

#### Problem Statement: Case study

Three alternatives are defined as three-expert judgments such that the set  $A = \{A_1, A_2, A_3\}$ , where  $A_1$  = Expert A's judgment;  $A_2$  = Expert B's judgment; and  $A_3$  = Expert C's judgment. Seven evaluation criteria are defined as the set  $C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7\}$ . Here  $C_1$  = PK = personal knowledge;  $C_2$  = SI = sources of information;  $C_3$  = UNB = degree of unbiasedness;  $C_4$  = IND = level of independence from the other experts;  $C_5$  = PI = personal interest in the study;  $C_6$  = PE = past experience; and  $C_7$  = PM = performance measures. The overall objective in the evaluation problem is to aggregate multi-expert opinions under evaluation criteria identified

and to obtain the aggregated estimates.

#### Fuzzy Aggregated Evaluation

In this case problem, the linguistic scales for the importance weight and the preference rating are corresponding to the five-member linguistic scales used in Ref. [Chang and Chen, 1994], respectively,

$$W = \{VL, L, M, H, VH\}, S = \{VP, P, F, G, VG\}, \quad (3-1)$$

where VL = very low, L = low, M = medium, H = high, and VH = very high, VP = very poor, P = poor, F = fair, G = good, and VG = very good.

In the present work, the assigned linguistic matrices available in the literature [Moon and Kang, 1998] are cited, as shown in Table 3-1, in order to exclude the possible reflection of author's subjective opinion in the assignment process of linguistic values. The corresponding trapezoidal fuzzy numbers are obtained from the scale conversion table shown in Tables 2-1.

For each alternative  $A_i$  ( $i = 1, 2, 3$ ), the fuzzy preference index obtained is given in Table 3-2. The overall utility values calculated using the present algorithm are shown in Table 3-3. The weighting values for three alternatives are given in Table 3-4. Finally, the aggregated percentile estimates, using the present model, are shown in Table 3-5.

Using non-fuzzy approaches such as the arithmetic mean method and the AHP method [Zio, 1996], the aggregated percentile estimates revised with Eq. (2-9b) are given in Table 3-6.

#### **4. Conclusive Remarks**

- (1) The overall utility value and the weighting value obtained using the present model are the same as that using the Liou/Wang model, unless the total risk attitude index used in each model is different;
- (2) The overall utility value and the weighting value obtained using the Chang/Chen model are the same as that using the Kim/Park model, unless the total risk attitude index used in each model is different.

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Table 2-1 A five-member scale conversion table [Chang/Chen 1994]

Decision variable	Conversion of linguistic scales to numerical scale				
Importance weight (W)	Very low (VL)	Low (L)	Medium (M)	High (H)	Very high (VH)
Preference rating (S)	Very poor (VP)	Poor (P)	Fair (F)	Good (G)	Very good (VG)
W or S	(0, 0, 0.25)	(0, 0.25, 0.5)	(0.25, 0.5, 0.75)	(0.5, 0.75, 1)	(0.75, 1, 1)

Table 3-1 Linguistic values for linguistic variables [Moon and Kang, 1998]

Criterion		$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
Importance weight		VH	H	H	L	M	L	M
Preference rating	Alternative $A_1$	G	F	G	VG	G	G	G
	Alternative $A_2$	F	P	F	F	F	G	F
	Alternative $A_3$	VG	G	G	G	F	G	G

Table 3-2 Fuzzy preference index of each alternative

Fuzzy number		Preference index
Alternative	$A_1$	(0.1429, 0.4107, 0.4107, 0.7500)
	$A_2$	(0.0625, 0.2679, 0.2679, 0.5714)
	$A_3$	(0.1786, 0.4464, 0.4464, 0.7589)

Table 3-3 Overall utility values of each alternative for various fuzzy models

Model	Risk attitude index	Overall utility values		
		$A_1$	$A_2$	$A_3$
Kim/Park model [Kim and Park 1990]	$\alpha = 1$	0.6638	0.5089	0.6903
	$\alpha = 0.5536$	0.5287	0.3834	0.5599
	$\alpha = 0.5$	0.5125	0.3683	0.5442
	$\alpha = 0$	0.3611	0.2277	0.3982
Liou/Wang model [Liou and Wang 1992]	$\alpha = 1$	0.5804	0.4196	0.6027
	$\alpha = 0.5536$	0.4448	0.3060	0.4731
	$\alpha = 0.5$	0.4286	0.2924	0.4576
	$\alpha = 0$	0.2768	0.1652	0.3125
Chang/Chen model [Chang and Chen 1994]	$\alpha = 0.5536$	0.5287	0.3834	0.5599
Present model	$\alpha = 0.5536$	0.4448	0.3060	0.4731

Table 3-4 Aggregation weighting values of each alternative for various fuzzy models

Model	Utility value	Risk attitude index		Aggregation weights		
				$A_1$	$A_2$	$A_3$
Kim/Park model [Kim and Park 1990]	R-L type m.f. grade	Given by DM	$\alpha = 1$	0.3563	0.2732	0.3705
			$\alpha = 0.5536$	0.3592	0.2605	0.3803
			$\alpha = 0.5$	0.3596	0.2585	0.3819
			$\alpha = 0$	0.3659	0.2307	0.4034
Liou/Wang model [Liou and Wang 1992]	R-L type integral value	Given by DM	$\alpha = 1$	0.3621	0.2618	0.3761
			$\alpha = 0.5536$	0.3634	0.2500	0.3866
			$\alpha = 0.5$	0.3636	0.2481	0.3883
			$\alpha = 0$	0.3669	0.2189	0.4142
Chang/Chen model [Chang and Chen 1994]	R-L type m.f. value	Calculated	$\alpha = 0.5536$	0.3592	0.2605	0.3803
Present model	R-L type integral value	Calculated	$\alpha = 0.5536$	0.3634	0.2500	0.3866

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Table 3-5 Aggregated percentile estimates using various fuzzy aggregation methods

			Percentile pressure rise (bar)					
			5 %	25 %	50 %	75 %	95 %	
Alternative	$A_1$		3.60	4.00	4.50	5.46	6.22	
	$A_2$		1.6	-	4.0	-	10.8	
	$A_3$		3.0	4.0	5.0	6.5	8.0	
Fuzzy aggregation approach	Kim/Park model [Kim and Park, 1990]	$\alpha = 1$	2.8313	4	4.5487	5.9902	8.1307	
		$\alpha = 0.5536$	2.8508	4	4.5599	5.9949	8.09	
		$\alpha = 0.5$	2.8539	4	4.5617	5.9956	8.0836	
		$\alpha = 0$	2.8965	4	4.5863	6.0054	7.9948	
	Liou/Wang model [Liou and Wang, 1992]	$\alpha = 1$	2.8507	4	4.5571	5.9898	8.0886	
		$\alpha = 0.5536$	2.8680	4	4.5683	5.9960	8.0532	
		$\alpha = 0.5$	2.8708	4	4.5701	5.9970	8.0474	
		$\alpha = 0$	2.9136	4	4.5976	6.0115	7.96	
	Chang/Chen model [Chang and Chen 1994]			2.8508	4	4.5599	5.9949	8.09
	Present model			<b>2.8680</b>	<b>4</b>	<b>4.5683</b>	<b>5.9960</b>	<b>8.0532</b>

Table 3-6 Aggregated percentile estimates using non-fuzzy aggregation methods

			Percentile pressure rise (bar)				
			5 %	25 %	50 %	75 %	95 %
Alternative	$A_1$		3.60	4.00	4.50	5.46	6.22
	$A_2$		1.6	-	4.0	-	10.8
	$A_3$		3.0	4.0	5.0	6.5	8.0
Non-fuzzy aggregation approach	Arithmetic mean [Zio, 1996]		2.73	4.00	4.5	5.98	8.34
	AHP [Zio, 1996]		2.868	4.00	4.56	5.984	8.034
	AHP [Zio, 1996] revised		2.870	4.00	4.567	5.992	8.049