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Correction of the Control Rod Cusping Effect Using One-Dimensional Fine Mesh Flux Profiles

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Abstract

The control rod cusping effect occurs when the heterogeneity within a partially rodded node (PRN) is not properly incorporated into the nodal calculation involving large, homogeneous nodes. In the present paper a new rod cusping correction method is proposed. This method uses fine mesh flux solutions obtained from two one-dimensional, three-node problems for each PRN. The heterogeneity within the PRN is explicitly kept and the flux-weighting factor is calculated from the resulting fine-mesh flux profile. The axial discontinuity factors are then generated with the homogenized cross section in the PRN for the subsequent nodal calculations. The result of this method corresponds with the reference result and the computation time spent in the rod cusping correction is less than 2% of the total neutronic calculation time.

I. Introduction

The control rod cusping effect occurs when the heterogeneity within a partially rodded node (PRN) is not properly incorporated into the nodal calculation involving large, homogeneous nodes. In the transient calculation for a rod withdrawal event, unphysical variation of reactivity due to rod cusping leads to large errors in the predicted core power. Rod cusping may also cause intolerable errors even in some eigenvalue calculations as shown in Figure 1. One such example is a rodded depletion calculation for a 330MWt soluble boron free PWR, SMART whose development is currently underway by Korea Atomic Energy Research Institute.

In order to resolve the rod cusping problem, it is necessary, first of all, to determine the axial flux profile within a PRN. The flux profile can be used to calculate flux-volume weighted homogenized cross sections or axial discontinuity factors. In the previously known rod cusping correction methods,[1-2] approximations were introduced in the determination of the axial flux profile. Interface displacement[1], quadratic flux representation and forward flux adjoint flux bilinear weighting method[2] are the examples of such approximations. Although the consequence of these approximations may not be significant in most practical calculations, they could cause unbearable errors in heavily rodded cases and transient calculations.

In the present paper a new rod cusping correction method is proposed that uses fine mesh flux solutions obtained from two one-dimensional, three-node problems for each PRN. The three-node problem consists of the PRN itself and its upper and lower nodes. In the first three-node problem, the heterogeneity within the PRN is explicitly kept and the flux-weighting factor is calculated from the resulting fine-mesh flux profile. The second problem is then solved with the homogenized cross section in the PRN to determine the axial discontinuity factor to be used in the subsequent nodal calculations.

II. Method

Considering that the axial effect is dominant over the radial one in a rod cusping problem, one needs to solve just a one-dimensional problem, which is obtainable by the transverse integration of the three-dimensional diffusion equation, to resolve the rod cusping problem. As well known, the one-dimensional problem is given as:

$$-D_g \frac{d^2 f_g(z)}{dz^2} + \Sigma_{rg} f_g(z) - Q_g(z) = S(z) - L_g^{xy}(z) \qquad (1)$$

where $Q_g(z)$ is the fission or scattering source term depending on the group index, $S(z)$ is the external source term. As the usual practice in the transverse-integrated nodal method, it is sufficient to represent the transverse leakage source as a quadratic polynomial. The quadratic polynomial can be determined at each node using the node average values of transverse leakage available from the previous iteration of the nodal calculation. With the transverse leakage polynomial on the right hand side, Eq. (1) becomes a one-dimensional fixed source problem. In the present method, the one-dimensional problem is solved for a three-node geometry having the PRN in the middle. If the heterogeneity is explicitly represented within the PRN, there are four distinctive regions in the three-node problem.

Since Eq. (1) is a second order ordinary differential equation, it can be solved only when two boundary conditions or constraints are specified. As the two constraints, one can take the node average fluxes of the top and bottom nodes determined in the previous nodal iteration, i.e.:

$$\frac{1}{\Delta z_a}\int_{\Delta z_a} f_{ga}^{n}(z)dz = \overline{f}_{ga}^{n-1} \quad , \quad a \in \{b,t\} \tag{2}$$

where n is the iteration index.

With the constraint of Eq. (2), Eq. (1) can be solved, in principle, analytically given the cross sections assigned to each of the four regions. However, the three-node problem is solved here numerically using the fine mesh finite difference scheme in order to avoid the complexity of the analytic solution. In the discretization, the mesh sizes are determined for each region with the same number of meshes per node. Since this is a one-dimensional problem, the time for numerical solution is trivial.

Once the fine mesh solution is determined first for the heterogeneous three-node problem, the flux weighting factor can be readily obtained as the ratio of the average flux of the rodded region in the PRN to that of the entire PRN. The flux weighting factor *(w_g)* is then used in the following way to obtain the homogenized cross section ($\tilde{\Sigma}_g$) for the next nodal calculation as well as for the homogeneous three-node problem:

$$\tilde{\Sigma}_g = \Sigma_g + w_g x \Delta\Sigma_g^r \tag{3}$$

where ξ is the volume fraction of the rodded portion and $\Delta\Sigma_g^r$ is the control rod cross section.

Figure 2 illustrates the fine mesh flux solutions obtained for the two three-node problems. As compared to the heterogeneous flux, the homogeneous flux has quite different slopes as well as values at the boundaries of the PRN. In the nodal calculation in which the heterogeneity information is lost, therefore, the use of homogenized cross section might lead to significant errors in the interface current. This problem can be alleviated if the discontinuity factor obtained as the ratio of the fine-mesh heterogeneous flux to the fine-mesh homogeneous flux is used in the nodal calculation.

III. Results

In order to demonstrate the effectiveness of the rod cusping correction method presented above, a simplified rod-withdrawal problem consisting of a checkerboard array at hot zero power was solved first using the PARCS code[3]. The checkerboard problem was created from the NEA/NSC bank withdrawal benchmark problem[4] by taking only the center portion of the core. The model core consists of 2x2x18 nodes with the reflective boundary condition at the radial boundaries. The control rod is inserted into the center assembly such that a node located near the mid-plane is partially rodded by 50%. The reference solution was obtained from a model that had a ten times more refined axial mesh structure. The boron concentration was adjusted such that the core was critical by the reference model. In the rod cusping correction calculation, ten fine meshes per node were used. As shown in Table 1 when no rod cusping correction was introduced, the k_{eff} was calculated as 0.99888 so that the error in k_{eff} was –112 pcm. The error is quite large in this problem because the control rod had a serious impact in this checkerboard problem. The rod cusping correction for this problem was examined in two steps. It should be noted that no correction means the simple volume weighting. The first step used only the flux-weighting factor and the second step used both the axial discontinuity factor and the flux-weighting factor. At the first step the error was reduced to –26 pcm and it was further reduced to –5 pcm with the discontinuity factor.

The transient rod withdrawal calculation was examined in the similar way. Figure 3 shows the power vs. time curves, up to 30 seconds into the transient, for the various cases. Cusp-shaped variations are observed twice in the figure as the rod pass through the plane boundaries even in the logarithmic scale (marked with the + sign) and the error is very severe. It is clear from the figure that the present correction method improves the error significantly.

The second problem was solved for the steady state of the NEACRP PWR rod ejection benchmark problem case A2[5] which is a heavily rodded hot full power case. The reference for this case was obtained by dividing the plane containing the partially rodded nodes into two planes such that there were no partially rodded nodes in the reference model. In this case the error in eigenvalue was reduced from –53 to –11 pcm.

In both test cases, the computation time spent in the rod cusping correction was less than 2% of the total neutronic computation time. Considering the total computation time that includes the times for T/H calculation and the cross section feedback, the overhead associated with cusping correction is insignificant. The rod cusping correction method presented in this paper reduces the errors remarkably and is efficient.

IV. Acknowledgement

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Table 1. Comparison of Core Eigenvalue(K-eff) with Various Cusping Correction Methods for the Modified NEA/NSC Bank Withdrawal Benchmark Problem

Correction Method	Keff	Error(pcm)*
Reference	1.00000	-
VW	0.99888	-112
FVW only	0.99974	-26
FVW+Discontinuity Factor	0.99995	-5

* : Error = (Reference – Correction Method)

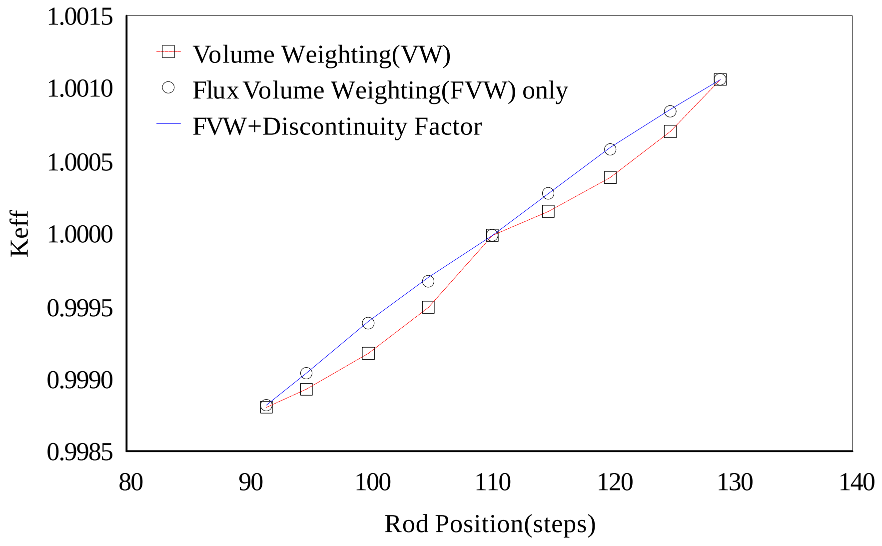

Figure 1. K-eff vs. Control Rod Insertion Depth

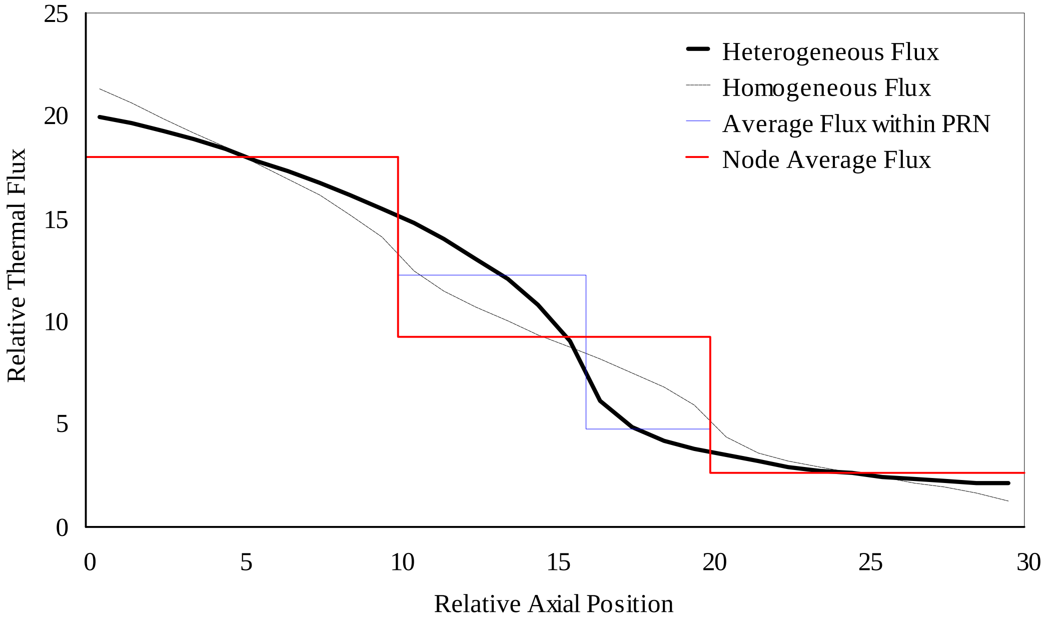

Figure 2. Axial Flux Profiles Around a Partially Rodded Node

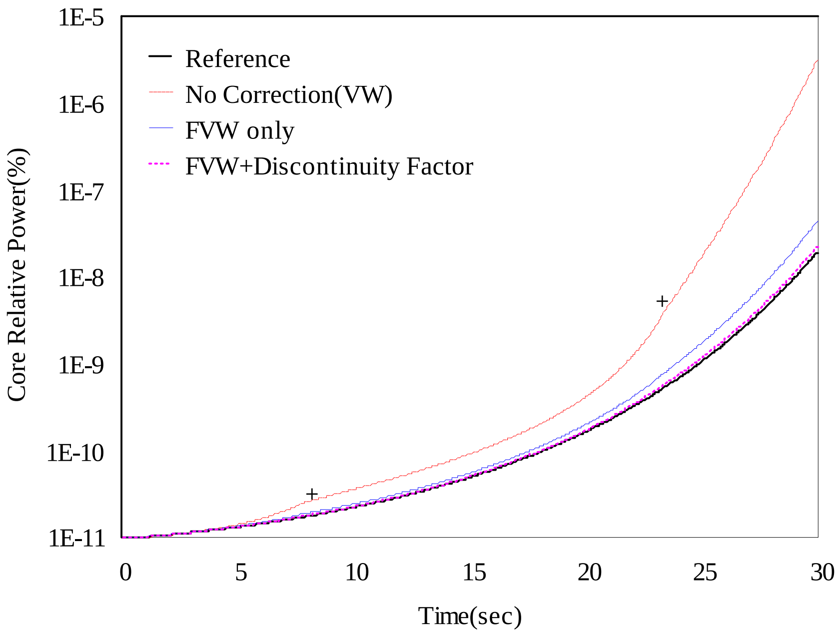

Figure 3. Comparison of the Transient Core Power Variations Obtained with Various Cusping Correction Options for the Checkerboard Rod Withdrawal Problem