p-CMFD Acceleration and Nonoverlapping Local/Global Iterative Methods with 2-D/1-D Fusion Kernel for Whole-Core Transport Calculation

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1. Introduction

Conventional reactor core analysis, based on isolated-single assembly lattice calculation and diffusion nodal core calculation, has the limitation due to the absence of treatment of accurate inter-assembly transport effect. To overcome the limitation of conventional method, we consider two approaches: One is whole-core fine-group deterministic transport calculation accelerated by partial current-based coarse-mesh finite difference (p-CMFD) method [1], and the other is nonoverlapping local/global (NLG) iteration [2]. In this paper, we apply both approaches to 3-D heterogeneous reactor problems. To deal with 3-D transport problems, 2-D/1-D fusion method [3-5] is used as solution kernel in both approaches.

2. Theory and Method

In this section, three methods are briefly described; p-CMFD, NLG, and 2-D/1-D fusion method.

2.1 p-CMFD acceleration

There are various acceleration schemes that are developed to solve the neutron transport equation. Coarse-mesh finite difference (CMFD) method is a popular acceleration method but it has divergence problem for optically thick coarse-mesh size. p-CMFD method is a modification of CMFD. A key feature of p-CMFD is the use of partial currents instead of net currents in CMFD. A detail scheme of p-CMFD is described in [1]. The theoretic and numerical results show that p-CMFD is a more stable acceleration method than CMFD.

2.2 Nonoverlapping local/global iteration

In NLG [2], whole-core domain is decomposed into nonoverlapping local problems which are governed by fine-group neutron transport equation (fixed-k problem formulation), and the local problems are wrapped around by global p-CMFD equation. After the global calculation, we update incoming angular flux boundary condition on each local problem for next local/global iteration:

$$\psi_{\pm n,g}^{new,local}(\vec{r}) = \frac{\psi_{\pm n,g}^{local}(\vec{r})}{\bar{J}_G^{\pm,local}} \bar{J}_G^{\pm,global}. \qquad (1)$$

In Eq. (1), NLG uses local angular flux and local partial current which come from the outgoing angular flux and the outgoing partial current in neighboring local domain. Since p-CMFD provides global partial currents, p-CMFD is naturally appropriate for update of angular flux at local problem boundary.

NLG has the following advantages for parallel computing: 1) Local problems are calculated independently. 2) Only boundary angular flux and homogenized p-CMFD parameters are needed for data communication. 3) Global calculation occupies small portion in NLG calculation.

2.3 2-D/1-D fusion method

2-D/1-D fusion method [3-5] is developed to deal with heterogeneous 3-D reactor problems. In this method, "consistent directional decomposition and integration" transforms a 3-D transport problem to 2-D and 1-D problems. A realistic reactor is radially heterogeneous so we use MOC method which can treat unstructured meshes. In axial direction, a typical reactor of current generation is usually piecewise homogeneous so we use S_N-like method. Both systems of 2-D and 1-D problems are coupled by axial and radial leakage source terms. Because 2-D and 1-D problems are solved by transport methods, the result is 3-D transport solution. Modular cell homogenization can be performed to reduce the storage. But more accurate solution is obtained if we do not use the modular cell homogenization. A detail scheme of 2-D/1-D fusion method is described in [5].

3. Numerical Results

To test p-CMFD acceleration and NLG iteration for 3-D reactor problems, OECD/NEA C5G7 benchmark problems [6] are considered.

3.1 Description of problems

In [6], there are three cases; unrodded, rodded A, rodded B, which are described in Fig. 1 and 2. These are seven energy group problems. Axially, fuel region is 42.84 cm, and reflector region is 21.42 cm. Control rods are inserted to the reflector region which is above the fuel region for unrodded case. In the other cases, control rods are inserted deeper than unrodded case. More details of problem description are available in [6]. For

computation, relative error criteria are 10^{-6} for multiplication factor and 10^{-4} for fission source. To compare the computing performance between whole-core p-CMFD acceleration and NLG iteration, we use 9 computing cores (CPU: Intel Xeon X5670 @ 2.93GHz×2) for parallel computation. The reference result is MCNP calculation which is presented in [6].

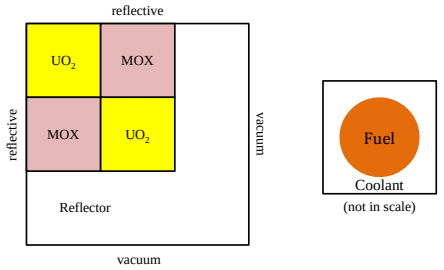

Fig. 1. Core and pin configurations of test problems (xy-plane) [6]

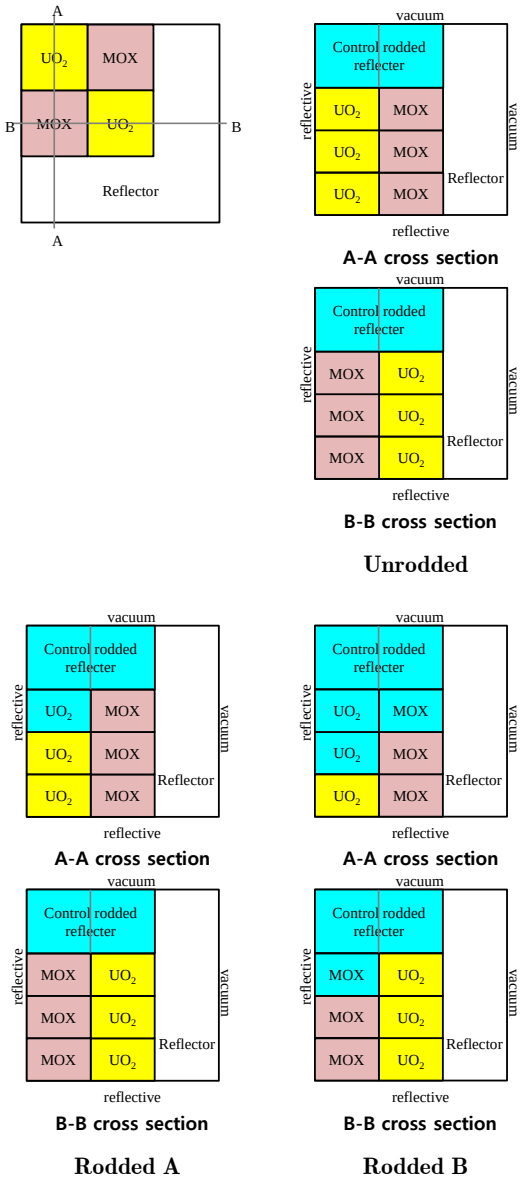

Fig. 2. Cut-away views of test problems [6]

3.2 Numerical results and discussion

First, we compare the NLG iteration to reference solution. Table I shows the multiplication factors from numerical calculation.

The multiplication factor of NLG iteration converges to the solution of whole-core 2-D/1-D fusion method whether modular cell homogenization is performed or not. We verify that the other values (maximum pin power, assembly pin power, etc.) of NLG iteration also converge to the results of whole-core 2-D/1-D fusion method. That means NLG iteration also converges to the 2-D/1-D fusion method. The 2-D/1-D fusion method gives accurate multiplication factor in all three problems. Especially, without modular cell homogenization, multiplication factor is best among the above methods.

Now we compare the computing performance of NLG and whole-core p-CMFD acceleration. Table II is the computing performance results for unrodded case. Table II shows that modular cell homogenization reduces the computing burden that is the expected result. We found that p-CMFD acceleration gives better computing time than NLG iteration. But computing time per iteration is similar for both methods. In early iterations, local problems of NLG are weakly coupled compared to whole-core p-CMFD acceleration. This characteristic makes the difference in the number of iterations. We expect that these differences disappear if we give appropriate initial guess (from, for example, crude diffusion calculation).

4. Conclusions

To overcome the limitation of conventional method, p-CMFD acceleration and NLG iteration are proposed. To deal with 3-D transport problem, 2-D/1-D fusion method is used as the solution kernel. These schemes are tested on 3-D OECD/NEA benchmark problems. NLG with the 2-D/1-D fusion method gives same solutions with those of the 2-D/1-D fusion method with whole-core p-CMFD acceleration. Compared to the whole-core p-CMFD acceleration using the same computing conditions, NLG requires more computational load. We believe that the rough initial guess makes this difference. So if an appropriate initial guess is used, computational burden will be reduced. We expect that NLG is more appropriate than whole-core p-CMFD acceleration on a highly parallel computing system. In a tentative conclusion, NLG with 2-D/1-D fusion method will be a useful computational framework for efficient and accurate reactor core design analysis.

References

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Table I. Multiplication factors of test problems

Method	Unrodded (relative error)	Rodded A (relative error)	Rodded B (relative error)
Reference MCNP	1.14308 (±6 pcm*)	1.12806 (±6 pcm*)	1.07777 (±6 pcm*)
2D/1D+Hom+ACC	1.14327 (16.47 pcm)	1.12875 (61.61 pcm)	1.07871 (87.25 pcm)
2D/1D+Hom+NLG	1.14327 (16.47 pcm)	1.12875 (61.61 pcm)	1.07871 (87.25 pcm)
2D/1D+Het+ACC	1.14301 (-6.42 pcm)	1.12819 (11.27 pcm)	1.07785 (7.76 pcm)
2D/1D+Het+NLG	1.14301 (-6.42 pcm)	1.12819 (11.27 pcm)	1.07785 (7.76 pcm)
3D MOC**	1.14165 (-125.10 pcm)	1.12638 (-148.93 pcm)	1.07530 (-229.18 pcm)

2D/1D: 2-D/1-D fusion method, Hom: modular cell homogenization, Het: no modular cell homogenization,
ACC: p-CMFD acceleration, NLG: NLG iteration
* 98% confidence interval of the reference MCNP solution [6], ** 3D MOC results from [7]

Table II. Computing performance results for unrodded case

Method	# of outer or local/global iterations	Computing time* (sec)	Computing time per iteration (sec)
2D/1D+Hom	76	33627.8	442.47
2D/1D+Hom+ACC	20	9820.9	491.05
2D/1D+Hom+NLG	30	14062.6	468.75
2D/1D+Het	76	38527.7	506.94
2D/1D+Het+ACC	20	11401.8	570.09
2D/1D+Het+NLG	30	17186.1	572.87

2D/1D: 2-D/1-D fusion method, Hom: modular cell homogenization, Het: no modular cell homogenization,
ACC: p-CMFD acceleration, NLG: NLG iteration
* Computer CPU: Intel Xeon X5670 @ 2.93GHz×2