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Asymptotic Method for Cladding Stress Evaluation in PCMI

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1. Introduction

PCMI (Pellet Cladding Mechanical Interaction) has been referred to as one of the widely spread fuel rod failures. A PCMI failure is a fracture of a cladding tube where, in many cases, a sharp edge of a cracked pellet contacts with the tube as shown typically in Fig. 1. The mechanical contact is caused by pellet swelling due to thermal expansion as well as fission gas release during normal and/or accident reactor conditions. Stress corrosion cracking (SCC) is generally known as the root cause of PCMI failure: high stress concentration under an iodine environment.

A PCMI failure was first reported in the GETR (General Electric Test Reactor) at Vacellitos in 1963 [1], and such failures are still occurring. Since the high stress values in the cladding tube has been of a crucial concern in PCMI studies, there have been many researches on the stress analysis of a cladding tube pressed by a pellet. Typical works can be found in some references [2-5]. It has often been assumed, however, that the cracks in the pellet were equally spaced and the pellet was a rigid body. In addition, the friction coefficient was arbitrarily chosen so that a slipping between the pellets and cladding tube could not be logically defined. Moreover, the stress intensification due to the sharp edge of a pellet fragment has never been realistically considered.

These problems above drove us to launch a framework of a PCMI study particularly on stress analysis technology to improve the present analysis method incorporating the actual PCMI conditions such as the stress intensification, arbitrary distribution of the pellet cracks, material properties (esp. pellet) and slipping behavior of the pellet/cladding interface. As a first step of this work, this paper introduces an asymptotic method that was originally developed for a stress analysis in the vicinity of a sharp notch of a homogeneous body [6]. The generalized stress intensity factor, describing the intensification of the stresses at the sharp contact edge of the pellet fragment, is also explained. It will be expanded to obtain a more accurate stress field in the vicinity of the pellet fragment edge.

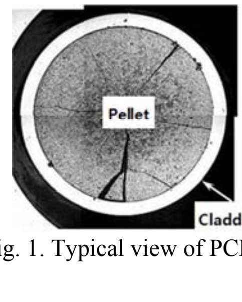


Fig. 1. Typical view of PCMI failure.

2. Asymptotic method

2.1 Formulation

Williams exploited the Airy stress potential, from which the intensified stress field at a sharp notch can be derived, as follows [6].

$$\Phi = r^{\lambda+1}\{a\cos(\lambda+1)\theta + b\sin(\lambda+1)\theta + c\cos(\lambda-1)\theta + d\sin(\lambda-1)\theta\} \tag{1}$$

where, Φ is the Airy stress potential, (r,θ) is the polar coordinates with the origin at the notch tip, λ is a characteristic value that determines the stress singularity, and *a*-*d* are the unknown constants to be determined corresponding to the boundary conditions of the problem.

The stress and displacement components are to be derived from the Airy stress potential using the well-known formulae [7], and thus they are not reproduced here. However, it should be noted that the stresses always have a factor of $r^{\lambda-1}$ so it will be singular if $\lambda < 1$ and as $r \to 0$. Now, if we focus on the very small region around the contact edge of the pellet fragment and cladding, the geometrical configuration can be illustrated as shown in Fig. 2. Note that the cladding tube and pellet fragment are designated as 'Body 1' and 'Body 2', respectively. The boundary conditions are written as follows, if we assume that the contact interface is adhered.

$$\sigma^1_{\theta\theta}(r,-\pi) = 0,\ \sigma^1_{r\theta}(r,-\pi) = 0, \tag{2a}$$

$$\sigma^2_{\theta\theta}(r,\varphi) = 0,\ \sigma^2_{r\theta}(r,\varphi) = 0, \tag{2b}$$

$$\sigma^1_{\theta\theta}(r,0) = \sigma^2_{\theta\theta}(r,0),\ \sigma^1_{r\theta}(r,0) = \sigma^2_{r\theta}(r,0), \tag{2c}$$

$$u^1_r(r,0) = u^2_r(r,0),\ u^1_\theta(r,0) = u^2_\theta(r,0) \tag{2d}$$

where, σ_{ij}^m and u_i^m are the *ij*-component of the stress, and the *i*-component of the displacement of body *m*, respectively.

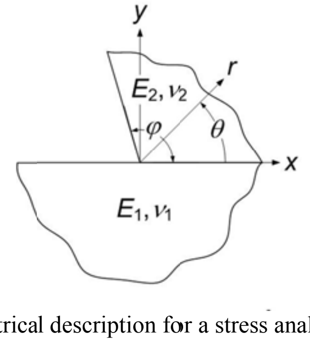


Fig. 2. Geometrical description for a stress analysis in the case of PCMI.

2.2 Eigensolution

Eq. (2) constitutes a simultaneous equation consisting of eight homogenous equations. Algebraically, the solution of a_1-d_1 and a_2-d_2 (i.e., a total of eight unknown constants) will not be trivial only if the determinant of the coefficient matrix (8×8) is zero. This yields the values of λ, in other words, the eigenvalue. When it is substituted into the original simultaneous equation, the eigenvectors (i.e., stresses) are finally obtained.

For the present PCMI case, it may be assumed that the contact interface between the pellet and cladding tube is adhered when the diameter of pellet increases owing to swelling and simultaneously contacts with the cladding tube. In this case, two eigenvalues (designated as λ_I, λ_{II}) appear, and those are within the range of $0 < \lambda_I$, $\lambda_{II} < 1$. This implies that the stress becomes singular as $r \rightarrow 0$. When we define $\lambda_I < \lambda_{II}$, each stress component consists of a stronger singularity of the order, O(λ_I-1), and a weaker singularity of the order, O(λ_{II}-1).

The above statement is validated when the material properties of the pellet and cladding tube are used, which are shown in Table 1 (data at 400°C for the cladding tube, and at 500°C for the pellet). The eigenvalues are then evaluated as provided in Table 2, depending on the contact angle (φ in Fig. 2). It may be noted that the difference between the stronger and weaker singularity orders reduces as the contact angle increases. It will be zero when the angle reaches 180°, which is then the same as a crack problem, since $\lambda_I = \lambda_{II} = 0.5$ there.

Table 1. Material properties of the pellet and cladding tube used for the present analysis

Component	Material	E (MPa)	ν
Pellet	UO_2	185288	0.316
Cladding tube	Zry-4	72076	0.34

Table 2. Eigenvalues evaluated for some contact angles of the pellet fragment against the cladding tube

Contact angle (φ)	λ_I	λ_{II}
90°	0.509816	0.791709
120°	0.500006	0.64112
150°	0.515968	0.53953

As a result, the asymptotic expression of the stresses in the vicinity of the sharp contact edge is written as follows.

$$\sigma_{ij}(r,\theta) = K_I r^{\lambda_I - 1} f_{ij}^{I}(\theta) + K_{II} r^{\lambda_{II}-1} f_{ij}^{II}(\theta) \tag{3}$$

where K_I and K_{II} are the generalized stress intensity factors of mode I and mode II, respectively. f_{ij}^{k} ($i,j = r,\theta$; $k = I$, II) is the function of angle only (referred to as 'angle function' *ad hoc*), which occurs owing to the trigonometric functions in Eq. (1).

3. Stress evaluation

3.1 Generalized stress intensity factors

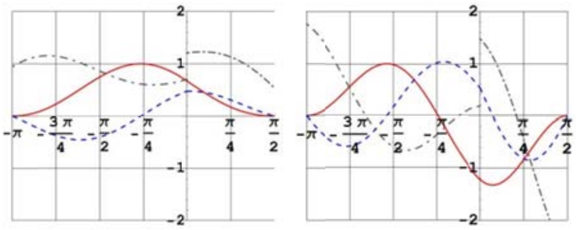
K_I and K_{II} in Eq. (3) take the role of a calibration factor that calibrates the characteristic solution to a finite and actual problem solution. They are defined as follows.

$$K_I = \lim_{r \to 0}\left[\sigma_{\theta\theta}(r,\theta_I)\cdot r^{1-\lambda_I}\right], \tag{4a}$$

$$K_{II} = \lim_{r \to 0}\left[\sigma_{r\theta}(r,\theta_{II})\cdot r^{1-\lambda_{II}}\right] \tag{4b}$$

where θ_I and θ_{II} are termed as 'mode separation angles', and are obtained from $f_{r\theta}^{I}(\theta_{II}) = 0$ and $f_{\theta\theta}^{II}(\theta_I) = 0$.

Fig. 3 shows the angle functions in the case of φ = 90°. The corresponding mode separation angles are found as θ_I = -43.74° and θ_{II} = -47.74°.



(a) $f_{ij}^{I}(\theta)$ (b) $f_{ij}^{II}(\theta)$

Fig. 3. Angle functions when φ = 90°; red: $f_{\theta\theta}(\theta)$, blue: $f_{r\theta}(\theta)$, black: $f_{rr}(\theta)$.

3.2 Finite element analysis

If an actual contact force is given in addition to the geometry and dimension, K_I and K_{II} can be readily calculated using a simple finite element method. In this paper, it is supposed that only the normal contact pressure is exerted to the cladding tube during the pellet swelling without any slipping on the contact interface.

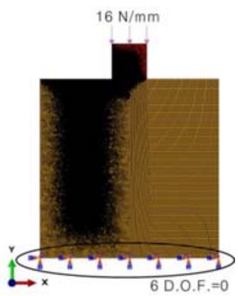


Fig. 4. Finite element model for evaluating K_I and K_{II} of the present example problem (case of φ = 90°).

Fig. 4 illustrates the present finite element model. A square block of 2a (regarded as a pellet fragment) is bonded to a rectangle of 10a×10a (regarded as a cladding tube). As for the contact force, a uniform pressure of 16 MPa is set to be applied (note that a plane strain condition is assumed). As a result, it is obtained that K_I = -18.386 and K_{II} = 6.489 by reflecting Eq. (4), and those behaviours are depicted in Fig. 5.

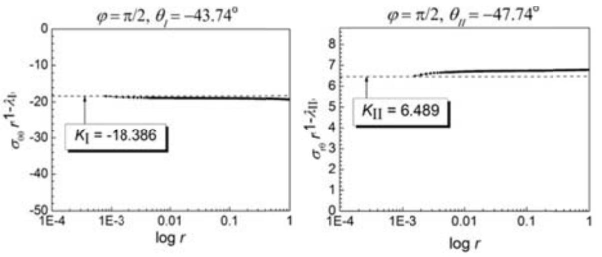


Fig. 5. K_I and K_{II} results from the finite element analysis.

As a result, if the pellet crack is perpendicular to the cladding inner surface (φ = 90°), the stress in the asymptotic field around the contact edge between the pellet fragment and cladding tube is expressed in MPa as follows.

$$\sigma_{ij}(r,\theta) = -18.386 r^{-0.490} f_{ij}^{I}(\theta) + 6.489 r^{-0.208} f_{ij}^{II}(\theta) \quad (5)$$

where the values of $f_{ij}^{k}(\theta)$ can be obtained from Fig. 3. For different contact angles, the procedure of stress calculation is exactly the same as written above.

4. Discussion

It is crucial to evaluate the stress values in the vicinity of the contact edge of the pellet fragment and cladding tube to analyze the cladding cracking failure in PCMI. This paper applies an asymptotic analysis to this interesting problem, which can describe the stress singularity at the sharp edge of a complete contact problem.

It was found that the method was successfully implemented when the pellet was assumed to be adhered to the cladding tube. However, this may cause some debates since a slipping may be supposed to occur on the interface during the pellet swelling. It may be tackled by consulting a recently developed theory [8], which states that a slipping takes place in the contact region, where $\sigma_{r\theta}/\sigma_{\theta\theta} \geq \mu$, in which μ is the coefficient of friction of the interface.

If this is applied to the present example case (section 3), $\sigma_{r\theta}/\sigma_{\theta\theta}$ = 0.704 at r = 0 [9]. This implies that if the coefficient of friction of the pellet/cladding tube interface is smaller than 0.704, some slipping will occur from the contact edge. The slipping region increases as μ decreases. However, it may be noted that an adhered contact problem should be analyzed *a priori*, even though such a slipping behaviour is attempted to be obtained.

In addition, the actual contact force between the pellet and cladding tube needs to be determined to obtain the actual stress values in the cladding tube. This is because the force should be given when the generalized stress intensity factors are evaluated. However, it has not been provided yet, to the authors' best knowledge, with the acceptable assumptions and relevant validation. It will be attempted within the present framework together with the above mentioned concern of slipping behaviour.

5. Conclusions

An asymptotic method is firstly applied to evaluate the accurate stresses in the cladding tube subject to PCMI. The intrinsic reason for applying this method is to simulate the stress singularity that is expected to take place at the sharp edge of a pellet fragment due to cracking during irradiation. As a first attempt of this work, an eigenvalue problem is formulated in the case of adhered contact, and the generalized stress intensity factors are defined and evaluated. Although some works obviously remain to be accomplished, for the present framework on the PCMI analysis (e.g., slipping behaviour, contact force etc.), it was addressed that the asymptotic method can produce the stress values that cause the cladding tube failure in PCMI more accurately than the previously published ones since none of them have ever simulated a stress singularity.

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