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An Inverse Function Least Square Fitting Approach of the Buildup Factor for Radiation Shielding Analysis

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1. Introduction

Buildup factor is widely used to obtain fast solution of the shielding analysis based on the point-kernel method. It is defined as the total flux including scattering and un-scattering events to un-scattered flux. Dose absorption and energy absorption buildup factors are widely used in the shielding analysis. The dose rate of the medium is main concern in the dose buildup factor, however energy absorption is an important parameter in the energy buildup factors.[1] The buildup factors starts from the Goldstein and Wilkins empirical data. From then lots of scientists suggested fitted approaches to obtain valid data and have tried to simulate the real application based on the advanced computational tools. ANSI/ANS-6.4.3-1991 standard data is widely used based on interpolation and extrapolation by means of an approximation method.[2] Recently, Yoshida's geometric progression (GP) formulae[3] are also popular and it is already implemented in QAD code[4]. In the QAD code, two buildup factors are notated as DOSE for standard air exposure response and ENG for the response of the energy absorbed in the material itself.

In this paper, a new least square fitting method is suggested to obtain a reliable buildup factors proposed since 1991. Total 4 datasets of air exposure buildup factors are used for evaluation including ANSI/ANS-6.4.3-1991, Taylor, Berger, and GP data.[5][6] The standard deviation of the fitted data are analyzed based on the results. A new reverse least square fitting method is proposed in this study in order to reduce the fitting uncertainties. It adapts an inverse function rather than the original function by the distribution slope of dataset. Some quantitative comparisons are provided for concrete and lead in this paper, too.

2. An Inverse Function Least Square Fitting Methods of Buildup Factors

From the literature, datasets of buildup factors are gathered as a function of mean free path when a certain energy is given for different materials. Table I shows the typical buildup factors for concrete and lead. From the table, a least square fitting in the third order polynomial can be applied such as

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & x_{n3} & x_{n4} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \quad (1)$$

where x_{ij} is a mean free path, a_j is a coefficient of x^{j-1} polynomial to be determined, and y_i is a buildup factor. In the case of the 3rd order polynomial fitting, x_{ij} is expressed as $x_{ij} = x_i^{j-1}$.

The equation is written as in the matrix form as follows

$$XA = Y \quad (2)$$

Equation (2) is solved easily with the standard least square fitting by multiplying the transpose matrix of X on both terms.

$$X^T XA = X^T Y \quad (3)$$

Then the coefficients are obtained as

$$A = \left(X^T X\right)^{-1} X^T Y \quad (4)$$

and their variances are also obtained as

$$Var(A_j) \approx \frac{R}{n-m} \left(X^T X\right)_{jj}^{-1} \quad (5)$$

where $R = \sum_i (y_i - \hat{y}_i)^2$, $\hat{y}$ is an estimate of the given buildup factor of y, and m, n are the fitting order and number of data sets, respectively. In this application, $n = 52, m = 4$.

In general, when evaluating the fitting variance, it is assumed that the variable of X has no errors. But it is not realistic in the area of application. Even a small deviation in X (mean free path) affects Y (buildup factor) depending on the slope of relationships between two parameters. When the shape is convex ($f''(x) < 0$), the deviation of Y will increase clearly. Conceptually, it is expressed as the Taylor series as follows,

$$f(x+h) - f(x) = hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f^{(3)}(x) + \cdots \quad (6)$$

The first term is dominant when small perturbation happens in X term. Therefore, if the shape of $f(x)$ is concave ($f''(x) > 0$), the deviation of Y due to X will decrease absolutely. An inverse function fitting is named to satisfy this condition. If the general trend of data set is an increasing slope, then the inverse function is used to fit the model to obtain more reliable fitting equations. If not, the usual least square fitting method is applied.

3. Application to the Buildup Factors of Lead and Concrete

Figure 1 shows the distribution of buildup factors of lead and Figure 2 depicts buildup factor of concrete from the obtained data set. For the convex data case (lead), the general least square fitting is adequate. However, the concave case (concrete), the inverse fitting is more adequate and the relative residue becomes much smaller in this case. The results of the 3rd order polynomial least fittings are provided in Table II. It is also included variance of each coefficients. However, the buildup factor of concrete is quite different trend in this test. The shape of concrete is concave as shown in Figure 2. For this case, the inverse function least square fitting method is applied. That is, X becomes buildup factor and Y will be mean free path, respectively. Thus, the fitting results of concrete and lead are provided in Table II. The relative residue of the inverse least square fitting for concrete decreases significantly when comparing the normal least square fitting. The relative residue represents the goodness of non-linear regression in the applications. In the case of concrete, the inverse fitting is more adequate than the normal fitting as expected. The coefficients of determination are also provided, which express the reliable probability of the fitted model. Thus, the inverse fitting of concrete represent data with a high probability of 99.6%. Additionally, the buildup factor can be easily obtained by standard numerical search methods such as the bi-section method or the false position method.[6] Table III summarizes the buildup factor of concrete obtained through the bi-sectional search method when the inverse least square fitting method is applied. The search criterion of bi-section is given as 1E-5 of relative difference. The results are converged very quickly within 20 iterations. The differences of buildup factors between the original and the inverse least square fittings are almost negligible except for large mean free path region about 15 mfpôs. The reason of large errors are mainly contributed from the error propagation toward outside in the general least square fitting method.

4. Conclusions

This study is focused on the least square fitting of existing buildup factors to be utilized in the point-kernel code for radiation shielding analysis. The inverse least square fitting method is suggested to obtain more reliable results of concave shaped dataset such as concrete. In the concrete case, the variance and residue are decreased significantly, too. However, the convex shaped case of lead can be applied to the usual least square fitting method.

In the future, more datasets will be tested by using the least square fitting. And the fitted data could be implemented to the existing point-kernel codes. The error of buildup factor is propagated into the final dose rate. Therefore, it is worthy to quantify the uncertainty of buildup factors and more data will be accommodate in order to reduce uncertainties.

Table I. Buildup Factors for Concrete and Lead for 1 MeV Photon Source

MFP	Concrete				Lead			
	ANSI/ ANS	Taylor	Berger	GP	ANSI/ ANS	Taylor	Berger	GP
0.5	1.45	1.733	1.645	1.450	1.20	1.158	1.149	1.195
1	1.98	2.488	2.311	1.982	1.38	1.312	1.296	1.367
2	3.24	4.069	3.708	3.233	1.68	1.612	1.582	1.675
3	4.72	5.746	5.194	4.711	1.95	1.900	1.860	1.952
4	6.42	7.524	6.774	6.405	2.19	2.176	2.130	2.206
5	8.33	9.409	8.452	8.308	2.43	2.441	2.392	2.444
6	10.40	11.406	10.233	10.412	2.66	2.697	2.645	2.670
7	12.70	13.520	12.122	12.713	2.89	2.942	2.891	2.886
8	15.20	15.757	14.124	15.202	3.10	3.178	3.129	3.095
10	20.70	20.624	18.490	20.718	3.51	3.626	3.582	3.495
15	37.20	35.402	31.786	37.287	4.45	4.616	4.593	4.426
20	57.10	54.690	49.171	57.153	5.27	5.462	5.445	5.275
25	80.10	79.676	71.661	79.966	5.98	6.200	6.155	6.014

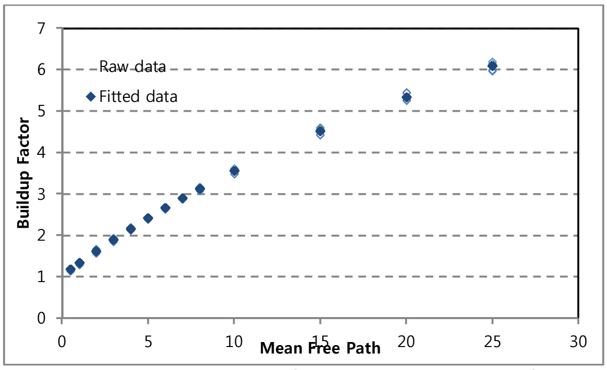

Figure 1. Buildup Factor of Lead as a Function of Mean Free Path when Photon Energy is 1 MeV.

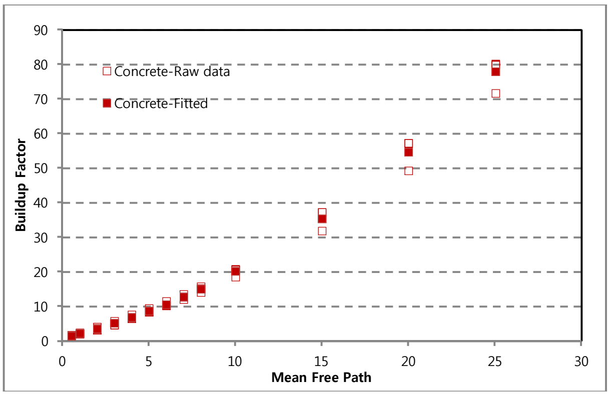

Figure 2. Buildup Factor of Concrete as a Function of Mean Free Path when Photon Energy is 1 MeV

Table II. Coefficients of Least Square Fitting for Buildup Factors for Concrete and Lead for 1 MeV Photon Source

Coef.	Concrete			
	Original Fitting	Variance	Inverse Fitting	Variance
A1	1.250E-04	4.522E-07	3.717E-05	4.248E-11
A2	7.295E-02	6.133E-04	-6.971E-03	5.852E-07
A3	1.174E+00	5.987E-02	6.386E-01	5.531E-04
A4	9.321E-01	3.846E-01	-1.854E-01	2.367E-02
Relative residue*	0.32		0.15	
Coefficient of Determination**	0.99518		0.99633	
Coef.	Lead			
	Org. Fitting	Var.	Inv. Fitting	Var.
A1	6.766E-05	5.284E-10	6.652E-03	2.669E-04
A2	-5.715E-03	7.166E-07	3.076E-01	3.011E-02
A3	3.025E-01	6.996E-05	2.424E+00	3.057E-01
A4	1.044E+00	4.494E-04	-2.799E+00	2.770E-01
Relative residue	0.016		0.07	
Coefficient of Determination	0.99871		0.99827	

* $R/\bar{y}^2$ * $r^2 = (S_t - R)/S_t,\ S_t = \sum (y_i - \bar{y})^2$

Table III. Buildup Factors of Concrete by Two Different Least Square Fitting Approaches

mfp	Original Least Square Fitting		Inverse Least Square Fitting			Difference |B0-B1|
	Buildup factor (B0)	Standard deviation	Buildup factor by Bi-section (B1)	Buildup factor with standard deviation(B2)	Sigma1 (B2-B1)	
0.5	1.5374	0.7488	1.0862	1.3914	0.3052	0.451
1	2.1793	0.8903	1.8953	2.2285	0.3332	0.284
2	3.5731	1.2140	3.5579	3.9634	0.4055	0.015
3	5.1144	1.5953	5.2844	5.7847	0.5003	0.170
4	6.8037	2.0382	7.0808	7.7051	0.6243	0.277
5	8.6420	2.5468	8.9535	9.7383	0.7848	0.312
6	10.6299	3.1251	10.9097	11.8982	0.9885	0.280
7	12.7682	3.7771	12.9584	14.2126	1.2542	0.190
8	15.0576	4.5069	15.1091	16.7065	1.5974	0.052
10	20.0928	6.2159	19.7645	22.3662	2.6017	0.328
15	35.3784	12.1320	34.241	43.3379	9.0969	1.137
20	54.5925	20.7993	54.8352	79.5134	24.6782	0.243
25	77.8289	32.7222	78.6376	110.2435	31.6059	0.809

* $B_1 = f^{-1}(x),\ B_2 = f^{-1}(x+\sigma)$

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