Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 29-30, 2014

Implicit Wall Heat Flux Model for the SPACE Code

Seung Wook Lee*, Jong Hyuk Lee, Kyung Doo Kim
Thermal-Hydraulic Safety Research Division, Korea Atomic Energy Research Institute, 111 Daedeok-Daero 989beon-gil Yuseong-gu, Daejoen, Korea
**Corresponding author: nuclist@kaeri.re.kr*

1. Introduction

The SPACE code [1], which is a two-fluid and three-field system analysis code, adopts an explicit wall heat flux model. This method has an advantage of making the source terms of the energy equations simple. However, a numerical instability will be caused because a hydraulic temperature variation due to a large wall heat flux is not considered. This numerical instability can be removed by introducing an implicit coupling of the heat structures and hydraulic cells. For this purpose, the implicit wall heat flux model of the RELAP5 code [2] has been introduced and modified for the three-field governing equations of the SPACE code. In this coupling method, the wall heat flux can be naturally controlled by the variation of the fluid temperature because the wall heat flux is expressed as a function of fluid temperatures of each phase. Therefore, the implicit wall heat flux model makes it possible to increase the numerical stability in case of a large difference of the fluid temperature due to the wall heat flux.

2. Implicit Wall Heat Flux Model

The SPACE code has three heat transfer coefficients for each phase and the wall heat flux is express as follows [3]:

$$q_k = h_k\left(T_w - T_k\right) + h_{kT}\left(T_w - T_{sT}\right) + h_{kp}\left(T_w - T_{sp}\right)$$

T is the temperature and h is the heat transfer coefficient. Subscript w means the wall surface, k is the phase index (=l, g, d), sT means the saturated state for the total pressure, and sp is the partial saturation. Note that the T_w is the new time value but the temperatures related with the fluid (T_k , T_{sT} , T_{sp}) are the old time value in the explicit wall heat flux model.

With the assumption of a constant heat transfer coefficient during a time step, the implicit wall heat flux for each phase is described by using Taylor expansion as follows:

$$\begin{aligned} q_k^n &= h_k\left(T_w^n - T_k^n\right) + h_{kT}\left(T_w^n - T_{sT}^n\right) + h_{kp}\left(T_w^n - T_{sp}^n\right) \\ &= q_k^0 + \left(h_k + h_{kT} + h_{kp}\right)\Delta T_w \\ &\quad - h_k\Delta T_l - h_{kT}\Delta T_{sT} - h_{kp}\Delta T_{sp} \end{aligned} \quad (1)$$

where,

$$q_k^0 = h_k\left(T_w - T_k\right) + h_{kT}\left(T_w - T_{sT}\right) + h_{kp}\left(T_w - T_{sp}\right)$$

$\Delta T \equiv T^n - T$: time difference of temperature
n: new time step

Note that the explicit wall heat flux is not exactly same as but equivalent to term of $q_k^0 + \left(h_k + h_{kT} + h_{kp}\right)\Delta T_w$ and the remaining terms are the additional heat flux due to the implicit coupling.

The implicit heat flux related with both of the wall and fluid-related temperature and it will be applied into the heat conduction equation.

2.1 Implicit Heat Conduction Model

General expression of the one-dimensional heat conduction equation at the left boundary of the wall is described as follows:

$$b_1 T_1^{n+1} + c_1 T_2^{n+1} = d_1$$

where b_1 and c_1 are coefficient and d_1 is a source term including the explicit wall heat flux.

The implicit wall heat flux is substituted for the explicit one in the above equation and then, source term is changed as follows:

$$d_1' = d_1 + A_1\Delta t \sum_{k=l,d,g}\left(h_k\Delta T_k + h_{kT}\Delta T_{sT} + h_{kp}\Delta T_{sp}\right)$$

where A_1 is surface area and Δt is a time step.

As seen in the above equation, the implicit heat flux affects only the right hand side (RHS) of the equation and there is no change in the left hand side (LHS). If necessary, similar modification can be applied to the right boundary side.

The system matrix of the implicit heat conduction equation with implicit source terms at both the left and right side is as follows:

$$\begin{bmatrix} b_1 & c_1 & & & \\ a_2 & b_2 & c_2 & & \\ & \ddots & \ddots & \ddots & \\ & & a_{M-1} & b_{M-1} & c_{M-1} \\ & & & b_M & c_M \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_{M-1} \\ T_M \end{bmatrix} \begin{bmatrix} d_1 + \sum_k\left(h_k\Delta T_k + h_{kT}\Delta T_{sT} + h_{kp}\Delta T_{sp}\right)_L \\ d_2 \\ \vdots \\ d_{M-1} \\ d_M + \sum_k\left(h_k\Delta T_k + h_{kT}\Delta T_{sT} + h_{kp}\Delta T_{sp}\right)_R \end{bmatrix} \quad (2)$$

$$= \underline{D}$$

where vector ***D*** is the source term including implicit source.

The vector ***D*** at the RHS of the above equation can be divided into two parts whether each part contains the fluid-related information (ΔT_k , ΔT_{sT} and ΔT_{sp}) or not.

Therefore, vector $\boldsymbol{D}$ can be expressed as follows:

$$\underline{D} = \underline{D}^0 + \underline{D}^1$$

where vector $\boldsymbol{D}^0$ is the explicit source term which is the same as d_j in Eq. (2), and $\boldsymbol{D}^1$ is the implicit one.

Applying the Gauss elimination scheme to Eq. (2), we can get the final wall temperature of the mesh point j at the new time as follows [4]:

$$\begin{aligned} T_j^n = g_{j0} &+ \sum_{k=l,d,g} \left(g_{jk}\Delta T_k + g_{jkT}\Delta T_{sT} + g_{jkp}\Delta T_{sp} \right)_L \\ &+ \sum_{k=l,d,g} \left(g_{jk}\Delta T_k + g_{jkT}\Delta T_{sT} + g_{jkp}\Delta T_{sp} \right)_R \end{aligned} \tag{3}$$

where the coefficient g is the converted element of the vector $\boldsymbol{D}$ during Gauss elimination. For the g_{jk} factor, the first subscript indicates the mesh point number and the second subscript indicates the phase index. The g_{j0} means the new time temperature due to change of the wall condition only. Note that the change in hydrodynamic temperature ΔT_k, ΔT_{sT} and ΔT_{sp} are still unknown. These values can be known only after solving the system matrix of hydrodynamics. After then, the wall temperature at every mesh point should be updated.

2.2 Boundary Wall Temperature

As seen in Eq. (3), every temperature is affected by the fluid temperatures from both the left and right boundary cells. A special case where the two sides are connected to two different cells is a heat structure representing heat exchanger tubing. This situation would involve unknown temperatures from more than one cell in the mass and energy equations for each attached cell and thus would not be applicable to the hydrodynamic advancement scheme. To avoid this problem, the heat flux for one boundary ignores the effects of fluid temperature changes in the other cell. This is a reasonable approximation since the effects of temperature changes of the fluid on one side of the heat structure has a highly attenuated effect on the heat flux on the other side. Therefore, boundary wall temperature is described as a function of adjacent cell temperatures.

$$T_{w,i}^n = g_0 + \sum_{k=l,d,g} \left(g_k\Delta T_k + g_{kT}\Delta T_{sT} + g_{kp}\Delta T_{sp} \right)_i \tag{4}$$

where i is boundary index (left or right).

2.3 Wall Heat Flux and Mass Transfer Rate

From Eq. (1) and (4), the wall heat flux to each phase can be expressed as follows:

$$q_k^n = q_{k0} + \sum_{k=l,d,g} \left(q_k\Delta T_k + q_{kT}\Delta T_{sT} + q_{kp}\Delta T_{sp} \right) \tag{5}$$

where q_{k0} is the explicit heat flux and q in the summation is the coefficient composed of h in Eq. (1) and g in Eq. (4).

Mass transfer rate due to boiling and condensing phenomena can be expressed easily. However, it is assumed that vapor condensation to droplet phase is neglected and only the condensation from vapor to liquid phase is allowed. With this assumption, a mass transfer rate due to boiling and condensation in the cell is expressed as follows:

$$\Gamma_{boil}^n = \sum_{k=l,d} \gamma_k q_k^n A_s \quad \left(q_{k0} > 0\right) \tag{6}$$

$$\Gamma_{cond}^n = \sum_{k=g} \gamma_k q_k^n A_s \quad \left(q_{k0} < 0\right) \tag{7}$$

where A_s is a surface area and γ_k is a mass transfer coefficient from k to other phase, which is determined by wall heat transfer correlation.

From the wall heat flux and mass transfer rate, energy balance equation of each phase is determined through rearranging each term with ΔT and will be inserted into the mass and energy equations to constitute the implicit coupled scalar matrix of the hydrodynamic solver. For the detail of mass and energy balance, refer to Ref. 4 because it is too long to describe the detail in this paper.

2.4 Mass and Energy Equations

The wall heat flux and mass transfer rate are treated as source terms in mass and energy equation and should be replaced with implicit wall heat flux. The primitive variables of the SPACE code are non-condensable gas pressure ΔP_n, gas temperature ΔT_g, droplet temperature ΔT_d, liquid temperature ΔT_l, void fraction $\Delta\alpha_g$, droplet fraction $\Delta\alpha_d$ and total pressure ΔP. The scalar matrix of mass and energy equations is expressed as follows [5]:

$$\underline{\underline{A}} \cdot \left(\Delta P_n, \Delta T_g, \Delta T_d, \Delta T_l, \Delta\alpha_g, \Delta\alpha_d, \Delta P\right)^T = \underline{b}^T + convection$$

where $\underline{\underline{A}}$ is a 7x7 matrix.

$\underline{b}^T$ in the RHS of the above equation contains the wall heat flux and mass transfer terms. Using the implicit terms, the matrix elements of $\underline{\underline{A}}$ are converted to $\underline{\underline{A'}}$ and all elements of $\underline{\underline{A'}}$ are shown in Ref. 3. Note that ΔT_{sp} and ΔT_{sT} of are not shown any longer in the converted matrix $\underline{\underline{A'}}$ but are converted by terms of $\partial T_{sp} / \partial P_n$, $\partial T_{sp} / \partial P$ and dT_{sT} / dP because ΔT_{sp} and ΔT_{sT} are not the primitive variables of system matrix. Temperature derivatives with respect to partial and total pressure are also expressed as follows:

$$\partial T_{sp} / \partial P = -\partial T_{sp} / \partial P_n = T_{sp}\left(h_g^{sp} - h_l^{sp}\right) / \left(\upsilon_g^{sp} - \upsilon_l^{sp}\right) \quad (8)$$

$$\partial T_{sT} / \partial P = T_{sT}\left(h_g^{sT} - h_l^{sT}\right) / \left(\upsilon_g^{sT} - \upsilon_l^{sT}\right) \quad (9)$$

where h is the saturated enthalpy and υ is the specific volume.

Though the matrix $\underline{\underline{A}}$ has been converted to $\underline{\underline{A'}}$ as a result of substitution by the implicit terms, the source vector $\underline{b}^T$ is not changed compared with the explicit coupling because the explicit wall heat flux and mass transfer terms still remain in $\underline{b}^T$ after matrix conversion.

After solving hydrodynamic system equation, we can get the unknowns in Eq. (3) ($\Delta T_g, \Delta T_d, \Delta T_l, \Delta T_{sp}, \Delta T_{sT}$) in aid of Eq. (8) ~ (9) and finally update the wall temperatures in all mesh points.

3. Validation Test

In order to validate stability of the implicit wall heat flux model, a total of 111 problems related with the heat structure have been selected among SPACE assessment problems [6] and each problem has been evaluated by both the explicit and implicit coupling method. Table I shows the summary of the test matrix.

Table I: Validation Test List

Category	Experiment	Inputs
Subcooled boiling	DOBO, SUBO, Christensen	11
CHF	Bennett, Becker, Boil-off, etc	34
Reflood	Flecht-Seaset, RBHT, etc	19
Condensation	UCB, Reflux, etc	35
SP convection	Conceptual problem	12

Figure 1 shows the time step size for the reflux condensation problem (RC-16). As shown in figure, the Courant limit is almost same and stable in both test cases, however, the actual time step size of the explicit coupling is much smaller than that of implicit coupling. It means that the explicit coupling method is more unstable than the implicit one.

Figure 2 shows the comparison result of the time consumption for all test problems. As shown in the figure, there is no remarkable reduction of calculation time because the time step size is restricted by small maximum time step size specified by user in most test problems. If the time step size is equal, calculation time of the explicit method is shorter than that of implicit one because the implicit method requires an additional calculation of the heat structure solver as well as hydrodynamic solver.

For the accurate stability test for implicit coupling method, fail ratio, which is defined as the ratio of the number of failed steps to the total successful steps, has been compared. As shown in figure 3, fail ratio of the implicit method is as smaller as expected than the explicit method in most test cases. From this result, the newly implemented implicit wall heat flux model would enhance the numerical stability of the SPACE code.

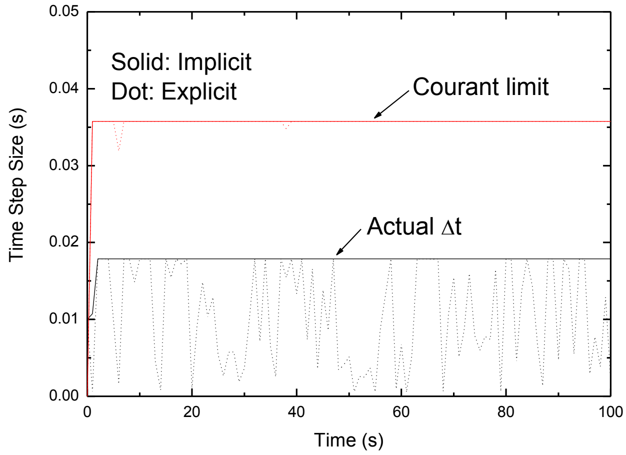

Fig. 1 Time step size for RC-16 Test

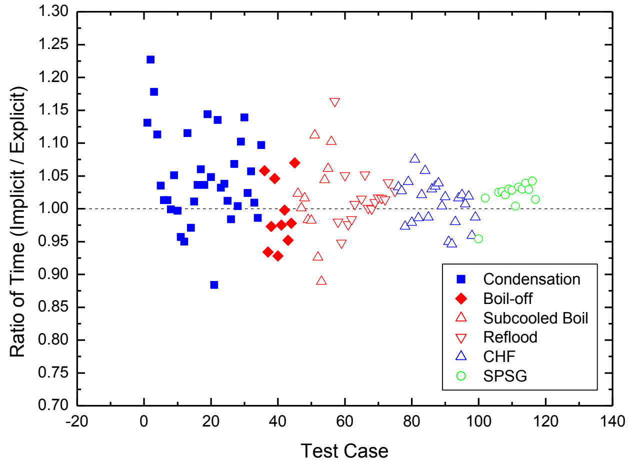

Fig. 2. Comparison of calculation time

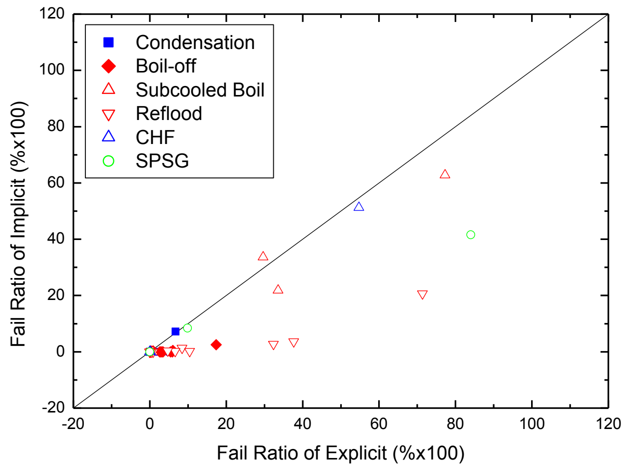

Fig. 3. Comparison of fail ratio

4. Conclusions

As an effort of enhancing numerical stability of the SPACE code, the implicit wall heat flux model similar to RELAP5 method has been implemented into the SPACE code. For this purpose, the explicit source terms of the heat conduction, mass and energy equations have been changed implicitly. In order to confirm the enhancement of numerical stability, a lot of validation tests have been performed and from test results, it is concluded that the implicit wall heat flux model is well incorporated in the code and improves numerical stability compared with the explicit method. However, it fails to reduce the calculation time because of the smaller user-specified maximum time step size in most test cases.

Acknowledgment

This work was supported by the Nuclear Power Technology Development Program of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea Government Ministry of Knowledge Economy.

REFERENCES

[1] Ha, Sang Jun *et al*., Development of the Space Code for Nuclear Power Plants, Nuclear Engineering and Technology, p.45 ~ p.62, Vol.43, No.1, Feb. 2011.

[2] RELAP5/MOD3.3 Code Manual, Volume I: Code Structure, System Models and Solution Methods, NUREG/CR-5535/Rev 1, INEEL, December (2001)

[3] S. K. Moon *et al*., Design Report of the Wall Heat Transfer Correlation of the SPACE Code, S06NX08-A-1-RD-11, Rev. 2, KAERI, 2012. 12.

[4] S. W. Lee *et al*., Implicit Coupling Method between Thermal-Hydraulic Cell and Heat Structure in the SPACE Code, KAERI/TR-5203/2013, KAERI, 2013. 11.

[5] SPACE 2.13 Manual: Volume 1 Theory Manual, S06NX08-E-1-TR-11, KHNP, 2013. 06.

[6] SPACE 2.13 Manual: Volume 4 Assessment Manual, S06NX08-K-1-TR-36, KHNP, 2013. 06.