A New Jacobian Matrix Method for Assessing Similarity between Critical Experiments and Real Reactor

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1. Introduction

For a metal fueled Sodium-cooled Fast Reactor (SFR), validation of the core neutronic characteristics is one of the most important issues. For a conventional Pressurized Water Reactor (PWR), experiment data can be easily secured from many operating reactors [1]. For innovative reactors such as Prototype Gen-IV Sodium-cooled Fast Reactor (PGSFR), unfortunately, experiment data from an operating reactor are unavailable because there are few operating reactors in the world. Hence, a critical experiment is the only way to obtain meaningful experiment data for the target core.

However, there is a considerable geometrical difference between the critical assembly for a critical experiment and the target core. The neutron characteristics of a system are influenced by the geometrical difference.

A number of researches have been performed to confirm the similarity between a critical experiment and a real reactor using a conventional representativity factor [2, 3, 4, 5, 6]. The conventional representativity factor defined as $\frac{\boldsymbol{S}_E^T \boldsymbol{U} \boldsymbol{S}_R}{\sqrt{\boldsymbol{S}_E^T \boldsymbol{U} \boldsymbol{S}_E}\sqrt{\boldsymbol{S}_R^T \boldsymbol{U} \boldsymbol{S}_R}}$, provides insight of similarity between two sensitivity vectors for cross-sections, but it did not provide a quantitative value [6, 7]. Hence, up to now, the influence of geometrical difference to the reactivity is believed to be negligible.

In this paper, a new Jacobian matrix method is proposed to provide a quantitative error for geometrical differences between two systems. In this method, reactivity of the critical assembly is decomposed into phenomenon-based reactivity and geometry-based reactivity. The reactivity is then transformed into a target core geometry using a Jacobian matrix. Then, non-linearity of two different systems can be derived by comparing the transformed reactivity with the original reactivity of the target core. The maximum error of the transformed reactivity can be used as an additional uncertainty of the geometrical difference.

2. Jacobian Matrix Method

Suppose that the reactivities of x phenomenon at the critical assembly, $\hat{\boldsymbol{\rho}}_{x,CA}$, and the reactivities of x phenomenon at a real reactor, $\hat{\boldsymbol{\rho}}_{x,R}$, can be written as

$$\hat{\boldsymbol{\rho}}_{x,CA} = \vec{\boldsymbol{\rho}}_x^T \vec{\boldsymbol{\rho}}_{CA}, \quad \textit{for all } \boldsymbol{\rho}_x \in \vec{\boldsymbol{\rho}}_{CA}, \tag{1}$$

$$\hat{\boldsymbol{\rho}}_{x,R} = \vec{\boldsymbol{\rho}}_x^T \vec{\boldsymbol{\rho}}_R, \quad \textit{for all } \boldsymbol{\rho}_x \in \vec{\boldsymbol{\rho}}_{CA}, \tag{2}$$

where the reactivity component of the x phenomenon is

$$\vec{\boldsymbol{\rho}}_x^T = \begin{bmatrix} \boldsymbol{\rho}_{L,x} \\ \boldsymbol{\rho}_{C,x} \\ \boldsymbol{\rho}_{F,x} \\ \boldsymbol{\rho}_{\nu,x} \end{bmatrix}^T, \tag{3}$$

and the geometrical reactivity component of the critical assembly is

$$\vec{\boldsymbol{\rho}}_{CA} = \begin{bmatrix} \boldsymbol{\rho}_{L,CA} \\ \boldsymbol{\rho}_{C,CA} \\ \boldsymbol{\rho}_{F,CA} \\ \boldsymbol{\rho}_{\nu,CA} \end{bmatrix}, \tag{4}$$

and the geometrical reactivity component of the real reactor is

$$\vec{\boldsymbol{\rho}}_R = \begin{bmatrix} \boldsymbol{\rho}_{L,R} \\ \boldsymbol{\rho}_{C,R} \\ \boldsymbol{\rho}_{F,R} \\ \boldsymbol{\rho}_{\nu,R} \end{bmatrix}. \tag{5}$$

We can then introduce a Jacobian matrix, which transforms the geometrical reactivity component from the critical assembly to a real reactor, such as

$$\begin{aligned} \hat{\boldsymbol{\rho}}_{x,CA} &= \vec{\boldsymbol{\rho}}_x^T \vec{\boldsymbol{\rho}}_{CA} \\ &= \vec{\boldsymbol{\rho}}_x^T \mathbf{G} \vec{\boldsymbol{\rho}}_R + \vec{\boldsymbol{\rho}}_x^T \vec{\boldsymbol{\Delta}}_{non-linear}, \quad \textit{for all } \boldsymbol{\rho}_x \in \vec{\boldsymbol{\rho}}_{CA}, \end{aligned} \tag{6}$$

where a Jacobian matrix $\mathbf{G}$ is

$$\begin{aligned} \mathbf{G} &= \begin{bmatrix} \boldsymbol{G}_{LL} & \boldsymbol{G}_{CL} & \boldsymbol{G}_{FL} & \boldsymbol{G}_{\nu L} \\ \boldsymbol{G}_{LC} & \boldsymbol{G}_{CC} & \boldsymbol{G}_{FC} & \boldsymbol{G}_{\nu C} \\ \boldsymbol{G}_{LF} & \boldsymbol{G}_{CF} & \boldsymbol{G}_{FF} & \boldsymbol{G}_{\nu F} \\ \boldsymbol{G}_{L\nu} & \boldsymbol{G}_{C\nu} & \boldsymbol{G}_{F\nu} & \boldsymbol{G}_{\nu\nu} \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial \boldsymbol{\rho}_{L,R}}{\partial \boldsymbol{\rho}_{L,CA}} & \frac{\partial \boldsymbol{\rho}_{L,R}}{\partial \boldsymbol{\rho}_{C,CA}} & \frac{\partial \boldsymbol{\rho}_{L,R}}{\partial \boldsymbol{\rho}_{F,CA}} & \frac{\partial \boldsymbol{\rho}_{L,R}}{\partial \boldsymbol{\rho}_{\nu,CA}} \\ \frac{\partial \boldsymbol{\rho}_{C,R}}{\partial \boldsymbol{\rho}_{L,CA}} & \frac{\partial \boldsymbol{\rho}_{C,R}}{\partial \boldsymbol{\rho}_{C,CA}} & \frac{\partial \boldsymbol{\rho}_{C,R}}{\partial \boldsymbol{\rho}_{F,CA}} & \frac{\partial \boldsymbol{\rho}_{C,R}}{\partial \boldsymbol{\rho}_{\nu,CA}} \\ \frac{\partial \boldsymbol{\rho}_{F,R}}{\partial \boldsymbol{\rho}_{L,CA}} & \frac{\partial \boldsymbol{\rho}_{F,R}}{\partial \boldsymbol{\rho}_{C,CA}} & \frac{\partial \boldsymbol{\rho}_{F,R}}{\partial \boldsymbol{\rho}_{F,CA}} & \frac{\partial \boldsymbol{\rho}_{F,R}}{\partial \boldsymbol{\rho}_{\nu,CA}} \\ \frac{\partial \boldsymbol{\rho}_{\nu,R}}{\partial \boldsymbol{\rho}_{L,CA}} & \frac{\partial \boldsymbol{\rho}_{\nu,R}}{\partial \boldsymbol{\rho}_{C,CA}} & \frac{\partial \boldsymbol{\rho}_{\nu,R}}{\partial \boldsymbol{\rho}_{F,CA}} & \frac{\partial \boldsymbol{\rho}_{\nu,R}}{\partial \boldsymbol{\rho}_{\nu,CA}} \end{bmatrix}, \end{aligned} \tag{7}$$

and errors coming from the non-linearity is

$$\vec{\Delta}_{non-linear} = \begin{bmatrix} \Delta_{L,non-linear} \\ \Delta_{C,non-linear} \\ \Delta_{F,non-linear} \\ \Delta_{\nu,non-linear} \end{bmatrix}. \quad (8)$$

Since directly calculating the exact Jacobian component in Eq. (6) is not easy, we introduce the following approximation:

$$\begin{aligned} \hat{\rho}_{x,CA} &= \vec{\rho}_x^T \mathrm{G} \vec{\rho}_R + \vec{\rho}_x^T \vec{\Delta}_{non-linear} \\ &= \vec{\rho}_x^T \tilde{\mathrm{G}} \vec{\rho}_R + \vec{\rho}_x^T \varepsilon \vec{\rho}_R + \vec{\rho}_x^T \vec{\Delta}_{non-linear}, \end{aligned} \quad (9)$$

in which the approximated Jacobian matrix $\tilde{\mathbf{G}}$ is

$$\tilde{\mathbf{G}} = \begin{bmatrix} \tilde{G}_{LL} & 0 & 0 & 0 \\ 0 & \tilde{G}_{CC} & 0 & 0 \\ 0 & 0 & \tilde{G}_{FF} & 0 \\ 0 & 0 & 0 & \tilde{G}_{\nu\nu} \end{bmatrix}, \quad (10)$$

and associated errors coming from the approximation is

$$\varepsilon = \begin{bmatrix} \varepsilon_{LL} & \varepsilon_{CL} & \varepsilon_{FL} & \varepsilon_{\nu L} \\ \varepsilon_{LC} & \varepsilon_{CC} & \varepsilon_{FC} & \varepsilon_{\nu C} \\ \varepsilon_{LF} & \varepsilon_{CF} & \varepsilon_{FF} & \varepsilon_{\nu F} \\ \varepsilon_{L\nu} & \varepsilon_{C\nu} & \varepsilon_{F\nu} & \varepsilon_{\nu\nu} \end{bmatrix}. \quad (11)$$

Eq. (9) can then be written as

$$\begin{aligned} \hat{\rho}_{x,CA} &= \begin{pmatrix} \rho_{L,x}\tilde{G}_{LL}\rho_{L,R} + \rho_{C,x}\tilde{G}_{CC}\rho_{C,R} \\ +\rho_{F,x}\tilde{G}_{FF}\rho_{F,R} + \rho_{\nu,x}\tilde{G}_{\nu\nu}\rho_{\nu,R} \end{pmatrix} \\ &+ \sum_{z=L,C,F,\nu} \rho_{z,x} \sum_{y=L,C,F,\nu} \varepsilon_{yz}\rho_{y,R} \\ &+ \sum_{y=L,C,F,\nu} \rho_{y,x}\Delta_{y,non-linear} \\ &= \begin{pmatrix} \tilde{G}_{LL}\rho_{L,x}\rho_{L,R} + \tilde{G}_{CC}\rho_{C,x}\rho_{C,R} \\ +\tilde{G}_{FF}\rho_{F,x}\rho_{F,R} + \tilde{G}_{\nu\nu}\rho_{\nu,x}\rho_{\nu,R} \end{pmatrix} \\ &+ \sum_{z=L,C,F,\nu} \rho_{z,x} \sum_{y=L,C,F,\nu} \varepsilon_{yz}\rho_{y,R} \\ &+ \sum_{y=L,C,F,\nu} \rho_{y,x}\Delta_{y,non-linear} \\ &= \begin{pmatrix} \tilde{G}_{LL}\rho_{x,L,R} + \tilde{G}_{CC}\rho_{x,C,R} \\ +\tilde{G}_{FF}\rho_{x,F,R} + \tilde{G}_{\nu\nu}\rho_{x,\nu,R} \end{pmatrix} \\ &+ \sum_{z=L,C,F,\nu} \rho_{z,x} \sum_{y=L,C,F,\nu} \varepsilon_{yz}\rho_{y,R} \\ &+ \sum_{y=L,C,F,\nu} \rho_{y,x}\Delta_{y,non-linear} \end{aligned} \quad (12)$$

where

$$\rho_{x,L,R} \equiv \rho_{L,x}\rho_{L,R}, \quad (13)$$

$$\rho_{x,C,R} \equiv \rho_{C,x}\rho_{C,R}, \quad (14)$$

$$\rho_{x,F,R} \equiv \rho_{F,x}\rho_{F,R}, \quad (15)$$

$$\rho_{x,\nu,R} \equiv \rho_{\nu,x}\rho_{\nu,R}. \quad (16)$$

Eq. (2) can be re-written using the above-defined constants as

$$\begin{aligned} \hat{\rho}_{x,R} &= \rho_{L,x}\rho_{L,R} + \rho_{C,x}\rho_{C,R} + \rho_{F,x}\rho_{F,R} + \rho_{\nu,x}\rho_{\nu,R} \\ &= \rho_{x,L,R} + \rho_{x,C,R} + \rho_{x,F,R} + \rho_{x,\nu,R}. \end{aligned} \quad (17)$$

Eq. (17) is the reactivity decomposition used in reference [].

Now, the approximated Jacobian components are obtained following the least-square method for all reactivities as

$$J = \sum_x \left(\hat{\rho}_{x,CA} - \begin{pmatrix} \tilde{G}_{LL}\rho_{x,L,R} + \tilde{G}_{CC}\rho_{x,C,R} \\ +\tilde{G}_{FF}\rho_{x,F,R} + \tilde{G}_{\nu\nu}\rho_{x,\nu,R} \end{pmatrix} \right)^2, \quad (18)$$

$$\frac{\partial J}{\partial \tilde{G}_{LL}} = 0, \quad (19)$$

$$\frac{\partial J}{\partial \tilde{G}_{CC}} = 0, \quad (20)$$

$$\frac{\partial J}{\partial \tilde{G}_{FF}} = 0, \quad (21)$$

$$\frac{\partial J}{\partial \tilde{G}_{\nu\nu}} = 0. \quad (22)$$

The solutions can then be obtained in the following matrix form:

$$\begin{aligned} &\begin{bmatrix} \tilde{G}_{LL} \\ \tilde{G}_{CC} \\ \tilde{G}_{FF} \\ \tilde{G}_{\nu\nu} \end{bmatrix} \\ &= \begin{bmatrix} \sum_x \rho_{x,L,R}^2 & \sum_x \rho_{x,C,R}\rho_{x,L,R} & \sum_x \rho_{x,F,R}\rho_{x,L,R} & \sum_x \rho_{x,\nu,R}\rho_{x,L,R} \\ \sum_x \rho_{x,L,R}\rho_{x,C,R} & \sum_x \rho_{x,C,R}^2 & \sum_x \rho_{x,F,R}\rho_{x,C,R} & \sum_x \rho_{x,\nu,R}\rho_{x,C,R} \\ \sum_x \rho_{x,L,R}\rho_{x,F,R} & \sum_x \rho_{x,C,R}\rho_{x,F,R} & \sum_x \rho_{x,F,R}^2 & \sum_x \rho_{x,\nu,R}\rho_{x,F,R} \\ \sum_x \rho_{x,L,R}\rho_{x,\nu,R} & \sum_x \rho_{x,C,R}\rho_{x,\nu,R} & \sum_x \rho_{x,F,R}\rho_{x,\nu,R} & \sum_x \rho_{x,\nu,R}^2 \end{bmatrix}^{-1} \\ &\times \begin{bmatrix} \sum_x \rho_{x,L,R}\hat{\rho}_{x,CA} \\ \sum_x \rho_{x,C,R}\hat{\rho}_{x,CA} \\ \sum_x \rho_{x,F,R}\hat{\rho}_{x,CA} \\ \sum_x \rho_{x,\nu,R}\hat{\rho}_{x,CA} \end{bmatrix}. \end{aligned} \quad (23)$$

Finally, we can obtain the errors coming from non-linearity originating from the geometrical difference with an additional error, $\vec{\rho}_x^T \varepsilon \vec{\rho}_R$, as

$$\vec{\rho}_x^T \vec{\Delta}_{non-linear} + \vec{\rho}_x^T \varepsilon \vec{\rho}_R = \hat{\rho}_{x,CA} - \vec{\rho}_x^T \tilde{\mathrm{G}} \vec{\rho}_R. \quad (24)$$

Although the additional error coming from the approximated Jacobian matrix is still included in Eq. (24), we can obtain the similarity between the two systems as a quantitative error form.

3. Models for Critical Assembly and Target Core

The BFS-109-2A critical assembly model, the target U-Zr fueled core model, and the uranium nitride (UN) fueled core are considered. The configurations of the critical assembly and target U-Zr fueled core models are described in reference [6]. The uranium nitride fueled core is configured similarly as the target U-Zr core except the equivalent diameter and height of the active core. The property of the uranium nitride fuel is based

on reference [9]. The configuration data of the three cores are shown in Table I.

Table I: Geometrical parameters of the considered cores

	Critical assembly	Target U-Zr core	UN core
Equivalent diameter of core(D)	124.33	142.79	171.40
Height of active core(H)	95.43	109.599	100
D/H Ratio	1.30	1.30	1.71

The Jacobian matrix method was applied for fuel axial reactivity phenomena. Fuel rods in the target U-Zr core and UN core were elongated 7.848% axially, which is the identical expansion rate to the modified fuel cell in the BFS-109-2A model [10]. Seven radial regions were considered for local expansion reactivities, as shown in Table II. The elongated fuel rods or assemblies were applied from the central region of the core to the peripheral region of the core.

Table II: Number of elongated fuel subassemblies/rods for each fuel expansion region

	Critical assembly	Target U-Zr Core and UN core	Volume ratio of elongated region
Reference	0	0	0.00
Region 1	112	12	25.00
Region 2	168	18	37.50
Region 3	224	24	50.00
Region 4	280	30	62.50
Region 5	336	36	75.00
Region 6	392	42	87.50
Region 7	448	48	100.00

4. Analysis Results

In this study, the MCNP5 code [11] was used with the continuous energy ENDF/B-VII.0 library to omit uncertainties in a multi-group cross-section.

Neutron spectrums in the central core region are shown in Fig. 1. As expected, the nitride fueled core showed considerable differences in neutron spectrum, while the target U-Zr core showed good agreement.

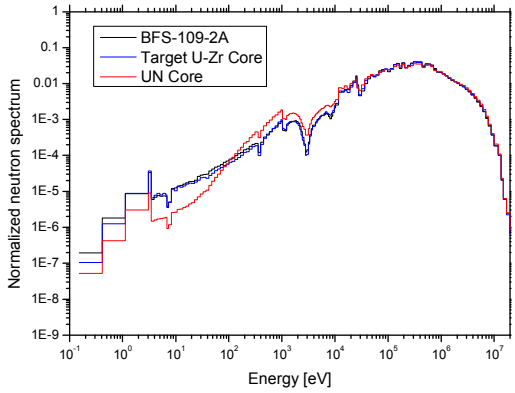

Fig. 1. Neutron spectrums in the central core region

The components of the fuel axial expansion reactivities for the considered regions are shown in Figs. 5, 6, and 7.

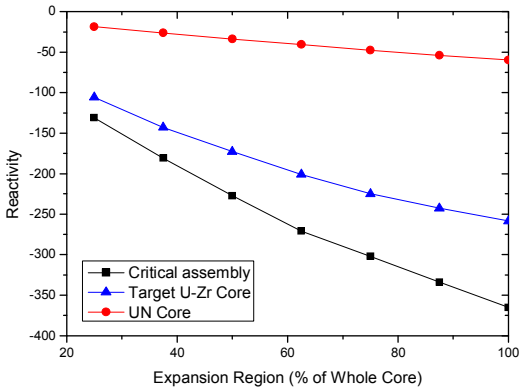

Fig. 2. Distributions of leakage reactivities in various regions

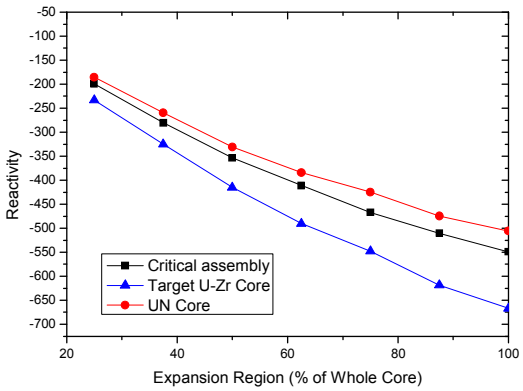

Fig. 3. Distributions of capture reactivities in various regions

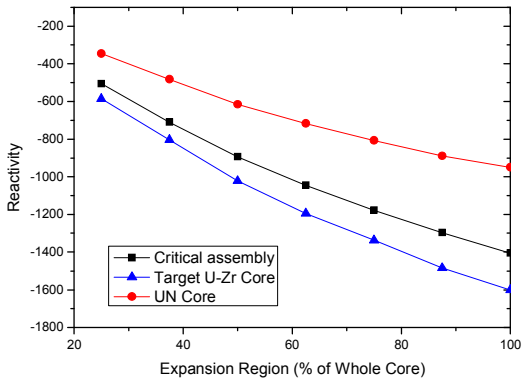

Fig. 4. Distributions of fission reactivities in various regions

Unlike neutron spectrums at the core central region, the UN core showed more similar capture reactivity distributions compared to the critical assembly, whereas the target U-Zr core showed more similar fission and leakage distributions compared to the critical assembly.

However, from the above reactivity distributions and neutron spectrums, it is still difficult to decide on the uncertainty increment when the BFS-109-2A experiments are used to target the U-Zr core or UN core.

Table III and Table IV showed transformed reactivities from the target U-Zr and UN cores, respectively, using the Jacobian matrix method. For fuel axial expansion reactivity, the target U-Zr core can be transformed into a critical assembly with a maximum error of 4.2%, while the UN core can be transformed into a critical assembly with a maximum error of 13.8%. As mentioned in the previous chapter, this error comes from the non-linearity of the geometry and approximated Jacobian matrix.

Table III: Transformed reactivities from the target U-Zr Core

	Critical assembly	Transformed from U-Zr Core	Difference [pcm]	Difference [%]
Region 1	-835.9	-835.9	0.0	0.0
Region 2	-1170.4	-1170.4	0.0	0.0
Region 3	-1474.3	-1474.3	0.0	0.0
Region 4	-1726.3	-1758.9	32.5	-1.9
Region 5	-1946.6	-1960.6	14.0	-0.7
Region 6	-2140.7	-2231.3	90.6	-4.2
Region 7	-2320.4	-2392.2	71.8	-3.1

Table IV: Transformed reactivities from the UN Core

	Critical assembly	Transformed from U-Zr Core	Difference [pcm]	Difference [%]
Region 1	-835.9	-835.9	0.0	0.0
Region 2	-1170.4	-1170.4	0.0	0.0
Region 3	-1474.3	-1474.3	0.0	0.0
Region 4	-1726.3	-1670.6	-55.7	3.2
Region 5	-1946.6	-1746.1	-200.5	10.3
Region 6	-2140.7	-1940.2	-200.5	9.4
Region 7	-2320.4	-2000.8	-319.6	13.8

5. Conclusions and Discussions

In this paper, a new Jacobian matrix method was proposed to assess the similarity quantitively between two different systems for neutron characteristics. In addition, unlike other methods for similarity decision, the proposed method reflects the sensitivity of the geometry as well as sensitivity of the isotopes.

Although an unavoidable additional error still remains in the proposed method, the quantitative geometrical difference in the reactivity can be used as an additional uncertainty for validating the real reactor cores.

Application of the proposed method to the other reactivities and other critical assemblies, such as ZPPR series and BFS-73-1, remains as future work. Meanwhile, the proposed method will also be used to make a configuration of a mock-up experiment for the initial uranium core of the PGSFR.

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