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Numerical Study of Steady Sheet Cavitation in Venturi Nozzle

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Abstract The purpose of the present work was to numerically investigate the sheet cavitation in venturi geometry, using a homogenous mixture model. An implicit, dual time upwind non-MUSCL Total Variation Diminishing (TVD) scheme was applied to solve the three-phase compressible Reynolds-Averaged Navier-Stokes (RANS). A new cavitation model developed by Merkle et al. (2006) was used to mode the mass transfer between phases. The local void ratio and velocity fields for cold water flows were compared against experimental and other published results. The numerically predicted shedding is in fair agreement with the experimental data.

1. Governing Equations

The equations governed the compressible multiphase mixture flow are given as follows [2].

Mass conservations

$$\frac{\partial}{\partial t}(Y_L\rho_m)+\frac{\partial}{\partial x}(Y_L\rho_m u)+\frac{\partial}{\partial y}(Y_L\rho_m v)=(\dot{m}^+ + \dot{m}^-)-c_a\frac{Y_L\rho_m v}{y} \quad (1)$$

$$\frac{\partial}{\partial t}(Y_v\rho_m)+\frac{\partial}{\partial x}(Y_v\rho_m u)+\frac{\partial}{\partial y}(Y_v\rho_m v)=-(\dot{m}^+ + \dot{m}^-)-c_a\frac{Y_v\rho_m v}{y} \quad (2)$$

$$\frac{\partial}{\partial t}(Y_g\rho_m)+\frac{\partial}{\partial x}(Y_g\rho_m u)+\frac{\partial}{\partial y}(Y_g\rho_m v)=-c_a\frac{Y_g\rho_m v}{y} \quad (3)$$

Momentum conservations

$$\frac{\partial}{\partial t}(\rho_m u)+\frac{\partial}{\partial x}(\rho_m u^2)+\frac{\partial}{\partial y}(\rho_m uv)=-\bar{\beta}_p\frac{\partial p}{\partial x}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\rho_m g_x-c_a\left(\frac{\rho_m uv}{y}-\frac{\tau_{xy}}{y}\right) \quad (4)$$

$$\frac{\partial}{\partial t}(\rho_m v)+\frac{\partial}{\partial x}(\rho_m vu)+\frac{\partial}{\partial y}(\rho_m v^2)=-\bar{\beta}_p\frac{\partial p}{\partial y}+\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\rho_m g_y-c_a\left(\frac{\rho_m v^2}{y}-\frac{\tau_{yy}}{y}\right) \quad (5)$$

Energy Conservation

$$\frac{\partial}{\partial t}(\rho_m h_t-\bar{\beta}_p p)+\frac{\partial}{\partial x}(\rho_m h_t u)+\frac{\partial}{\partial y}(\rho_m h_t v)=\frac{\partial(u\tau_{xx}+v\tau_{xy}-q_x)}{\partial x}+\frac{\partial(u\tau_{yx}+v\tau_{yy}-q_y)}{\partial y}+c_a\frac{-\rho_m h_t v+u\tau_{xy}+v\tau_{yy}-q_y}{y} \quad (6)$$

where *p*, *u*, *v*, h_t, *ρ*, Y and g indicate the pressure, Cartesian or cylindrical components, total enthalpy, density, mass fraction and gravity acceleration, respectively; $\boldsymbol{\tau}$ is pseudo time; *t* is the physical time; τ and q is represented by viscous stress and heat flux; The *l*, *v* and *g* represent the liquid, vapor and non-condensable gas phase, respectively; m denotes the mixture phase and the second *t* in subscript denotes the turbulent state; $\dot{m}^+$ and $\dot{m}^-$ denotes transformation of vapor to liquid and liquid to vapor phase respectively. The governing equations can be written in the generalized curvilinear coordinates as follows:

$$\Gamma_e\frac{\partial\hat{Q}}{\partial t}+\Gamma\frac{\partial\hat{Q}}{\partial\tau}+\frac{\partial\hat{E}}{\partial x}+\frac{\partial\hat{F}}{\partial y}-\frac{\partial\hat{E}^v}{\partial x}-\frac{\partial\hat{F}^v}{\partial y}=\hat{S} \quad (7)$$

where $\hat{Q}$, $\hat{E}$ and $\hat{F}$, indicate the vector of primitive variables and the convective flux vectors and $\hat{E}^v$ and $\hat{F}^v$ the viscous flux vectors; $\hat{S}$ indicates the source term; the Γ_e indicates Jacobian matrix; Γ is preconditioning matrix [4] with were added to the system of equations (7) to handle the stiffness problem.

$$\hat{Q}=\frac{1}{J}(p,u,v,T,Y_v,Y_g)^T$$

$$\hat{E}=\frac{1}{J}(Y_L\rho_m U,\ \rho_m uU+\xi_x\bar{\beta}_p p,\ \rho_m vU+\xi_y\bar{\beta}_p p,\rho_m h_t U,Y_v\rho_m U,Y_g\rho_m U)^T$$

$$\hat{F}=\frac{1}{J}(Y_L\rho_m V,\ \rho_m uV+\eta_x\bar{\beta}_p p,\ \rho_m vV+\eta_y\bar{\beta}_p p,\ \rho_m h_t V,Y_v\rho_m V,Y_g\rho_m V)^T$$

$$\hat{E}^v=\frac{1}{J}(0,\xi_x\tau_{xx}+\xi_y\tau_{xy},\xi_x\tau_{yx}+\xi_y\tau_{yy},\xi_x(u\tau_{xx}+v\tau_{xy}-q_x)+\xi_y(u\tau_{yx}+v\tau_{yy}-q_y),0,0)^T$$

$$\hat{F}^v=\frac{1}{J}(0,\eta_x\tau_{xx}+\eta_y\tau_{xy},\eta_x\tau_{yx}+\eta_y\tau_{yy},\eta_x(u\tau_{xx}+v\tau_{xy}-q_x)+\eta_y(u\tau_{yx}+v\tau_{yy}-q_y),0,0)^T \quad (8)$$

$$\hat{S}=\frac{1}{J}\begin{pmatrix}\dot{m}^-+\dot{m}^+-c_a\frac{Y_L\rho_m v}{y},\ \rho_m g_x-c_a\left(\frac{\rho_m uv}{y}-\frac{\tau_{xy}}{y}\right),\rho_m g_y-c_a\left(\frac{\rho_m v^2}{y}-\frac{\tau_{yy}}{y}\right)\\ c_a\frac{-\rho_m h_t v+u\tau_{xy}+v\tau_{yy}-q_y}{y},-\dot{m}^- -\dot{m}^+-c_a\frac{Y_v\rho_m v}{y},\ -c_a\frac{Y_g\rho_m v}{y}\end{pmatrix}^T \quad (9)$$

$$\Gamma=\begin{bmatrix}Y_L\partial_p\rho_m' & 0 & 0 & 0 & Y_L\partial_T\rho_m & -\rho_m+Y_L\partial_{Yv}\rho_m & -\rho_m+Y_L\partial_{Yg}\rho_m\\ u\partial_p\rho_m' & \rho_m & 0 & 0 & u\partial_T\rho_m & u\partial_{Yv}\rho_m & u\partial_{Yg}\rho_m\\ v\partial_p\rho_m' & 0 & \rho_m & 0 & v\partial_T\rho_m & v\partial_{Yv}\rho_m & v\partial_{Yg}\rho_m\\ w\partial_p\rho_m' & 0 & 0 & \rho_m & w\partial_T\rho_m & w\partial_{Yv}\rho_m & w\partial_{Yg}\rho_m\\ h_t\partial_p\rho_m'+\rho_m\partial_p h-\bar{\beta}_p & \rho_m u & \rho_m v & \rho_m w & h_t\partial_T\rho_m+\rho_m\partial_T h & h_t\partial_{Yv}\rho_m+\rho_m\partial_{Yv}h & h_t\partial_{Yg}\rho_m+\rho_m\partial_{Yg}h\\ Y_v\partial_p\rho_m' & 0 & 0 & 0 & Y_v\partial_T\rho_m & \rho_m+Y_v\partial_{Yv}\rho_m & Y_v\partial_{Yg}\rho\\ Y_g\partial_p\rho_m' & 0 & 0 & 0 & Y_g\partial_T\rho_m & Y_g\partial_{Yv}\rho_m & \rho_m+Y_g\partial_{Yg}\rho_m\end{bmatrix} \quad (10)$$

$$\dot{m}^{-} = -k_v \frac{\rho_v \alpha_l}{t_\infty} \min\left\{1, \max\left(\frac{(p_v - p)}{k_p p_v}, 0\right)\right\}$$

$$\dot{m}^{+} = k_L \frac{\rho_v \alpha_v}{t_\infty} \min\left\{1, \max\left(\frac{(p - p_v)}{k_p p_v}, 0\right)\right\} \qquad (11)$$

2. Results

The present numerical results were compared with the experimental data and numerical computations by S.Barre et al. [3]. The flows conditions are as follows P_{inlet} =36000, U_{inlet} =10.8 m/s, T_{ref}=293 K.

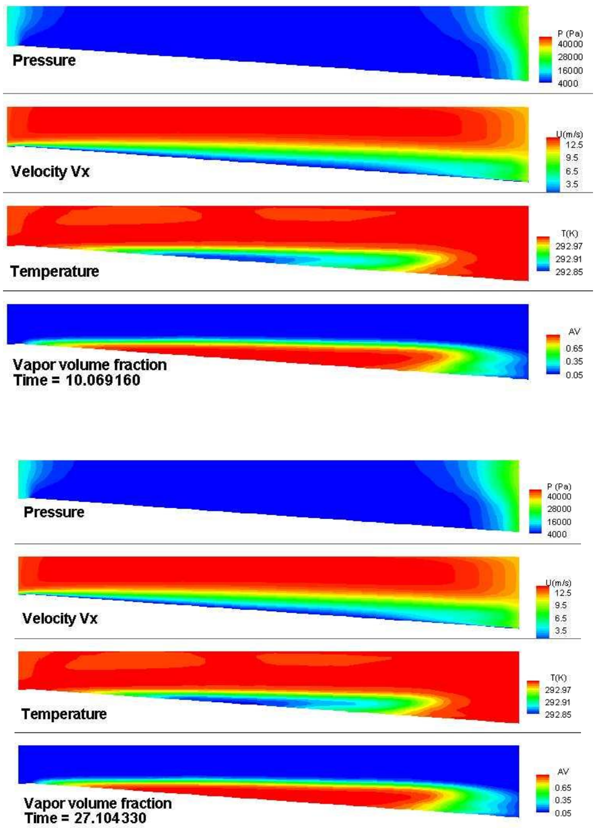
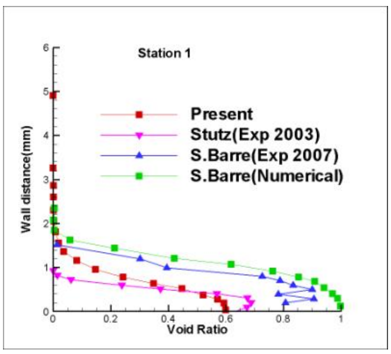
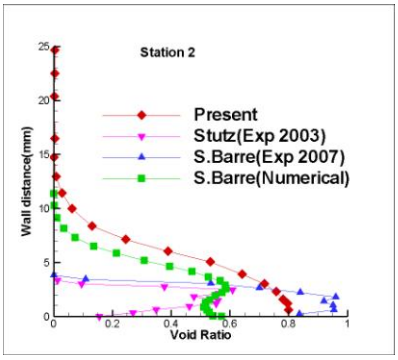


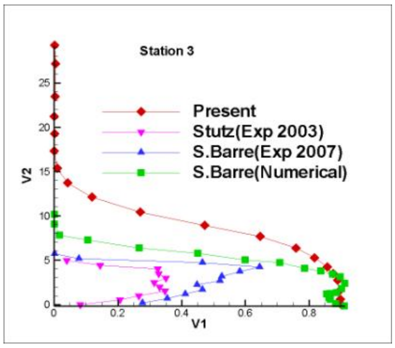
Fig.1. shows time sequence of the cyclic process of bubble formation for the cavitating flow over 0-caliber nozzle.



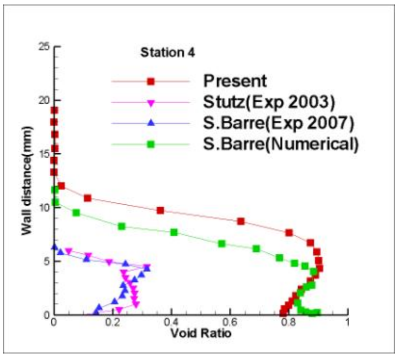
(a) Station1



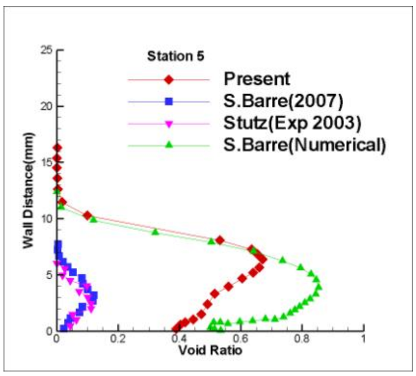
(b) Station2



(c) Station3



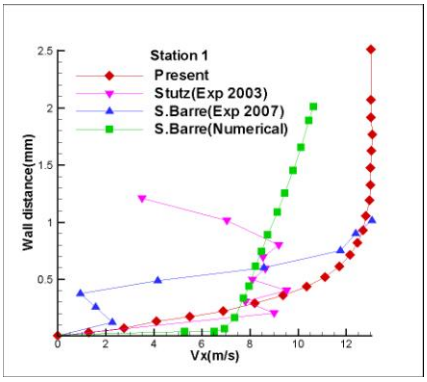
(d) Station4



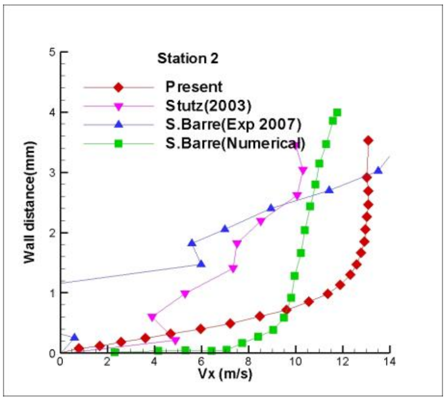
(e) Station5

Fig.2 Void ratio profile, comparison between present result and experimental results

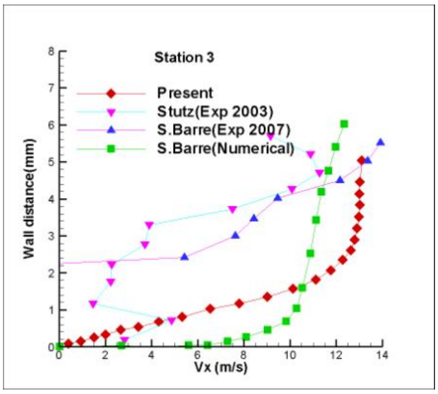
Figs. 2 (a)-(e) show the comparison of void ratio evolution with five probed station points. As can be seen in Figs.2 that the present numerical results and there by S.Barre et al. [3] are in agreement. However, an overestimation of void ratio and cavity thickness has been observed.



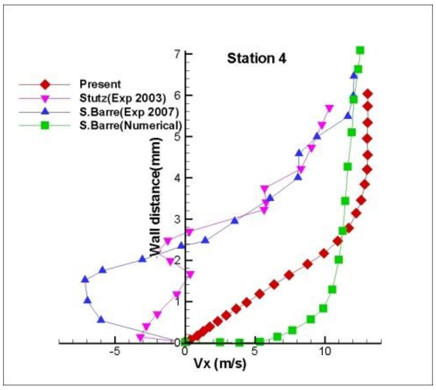
(a) Station1



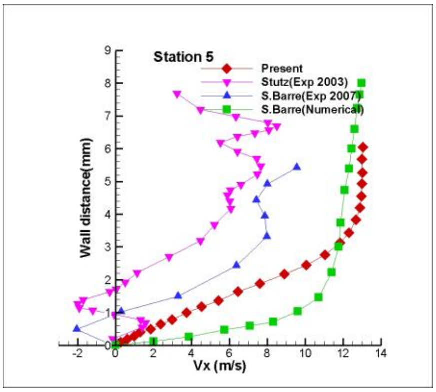
(b) Station2



(c) Station3



(d) Station4



(e) Station5

Fig.3. Velocity profile

Fig. 3 shows comparison of velocity profile against experimental data. Overall, the predicted results are quantitively matched with experiments.

3. Conclusions

A study of cavitation sheet in two-dimensional, multiphase venturi configuration was successfully performed by applying the compressible homogenous mixture model. Compared to previous experimental works done in the same geometry, reliable results concerning void ratio and velocity profile were obtained. Results show that the performed numerical simulations were able to describe qualitatively and quantitatively the steady behavior of the cavitation sheet.

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