Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 29-30, 2014

Use of Exponential Approximation on Numerical Analysis of Discrete Ordinates Equation

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1. Introduction

The discrete ordinates (S_N) method is an effective technique for deriving the numerical solution of the integro-differential transport equation [1]. It is based on the transport equation discretized both in the space and in the angular coordinates and an auxiliary relation to connect the cell average to the cell edge fluxes. The most widely used auxiliary relation is the so-called Diamond Difference (DD) relation that is assumed to be a linear relationship between the cell average and the cell edge fluxes. However, this relation does not match with the actual phenomena because the number of incident radiations is exponentially decreased for passing a specific material.

In this study, the Exponential Approximation (EA) different from existing methods (i.e., Exponential Characteristic [2] and Method [3]) is developed to overcome the above-mentioned problem. A simple benchmark calculation for deriving the scalar flux distribution in the X-Y geometry is performed to evaluate the accuracy and efficiency of numerical analyses applied with the EA and DD relation.

2. Methods and Materials

The 2-D neutron balance equation described by the discrete ordinates form is as follows:

$$\frac{\mu_n}{\Delta x_i}\left(\psi_{n,i+1/2,j}-\psi_{n,i-1/2,j}\right)+\frac{\eta_n}{\Delta y_j}\left(\psi_{n,i,j+1/2}-\psi_{n,i,j-1/2}\right)+\sigma_{ij}\psi_{nij}=q_{nij}$$

where Δx_i is a spatial interval between $x_{i+1/2}$ and $x_{i-1/2}$, and $\psi_{n,i\pm1/2}$ is the edge fluxes on the surface of $x_{i\pm1/2}$. In addition, q_{nij} is a sum of the scattering and external source (s_{nij}) terms. In order to solve this equation, the auxiliary relations are required with respect to the relationship between the cell average ($\psi_{n,ij}$) and cell edge ($\psi_{n,i\pm1/2}$ and $\psi_{n,j\pm1/2}$) fluxes. In this study, these relations are defined as follows:

$$\psi_{nij}=\frac{e^{-\sigma_{ij}\Delta x_i/2}}{1+e^{-\sigma_{ij}\Delta x_i}}\left(\psi_{n,i-1/2,j}+\psi_{n,i+1/2,j}\right)\qquad (s_{nij}=0)$$

$$\psi_{nij}=\frac{e^{-\sigma_{ij}\Delta y_j/2}}{1+e^{-\sigma_{ij}\Delta y_j}}\left(\psi_{n,i,j-1/2}+\psi_{n,i,j+1/2}\right)\qquad (s_{nij}=0)$$

$$\psi_{nij}=\frac{\cos\left(\dfrac{\pi\Delta x_i/2}{\Delta x_i+2/3\sigma_{ij}}\right)^{-1}}{2}\left(\psi_{n,i-1/2,j}+\psi_{n,i+1/2,j}\right)\qquad (s_{nij}\neq 0)$$

$$\psi_{nij}=\frac{\cos\left(\dfrac{\pi\Delta y_j/2}{\Delta y_j+2/3\sigma_{ij}}\right)^{-1}}{2}\left(\psi_{n,i,j-1/2}+\psi_{n,i,j+1/2}\right)\qquad (s_{nij}\neq 0)$$

These relations, called EA, can be divided according to the existence of the external source (s_{nij}) in spatial meshes. In the case of no external source in the mesh, this relation is determined from the ratio of exponential attenuation of entering the flux until the center of the mesh ($\Delta x_i/2$). In the other case, it is determined with reference to the equation for analyzing the flux distribution in an infinite slab bare reactor [4]. For $\mu_n > 0$, $\eta_n > 0$, the cell average flux can be consequently derived as follows:

$$\psi_{nij}=\left[\sigma_{ij}+\frac{|\mu_n|}{\Delta x_i}\left(\frac{1+e^{-\sigma_{ij}\Delta x_i}}{e^{-\sigma_{ij}\Delta x_i/2}}\right)+\frac{|\eta_n|}{\Delta y_j}\left(\frac{1+e^{-\sigma_{ij}\Delta y_j}}{e^{-\sigma_{ij}\Delta y_j/2}}\right)\right]^{-1}\times\left[\frac{2|\mu_n|}{\Delta x_i}\psi_{n,i-1/2,j}+\frac{2|\eta_n|}{\Delta y_j}\psi_{n,i,j-1/2}+q_{nij}\right]\qquad (s_{nij}=0)$$

$$\psi_{nij}=\left[\sigma_{ij}+\frac{2|\mu_n|}{\Delta x_i}\cos\left(\frac{\pi\Delta x_i/2}{\Delta x_i+2/3\sigma_{ij}}\right)+\frac{2|\eta_n|}{\Delta y_j}\cos\left(\frac{\pi\Delta y_j/2}{\Delta y_j+2/3\sigma_{ij}}\right)\right]^{-1}\times\left[\frac{2|\mu_n|}{\Delta x_i}\psi_{n,i-1/2,j}+\frac{2|\eta_n|}{\Delta y_j}\psi_{n,i,j-1/2}+q_{nij}\right]\qquad (s_{nij}\neq 0)$$

A well-known benchmark calculation given in a 2-D Cartesian coordinate is performed to evaluate the accuracy and efficiency of these relations [5]. In this problem, a flat isotropic source is located at the left-bottom corner of a square region (see **Figure 1**). The boundary conditions are also given as reflective on the bottom and left sides, and a vacuum on the top and right sides of the region.

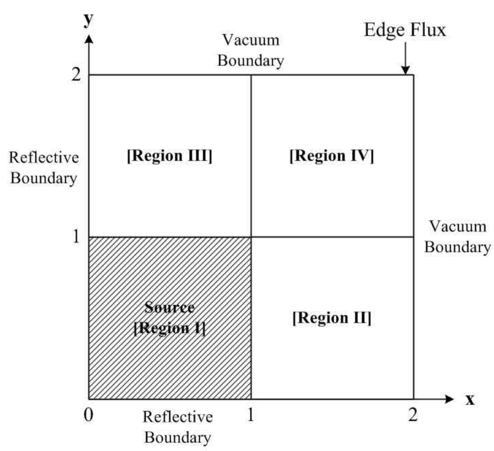


Figure 1. Benchmark Problem in X-Y Geometry

3. Results and Discussions

In **Figure 2**, the flux distribution is shown along the right edge of the benchmark problem. In this calculation, the X-Y geometry is divided into 10×10 meshes, and is based on the level symmetry quadrature sets. As shown in the figure, a numerical analysis applied with EA is exactly matched with the reference data, which are applied with the DD relation and divided into 100×100 meshes in the X-Y geometry. In addition, **Figure 3** shows the average scalar flux in region IV as a function of the number of meshes and iterations. Regardless of applying the relations, the average flux is gradually converged to the reference value with an increasing number of meshes (see **Figure 3 (a)**). In addition, the convergence speed of the numerical analysis applying EA is almost the same with the other case (see **Figure 3 (b)**). From these results, it is confirmed that this approximation is sufficiently applicable for deriving a numerical solution of the transport equation.

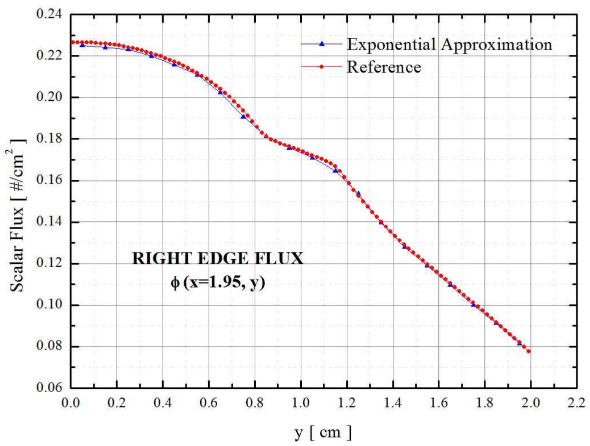
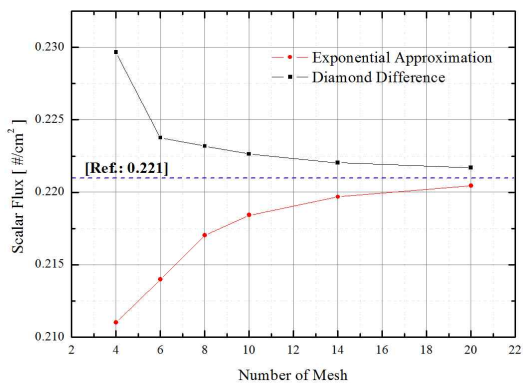
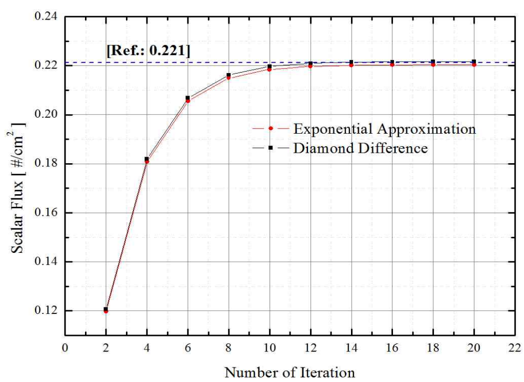


Figure 2. Flux Distribution along the Edge of the Calculation Model (Level Symmetry Quadrature Set, S_{16})



(a) Scalar Flux Vs Number of Meshes



(b) Scalar Flux Vs Number of Iterations

Figure 3. Scalar Flux depending on Calculation Condition

4. Conclusions

In this study, the auxiliary relations with respect to the cell average and cell edge fluxes are developed by considering the exponential attenuation of the entering flux. From the calculation results, it was confirmed that these relations are applicable for deriving a numerical solution of the transport equation. However, if the mesh space is too large, these relations produce negative fluxes on one or more edges of the cell. Therefore, it is necessary to perform additional research to overcome this problem.

REFERENCES

[1] E.E. Lewis and W.F. Miller, Computational Methods of Neutron Transport, American Nuclear Society, INC, Illinois, 1993.

[2] G.E. Sjoden, Exponential Characteristic Spatial Quadrature for Discrete Ordinates Neutral Particle Transport in Slab Geometry, Air Force Institute of Technology, AFIT/GNE/ENP/98M-10, 1992.

[3] P. Barbucci and F. D. Pasquantonio, "Exponential Supplementary Equations for S_N Methods: The One-Dimensional Case," *Nuclear Science and Engineering*, **63**, 179, 1977.

[4] M.M. El-wakil, Nuclear Heat Transport, American Nuclear Society, INC, Illinois, 1978.

[5] K.D. Lathrop, "Ray Effects in Discrete Ordinates Equations," *Nuclear Science and Engineering*, **45**, 255, 1971.